DESIGN OF MODERN STEEL STRUCTURES

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Structure

DESIGN OF
MODERN STEEL STRUCTURES

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stresses. The study of complete structures, which follows, includes the design of riveted and welded girders and roof trusses, a low-truss highway bridge, and a tall building frame. A concluding chapter on the design of continuous beams serves to introduce the subject of design for continuity. Since each subject is presented in a separate chapter, the order of study can be arranged to meet special purposes.

In general, the design problems given here make use of recent specifications. However, in a few cases either lower or higher working stresses than standard are employed in order to keep the student from becoming too restricted in his point of view. Design problems are worked out either in the text or on special design sheets. The design-sheet form is typical of office practice, but the more complex problems requiring detailed explanation are worked out in the text. The specifications used and given are those of the American Institute of Steel Construction (*AISC*), American Railway Engineering Association (*AREA*), American Association of State Highway Officials (*AASHO*), and American Welding Society (*AWS*). These specifications control a large part of all structural design work for buildings and bridges.

The points to be emphasized by the teacher in the design of major structures are the *functional aspects* which receive less than proper attention in all textbooks, including this one. The reason for building a bridge or a building exactly of the type chosen must be for it to serve best its intended function. Accordingly, function becomes the most important criterion of good design, although economy is certainly of almost equal importance. Neither the functionally perfect structure that costs too much nor the cheap one that cannot serve its intended purpose will be built.

The author hopes that this book may serve to help produce young engineers having a thorough background of elementary structural design. Such men may find an immediate position in engineering or they may prefer to develop a more scientific point of view by study for an advanced degree. The information presented here is essential for the structural engineer who chooses either road for his development.

Professors Merit P. White and R. L. Stevens generously offered many constructive suggestions that are incorporated in the book.

L. E. GRINTER

CHICAGO, ILLINOIS
May, 1941

LIST OF ABBREVIATIONS AND SYMBOLS

Abbreviations

AASHO	American Association of State Highway Officials.
AISC	American Institute of Steel Construction.
AREA	American Railway Engineering Association.
ASCE	American Society of Civil Engineers.
ASTM	American Society for Testing Materials.
AWS	American Welding Society.
C.G.; c.g.	center of gravity.
d.	penny designation for nail sizes.
D.L.	dead load.
k.	kips.
L.L.	live load.
N.A.	neutral axis.
WF	wide flange beam section.
W.L.	wind load.

Symbols

a	distance, length, or thickness.
A	area.
A_b	area of bearing.
A_f	area of flange.
A_s	area for shear.
A_w	area of web.
b	breadth or distance.
c	distance such as that to the extreme fiber.
c'	constant.
C	compressive force, arm of a couple.
d	diameter, depth, or distance.
D	deflection, also detrusion ratio.
e	eccentricity.
E	modulus of elasticity.
f	fiber stress.
F,F'	forces, also fatigue limits.
f_b	allowable beam stress.
f_c	allowable column stress.
g	gage distance.
G	torsional modulus of rigidity, also specific gravity.

Symbols — continued

h	height, also diameter of a rivet hole.
I	moment of inertia.
j	ratio.
J	polar moment of inertia.
k	ratio.
K	stiffness, also special factor defined in text.
l,L	lengths.
M	bending moment.
M_e	moment of eccentricity.
M_t	total moment.
n	number of bolts or rivets, also a ratio.
N	number of repetitions of the stress S.
p	allowable stress parallel to the grain in wood.
P	load.
P_n	wind load normal to the roof.
q	allowable stress perpendicular to the grain in wood.
Q	statical moment of an area.
r	radius of gyration, also a ratio.
R	rivet stress, radius, or reaction.
s	rivet spacing, also unit stress.
S	section modulus, also total stress.
s_b	bearing unit stress.
s_c	compressive unit stress.
s_n	normal unit stress.
s_s	shearing unit stress.
s_t	tensile unit stress, tangential stress.
t	thickness.
T	tensile force or total tension, also torque moment.
u	unit bond stress in reinforced concrete, also Poisson's ratio.
v	unit shearing stress in reinforced concrete.
V	total vertical shear.
w	uniform load per unit length or area.
W	total uniform load, also width.
x,y	coordinates or distances.
$\bar{y}$	distance to the center of gravity.
Δ	deflection.
θ	slope or angle.
Σ	summation.

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The highest attainment in design
is a simplicity approaching functional perfection.

DESIGN OF MODERN STEEL STRUCTURES

CHAPTER 1

PRACTICE VERSUS THEORY

1. Conflicting Points of View. We hear much of the conflict between theory and practice especially from the field man. Actually, of course, there will be no conflict between good theory and good practice although the two frequently seem at cross-purposes, particularly when both are bad. Bad theory develops from unjustifiably crude assumptions while bad practice follows unjustifiably crude methods. When theory can be based upon accurate information and practice can be controlled by one who understands the theory involved, the two will agree. Nevertheless, there are certain considerations of practice that must be allowed to control design because of cost and to facilitate construction. A few of the many problems that should influence the thinking of the designer and of the construction engineer will be discussed.

Art versus Science. Until modern times all engineering was art. The construction engineer not only erected the structure, but, through an exceptional development of faculties he intuitively analyzed, designed, and directed the fabrication of the individual parts on the job. Of course, his ability had been nurtured by long apprenticeship, although an occasional genius, such as Andrea Palladio or Leonardo da Vinci rose far above the level of his associates. *Construction* was therefore first. Crude *design* followed construction and was dependent upon a knowledge of geometry and an understanding of proportions. In other words, there came into existence designers or, more technically, detailers, who saw to it that blocks of stone and pieces of wood were fabricated of a proper size to fit into the structure at a given point. *Analysis,* naturally, developed last since structural analysis is an application of science. Structural analysis in the modern sense received little attention until after the year 1800. Hence, we may look upon it as the youngest of the triumvirate; construction, design, and analysis. As such, it was long discounted and often ridiculed.

A survey even as brief as this one points out the direction of evolution from art to science. Construction started as an art; design followed by

applying the most elementary scientific principles to the construction art; analysis finally entered the picture as almost pure science and has affected both construction and design. The obvious tendency is for science, through analysis, to influence more and more greatly the design and construction of structures. It would be rash to state that this evolution would ever reach the ultimate conclusion where science controlled such work completely — certainly development today is far short of this. Accordingly, the engineer must understand not only the science of analysis but the arts of design and construction if he intends to accomplish anything significant in this very practical field.

2. Analytical Calculations. Since analysis precedes design, it will be useful to think over the process of analysis from the point of view of the practical designer. Analysis, to serve a useful purpose, must finally reach expression in terms of *tons* of steel, *cubic yards* of concrete, and *board feet* of structural timber. It is useless for the analyst or the designer to expect the construction engineer to worry about increasing the unit stress in a steel beam to 300 lb. per sq. in. above the allowable stress by the shifting of a partition. The field man knows that there are decisions which he will have to make during erection that may influence the stress to a greater extent than the amount mentioned. For the same reason he is not likely to be sympathetic when the blueprint carries a statement that a field connection is to be welded at a distance of $5\frac{1}{16}$ in. from a sheared edge. The accuracy of field work is seldom greater than a tolerance of $\frac{1}{8}$ in. and a sheared edge is far from a planed edge at best. The designer will cultivate the respect of the field man by avoiding such inconsistencies.

With these considerations in mind, we may conclude that there is little reason for a designer to use log tables in making his usual calculations. A slide rule will provide all requisite accuracy and such calculations will actually command greater confidence. However, this does not justify the substitution of crude guesses for sensible analysis or for careful design calculations.

Tools of Analysis. There is often a choice of analytical tools to be made by the designer. For example, the design of a riveted plate girder may be made on the basis of several possible assumptions. (1) The " effective " depth may be guessed at. (2) The " effective " moment of inertia may be used and the neutral axis may be taken at the mid-depth. (3) The net moment of inertia may be used and the neutral axis may be taken at the center of gravity of the net section. The first method is a satisfactory one for use by a designer who has developed his sense of structural action through study and experience with methods two and three. The third method was long considered to be the most accurate, perhaps because it involves longer calculations, but the second method has been justified by

tests. The point of importance here is merely that there are often unknown elements influencing the action of a structure so that an elaborate analysis may result in no better *design* than a simple one. The designer should choose a method of analysis to agree with the best information available as to the action of the structure and not simply to satisfy what he considers to be the most erudite procedure.

3. Impractical Theory. Nothing discredits the usefulness of theory as a practical design tool so much as the use of theoretical toys. It is often true that theory tends to become an end in itself instead of a tool for practical use. The literature is full of formulas, graphs, and mathematical studies that are of interest mainly because of their intricacy. This criticism is in no way intended to discredit sound analytical studies, however complex. *Mathematics should neither be avoided nor displayed.*

Theory of Elasticity. There is no tool that has proved of greater value to the designer than the mathematical theory of elasticity. On the other hand, it is worth remembering that the significance of the word elasticity automatically rules out the effect of plastic flow or "yield." Hence, the picture of stresses presented by this theory is the picture that would apply before *any single particle* passed the yield point. As soon as any part of the structure begins to yield, the distribution of stresses will change. The accomplished designer will be able to interpret and use the results of mathematical studies based upon the theory of elasticity, but he will not fail to readjust his ideas of structural action to care for the influence of yielding beyond the elastic limit.

*Photoelasticity.** There is usually a reasonable check between the stresses obtained by the mathematical theory of elasticity and by photoelastic investigations. At least this correspondence should follow if the photoelastic material used is a brittle material (such as bakelite) which has a straight line stress-strain diagram up to the failure point. Again, this tool has proved extremely valuable although, when interpreted carelessly, it has confused about as many problems as it has clarified. For example, there is the case of the structural eye bar. For several generations of structural engineers the eye bar has served a useful purpose. Based upon actual tests to failure the standard design has become the usual rounded head with not more than 40 per cent excess area through the head. Photoelastically it has been shown that bakelite eye bars are stressed more heavily through the head than through the shank even when the area through the head is made fully 100 per cent greater than the area of the shank. Of course, such bakelite models break through the head while standard

* By the photoelastic method, stresses are determined in celluloid or bakelite models representing structural parts. The stress bears a relation to the number of dark lines or fringes that appear when polarized light is passed through the loaded model.

eye bars break through the shank. The reason is found in the *yielding of steel* that allows a *redistribution* and an ironing out of the high stresses around the hole. It seems evident to the author that the photoelastic study is a misrepresentation in this instance. Only a ductile transparent material with the dual characteristics of elasticity and yield offered by steel could be used photoelastically to determine the result to be expected of the steel eye bar. No such material exists at the present time.

4. Ductility. This property has been mentioned as one which helps to reduce stress concentrations. For instance, a small hole in a simple tension member is supposed to produce a stress concentration of three times the average unit stress in the member. Photoelastically it has been possible to measure stress concentrations around the hole of more than twice the average stress in the member. It is therefore surprising that rivet holes do not seem to reduce the ultimate *static* strength of a tension member (steel) by more than the influence of the reduction of effective area. The explanation must be, of course, that the steel around the rivet hole flows and thus permits a redistribution of stress so that the maximum unit stress at failure is little greater than the *average unit stress.* There are innumerable similar conditions to be evaluated in structural design. All " stress raisers " such as notches, holes, threads and cross-sectional changes are best eliminated, but, if they are unavoidable, some reduction of their objectionable features will be obtained from ductility. Such stress raisers are of the greatest significance when members are subjected to repeated stress, as will be shown.

Statistical Stresses. Perhaps one factor that tends to overcome the more serious consequences of stress raisers is the fact that steel is not a truly homogeneous material. It is therefore reasonably certain that the stress variation in a tension bar centrally loaded is far from uniform. The probability seems to be that adjacent finite particles will act under considerably different stresses and that adjacent microscopic particles may be extremely nonuniform in their resistances to stress. Therefore, we may prefer to think of numerous minute discontinuities that act as stress raisers (holes) and thus the addition of *another discontinuity, such as a rivet hole,* would have little influence. The term " statistical stress " has been used to represent the probable average stress over a finite area in contrast to the microscopical stress that is indeterminate and highly variable.

Repeated Stresses, Vibration, and Impact. There are two conceptions of impact that need study. One is that an impact stress may be applied too rapidly for the material to deform plastically before failure. The other assumes that impact is merely an increase of the static stresses and strains. Probably the latter definition is the correct one for most cases of impact, but it is possible to visualize stress applications (locally) of such speed that

deformation cannot follow freely. Evidently, under such rapid applications of stress there would be increased danger from stress raisers and failure might follow without the compensation of ductility. Similarly, when stresses are repeated without limit, failures occur at unit stresses below the ordinary limit of elasticity. Stress raisers are again significant and in such cases the maximum stresses found by the mathematical theory of elasticity or by the photoelastic apparatus are much more significant than for static loads. Tests on the fatigue of riveted joints have established this fact.

5. The Factor of Safety. This is a favorite subject for discussion and argument. Some writers have considered the factor of safety to be based upon *ultimate strength* while others feel that the ratio of the *elastic limit* to the working stress is in reality the factor of safety. The latter point of view is certainly the more significant, but neither presents a correct picture. The engineer is always willing to let the actual stress approach *nearly* to the elastic limit. The range between the working stress and the elastic limit is mainly an allowance to cover unknown or partially unknown stresses.

Knowledge of Loads. One of the undeterminable factors in design may be the loading itself. Dead load can be estimated quite accurately, but live loading, wind, and impact, as well as traction, sway, and other inertia forces are extremely variable. Then there is the influence of temperature and the action of settling supports that often damage an otherwise well designed structure. The engineering designer makes a sincere effort to evaluate the *probable* loads, but even his best judgment is unable to cope with the situation in all cases. One purpose then of the factor of safety is to provide some reasonable allowance for *possible* increase of loading when the structure may need to serve a purpose somewhat different from that intended.

Fabrication and Erection Stresses. It is no secret that structural steel is handled rather roughly in the shop and in the field. Rivet holes seldom line up perfectly and they must be *pulled* into line. Welding *warps* and *buckles* the structure and leaves high residual stresses. During fabrication, bent shapes are *straightened* as a standard part of the fabrication process and, of course, the elastic limit must be passed to accomplish this. The mere *punching* of a hole distorts the surrounding material and leaves high residual stresses. The writer is convinced that these processes will result in a structure having stresses, under the design loading, that reach the elastic limit over small areas. Such a structure would be highly unsafe if it were not constructed of a ductile material such as structural steel. When a minute flaw in the material coincides with the location of such a point of high residual stress, a failure is likely to result, particularly if the loading is of the repeating type. Failures have often been traced to such influences.

All things considered, it is remarkable that serious failures are so infrequent. It speaks well for the care exercised by the designer.

6. Fabrication Methods. It is the responsibility of the designer to understand fabrication methods and to fit his particular design to the fabrication facilities available. For instance, it is foolish to select a beam that is longer than rolled sections stocked in local warehouses or longer than the fabrication shop can handle properly. Yet this is a mistake expected of young designers. It is worth noting that each central warehouse provides the draftsmen in its vicinity with a list of maximum sizes of materials that are readily available. *Special sizes* may not be obtainable for months even at an *increased cost* per pound. The same criticism may be made of the use of angles of the less common sizes and of special beams. There are cases where it may actually cost less to use a 6-in. angle than a 5-in. angle of the same thickness.

The use of a multiple punch may be a cost-saving feature where duplication can be provided. However, more often than not the shop foreman decides that the cost of setting up the multiple punch is greater than the savings involved in a few duplications. The designer should work with the shop man so that the resulting structure will be economical. An edge can often be finished either by grinding, by milling, or, possibly, simply by burning. A thorough knowledge of relative costs is necessary if we are to reach a proper decision.

Field Erection. The designer usually has more difficulty in cooperating with the field organization than with the shop men. The reason is that field conditions are never under complete control. The weather, the soil, the kind of labor obtainable and the vagaries of nature all combine at times to plague the field engineer so that he finds it difficult if not impossible to follow the exact plan presented to him. On the other hand, construction engineers are so versatile that they can usually accomplish the result desired even though some changes become necessary. The responsibility again falls upon the designer to consider the influence of all possible field conditions upon his design. For example, a design that could not be completed in rainy weather would not be a practical one for most locations. Neither would a design that could not be carried out under extremes of temperature normal to the locality. Some designs must be made so that the structure can be erected by unskilled labor while other structures may be dependent upon the services of welders and craftsmen of highly specialized qualifications. The writer knows of one bridge that was designed for transportation on the backs of *camels* and another that was brought to the site by *airplanes*. Even freight car or truck transportation introduces certain limitations that must be observed as to the overall size or length of a given piece. When a member cannot be transported to the site or erected

with the tools and labor available, either the designer has made a serious error or the organization needs new channels of information between the field and the office.

7. Cost as a Major Factor. All of this discussion leads to the inevitable conclusion that only an economical design can be a good design. The designer will accomplish little if his structures are seldom built because of excessive cost. Therefore, the designer must balance himself nicely between the criticism or danger of unsafe practice on the one hand and, on the other hand, the inevitable lack of success if he is too conservative. His best approach to the solution of this problem is to learn everything possible from the detailer, the shop man, and the construction engineer. If he knows the *tolerances*, *clearances* and *allowances* introduced by the detailer, the *sizes*, *tools* and *methods* used by the shop, and the *shapes*, *weights* and *fits* desired by the field organization, his designing is likely to be successful. If he is not familiar with these factors, his ability in stress analysis and his careful choice of sections is still likely to result in friction throughout the entire construction job. The result is increased expense at every turn and ultimately a noncompetitive job.

In the study of costs, it is interesting to observe that certain structures commonly used in foreign countries are seldom used in the United States. We have the highest labor costs in the world, which explains our desire for machine production. Slender structures are most likely to be found in Europe where the high cost of material and low cost of labor make weight reduction important -- a fact that is particularly evident in the field of reinforced concrete.

8. Specifications. All structural design is controlled by specifications. Even if no limitation is placed upon the designer, he will still be very likely to depend upon a standard set of specifications for guidance. It is understood that the large cities all have building codes that specify not only working stresses and qualities of materials and workmanship but such general features as window area, hallway widths, and fire provisions for a building and similar features of other structures. The designer will follow the specifications of the local building code by necessity, but he will also usually follow the provisions of standard sets of specifications (*AREA*, *AISC*, *AWS*, *ACI*) for his own guidance. It is impossible for any one designer to have experienced all of the possible situations that may need to be controlled for absolutely safe structural design. Standard sets of specifications are prepared under the sponsorship of the technical societies. Over a period of years such specifications have been written and rewritten many times. The profession as a whole has used each specification and has either accepted or rejected it. Each time that a set of specifications is rewritten, many new ideas are introduced and old ones are removed. In this way a

standard set of specifications may be accepted to represent the best information available on the subject as of the date when it was written. Perhaps we had better say that it reflects some ideas that are a few years out of date, since the inevitable lag between the presentation of a new procedure and its general acceptance is as evident in structural design as elsewhere. This natural conservatism of the engineer is his safeguard against dangerous construction, but it naturally mitigates quite as effectively against the adoption of progressive ideas that are entirely sound.

Interpreting Building Codes. In order to interpret specifications properly, the designer must understand the purpose behind their development. It is the obligation of the specification committee to produce a set of specifications that will guard *the public safety* under unexpected as well as normal conditions. Hence, today we find the working stress for structural steel in tension to be set at about one half of the elastic limit. A higher working stress would be allowable in special cases, but the specification committee must make a reasonable allowance for the unknown characteristics of certain loadings, for the possibility of excessive impact, for residual stresses that fabrication may produce, for erection stresses that are unavoidable, and for accidents such as collisions or settlement that are ordinarily not considered in design.

Variation of Working Stresses. The writer knows of standard design work where the usual specified working stresses have been exceeded by fully 30 per cent. However, the case referred to was the design of supports for tanks that carried a depth of liquid as the only loading. Certain facts were evident. First, the design loading could not be exceeded because the tanks could carry no more liquid than their combined capacities. Second, the factors of impact and vibration were nonexistent and their unquestioned elimination added to the certainty of the loading. Third, such low tanks present no serious wind resistance problem so that another disturbing factor did not have to be considered. Fourth, the fabrication and erection of tank supports is such a simple job that there could be little in the nature of unusual fabrication and erection stresses to worry about. Accordingly, for this very simple case the designers felt justified in increasing standard working stresses in rivets and for structural sections by 30 per cent. Since the plant in question was not located in a city, the designers were free from the restrictions of a local building code. It is worth mentioning that the same designers reduced the working stresses for the steel supports of heavy machinery even below the allowance of standard specifications. The problems of impact, vibration, and repeated stresses seemed sufficiently serious there to justify a very conservative position. The writer is of the opinion that both of these instances were justified and that the attitude expressed is an entirely commendable one.

The designer must always be safe, but he cannot be careless with his employers' money.

9. Structural Failures. There are a great many minor structural failures, but, unless there is loss of life or other newsworthy features about the failure, it never comes to the attention of anyone except the firm that repairs the damage. Frequently, the owner requests that there be no publicity given to the failure. Many failures are caused by improper details. It has been a habit of " handbook designers " to select members of ample size and then to connect them together inadequately. Most building failures in tornadoes can be traced to this weakness. Undoubtedly, this is due to the fact that member selection is often quite simple while joint design requires a greater understanding of the problem.

Settlement. Certainly the most common source of all failures is foundation settlement. The science of soil mechanics is now centered in this study and it is hoped that rapid progress will continue to be made. The problem involved is not to prevent settlement, which can never be done, but to obtain uniform settlement so that the structure will not be stressed thereby. For instance, if all footings of a building settle the same amount, the building will be uninjured. Similarly, the settlement of one pier of a simple span bridge will be of little consequence unless it tips at the same time. Tipping piers, however, are a common sight and unequal settlement of building footings is evidenced by cracks in a large part of all public and private structures. If the structural designer does not have control of the foundation design or if he has inadequate data on which to base such a design, he must make an allowance for unequal settlement in his analysis. Hence, the ideal structure for such a location may be one that is flexible or deformable rather than rigid or even stiff. In bridge design the simple span structure has long been pointed to as the ideal where heavy settlement is anticipated. But on the other hand, the writer has seen large settlements of buildings (as much as two or three inches between columns produced by a near-by excavation) with little evident damage except to plaster and external covering. The structural steel frame seemed to absorb the deformation without obvious damage. Over greater lengths, buildings have shown unequal settlements of more than a foot with no greater distress than some destruction of outside trim and inside decoration.

Conclusion. These and other observations reemphasize the conflict that some engineers like to point to between theory and practice. Actually, no conflict exists. Steel is a ductile material that will absorb a great deal of punishment. Good theory may of necessity assume elastic conditions and certainly good practice ordinarily presupposes operating conditions that will not load a material beyond the elastic limit. But when the unexpected (or sometimes the anticipated) happens and excessive deformations occur,

both the designer and the construction engineer may place a justified faith in the fact that structural steel is ductile as well as elastic. Perhaps we may ultimately reach the point where our design calculations actually are based upon the stress-strain curve of the material we use. As yet we have no simple method of design that accounts for other than elastic conditions.*

* The paper of J. A. Van den Broek, *Theory of limit design*, Transactions, *ASCE*, 1940, pp. 638–661, treats indeterminate structures and considers the influence of inelastic deformations.

CHAPTER 2

RIVETED CONNECTIONS

10. Types of Rivets. The usual rivet for structural work has a button head of rounded shape with a diameter of $1.5D + \frac{1}{8}$ in. (Fig. 1) where D is the nominal diameter of the rivet. The height of the head is 0.425 times its diameter, which makes it somewhat less than a hemisphere. These button heads can be flattened to $\frac{1}{4}$ or $\frac{3}{8}$ in., countersunk for a projection of $\frac{1}{8}$ in., or countersunk and chipped flush as dictated by clearance requirements. Rivets countersunk and chipped flush do not have sufficient head to develop full strength. They should be discounted 50 per cent in design.

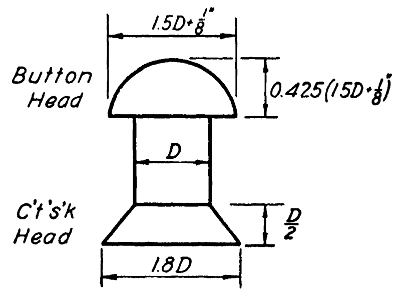


Fig. 1. Rivet Heads.

Conventional Signs for Riveting. The conventional symbols shown in Fig. 2 illustrate the standard method of indicating types of rivets on drawings.

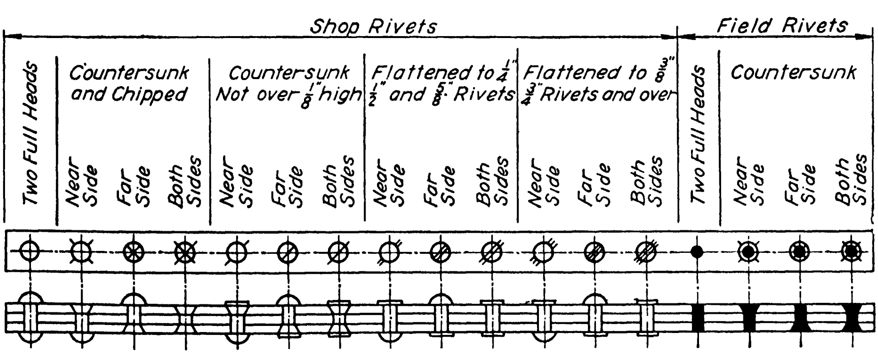


Fig. 2. Conventional Symbols for Riveting.

The appearance of a drawing will be much enhanced by care in the use of these symbols. It is sometimes preferable to make the size of the open circle denoting the rivet head slightly larger than it would be to scale, but the black solid dot indicating an open hole for a field rivet should not be enlarged. Such black spots appear prominently on the drawing and should be made no larger than 50 per cent of the diameter of the open circles.

Size and Weight of Rivets. The usual sizes of structural rivets are ¾ in. for mill buildings and light structures and ⅞ in. for ordinary bridges and office buildings. Tower structures and monumental bridges may require 1-in. or 1⅛-in. rivets, while light frames, such as short-span roof trusses, electric sign supports and power-line towers, may be made with ½-in. or ⅝-in. rivets. It is desirable to use one or not more than two sizes of rivets in a single member and as few sizes as possible in the entire structure. Nevertheless, even when the rivet size is standardized at ⅞ in., it will be necessary to use smaller rivets through the flanges of channels and the legs of small angles where *proper edge distance* limits the size of a punched hole. Such limitations are given in all structural steel handbooks.

Rivets are to be of such length that the part of the shank projecting beyond the face of the structure, when the rivet is in the hole, will just provide enough metal to form the head. Again, such data are given in the structural steel handbooks. The heads add to the weight of the structure and must be estimated as a part of the dead load. The heads for one hundred ¾-in. rivets weigh 16 lb. The corresponding weight is 24 lb. for ⅞-in. rivets. The weight of the rivet heads is an appreciable factor for a structure containing several thousand rivets.

11. Fabrication. *Punching Holes.* Rivet holes may be punched, sub-punched and reamed, or drilled. Punched holes are standard, but railway-bridge work has often been sub-punched and reamed. The expense of drilling has limited its use to thick material (thicker than the diameter of the hole) where punching is unsatisfactory because of excessive deformation of the surrounding metal.

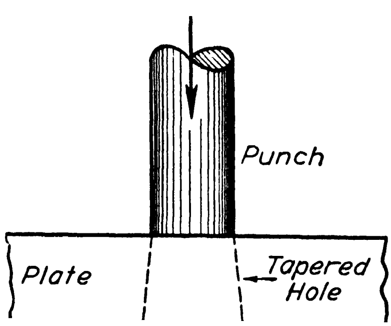


FIG. 3. TAPER OF RIVET HOLE.

A punched hole has a noticeable taper, as is indicated by Fig. 3, which increases with the thickness of the plate. This taper aggravates the problem of alignment. Holes cannot be expected to match perfectly in punched work because the action of the punch distorts the metal and *lengthens* the part being punched. The operator allows for this stretch by rule of thumb, but, nevertheless, holes aligned as well as those shown in Fig. 4 represent good workmanship. Hence, many of the holes must have a reamer passed through them before the rivet can be dropped in. The purpose of the reamer is not to produce a perfectly cylindrical hole and therefore this is not equivalent to "reamed work" for which the holes are punched $\frac{1}{16}$ in. or ⅛ in. undersize and reamed to size.

Driving Rivets. The rivet blank with one head already formed is heated until it glows and inserted in the hole. Then, either by direct pressure or by a series of blows, a second head is formed before the rivet becomes entirely black. The most satisfactory rivets are produced with direct pressure (air, hydraulic, or steam) by use of a power or bull riveter. The riveter grips the rivet between its jaws and produces the head by direct pressure of perhaps 50 tons or even more. The head is formed by *squeezing* the rivet. Naturally, the plastic rivet steel is squeezed out to fill the hole adequately. In close work, or where the riveter cannot reach around the member, and for field connections, the rivet head usually is formed by the air hammer. No distinction is made in regard to the strength of shop rivets made by direct pressure and by the pneumatic hammer, but the former are to be preferred. A hammered rivet may be "over driven," which means driven too cold, with the result that its head is easily knocked off. Hand hammered rivets were common in the early days of riveting, but they are unusual today. Bolts are considered at least equally effective and less expensive.

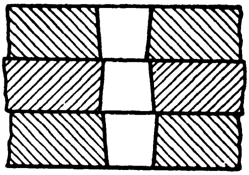


FIG. 4. MISALIGNMENT OF PUNCHED HOLES.

INTERNAL ACTION OF RIVETED JOINTS

12. Elastic Action of Rivets. *Shrinkage.* A proper rivet is completely driven before the metal turns black. Hence, the shrinkage from about 1000° F. to air temperature reduces both the length and the diameter of the shank. Evidently, lateral shrinkage tends to produce a loose rivet, but this effect is not very serious. Sections have been made through power riveted joints, cutting the rivets, which show almost no distinguishable line of demarcation between plate steel and rivet steel. It is probable that the pressure developed produced a rivet that was actually too large for the hole (subjected therefore to lateral compression), which still filled the hole after shrinkage. In fact, it is known that at 50 tons pressure a short rivet can be produced which is $\frac{1}{32}$ in. larger than the original hole. Even those rivets produced by air hammers appear to fill the holes adequately although an occasional poor rivet is probably unavoidable.

Initial Tension. The self-evident fact that a cooling rivet develops initial tension has been long recognized, but the extent of this initial tension was seriously questioned. An occasional rivet would fail under a light blow by popping off the head and this was taken to mean that the amount of initial tension was at least problematical. Finally, tests were made which showed that plate slip did not occur with proper fabrication until the stress in shear on the cross-sectional areas of the rivets reached about 11,500 lb.

per sq. in. *Plate friction* was the only force that could prevent slight slippage, and even at an initial rivet tension of 35,000 lb. per sq. in. the coefficient of friction would have to be approximately 0.33 to produce a frictional resistance equivalent to 11,500 lb. per sq. in. per rivet. Later, tests* were made which showed conclusively that the initial tension could be expected to approach the elastic limit and might safely be taken at *90 per cent of the elastic limit as an average value.* Since the coefficient of friction for plates with rough mill scale in contact might be expected to be higher than the usual value of 0.33 given for steel on steel, it seems reasonable to state that most riveted joints properly designed and not overloaded act elastically without any slip between the parts joined by the rivets. Under such conditions the rivets themselves would be unstressed except for their initial tensions.

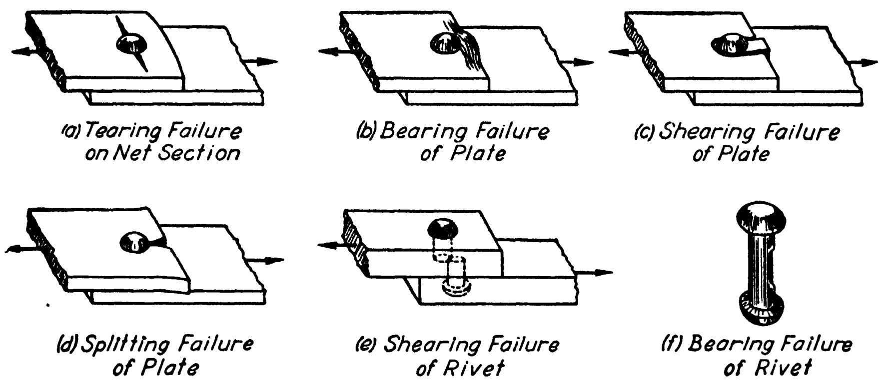


FIG. 5. FAILURES OF RIVETED JOINTS.

13. Failures of Riveted Joints. *Rivet Shear.* A riveted joint will deform greatly before final failure. (See Fig. 5.) There is a slip between the plates or structural shapes joined by the rivets and a shear is developed on the cross-sections of the rivets. As shown in Fig. 5(*e*) the load may shear the rivet off along the plane of slip, but this is only one possible method of failure.

Net Section. A tearing failure as illustrated in (*a*) of Fig. 5 may occur whenever the rivets are stronger than the plate. This possibility is enhanced by the fact that a hole not only reduces the effective section of the plate but also by the fact that a hole theoretically triples the maximum stress in a simple tension plate. Practically, the increase in stress seems to

* W. M. Wilson and W. A. Oliver, *Tension Tests of Rivets*, Bulletin 210, University of Illinois Engineering Experiment Station.

C. R. Young and W. B. Dunbar, *Permissible Stresses on Rivets in Tension*, Engineering Research Bulletin No. 8, University of Toronto.

be more nearly in the neighborhood of 100 per cent. In addition, it is known that plastic deformation beyond the yield point greatly reduces all stress concentrations, which are therefore usually neglected. Tension failure on the net section of the plate is considered in standard design simply by deduction of rivet holes from the gross section.* Whenever fatigue failure is a possibility, this method will not be safe because repeated stress concentrations around rivet holes may produce fatigue cracks before plastic flow occurs. This matter will be discussed later.

Edge Distance. A plate failure may also occur at right angles to the main direction of stress as illustrated in Fig. 5(d). Splitting failure may be caused by the internal pressure of an " over driven " rivet where adequate edge distance has not been provided. The minimum proper edge distance is 1.5 times the diameter of the rivet. Similarly, the double-shear failure of the plate, as shown in (c), will not occur when the proper edge distance is provided. The photograph, Fig. 6, illustrates such failures. The edge distance is measured from the center of the rivet. It should be increased ⅛ in. when measured to a sheared edge.

Bearing between Rivet and Plate. Bearing failure is a crushing of the plate and rivet around the half circumference. The result for an edge rivet may appear externally as in (b) of Fig. 5. If a rivet that has failed in bearing is removed and examined, it will show a flattened face as in (f) where the heaviest stressed (ordinarily the thinnest) plate pressed against it. The rivet shown in (f) joined three plates instead of two as for the other joints illustrated.

Rivet Resistance

14. Lap and Butt Joints. Riveted connections are of all degrees of complexity, but basically they may be pictured and studied as being of two types, lap joints and butt joints. In order to understand the usual design procedure for riveted joints, it is necessary to visualize the rivet areas subjected to failure in these two types of joints.

Shear on Rivets. If failure occurs by shear *on the rivet*, the area subjected to shear failure is the cross-section or *circular area* of the rivet. Failure occurs along one plane in the lap joint of Fig. 7(a) and the rivets are in *single shear*. The resistance of this joint in single shear would be the sum of the cross-sectional areas of the two rivets multiplied by the allowable shearing unit stress on the rivet, s_s. For ¾-in. rivets at 12,000 lb. per sq. in., the value of the joint is

$$2 \times 0.44 \times 12{,}000 = 10{,}560 \text{ lb.}$$

* Note that the " hole " here is a *rectangular* area equal to the diameter of the hole times the plate thickness.

FIG. 6. FAILURES OF RIVETED TEST SPECIMENS.

Bearing on Rivets. Bearing failure occurs between the rivets and the thinner plate in the lap joint of Fig. 7(*d*). The butt joint of Fig. 7(*e*) fails by bearing on the center plate provided that it is thinner than the total

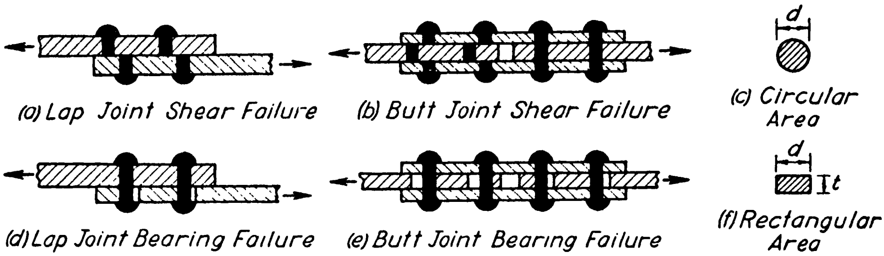


FIG. 7. FAILURES OF LAP AND BUTT JOINTS.

thickness of the two outside plates which resist it. The bearing area used in design in either case is a *rectangular area* equal to the diameter of the rivet times the thickness of the plate which fails by bearing or crushing. Actually, bearing stresses act in part radially, as indicated in Fig. 8, but such radial stresses are assumed to be equivalent to a uniform pressure on a diametral plane through the rivet. Since the bearing pressure is probably not hydrostatic, we can not prove this assumption to be exactly true, but it is always used in design computations.

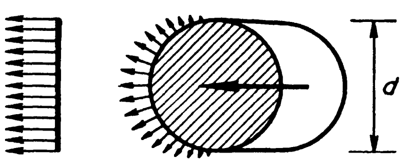


FIG. 8. BEARING STRESSES.

The *bearing value* for either the lap joint of Fig. 7(*d*) or for the butt joint of (*e*) may be computed from the following data: $d = \frac{3}{4}$ in., $t = \frac{1}{2}$ in. for the controlling plate; allowable bearing stress, $s_b = 24{,}000$ lb. per sq. in.

$$2 \times \tfrac{3}{4} \times \tfrac{1}{2} \times 24{,}000 = 18{,}000 \text{ lb.}$$

Actually, the lap joint of Fig. 7(*d*) is far more likely to fail in bearing than the butt joint (*e*). The reason is that there is a tendency for the eccentric forces to *bend the plates* in a lap joint as illustrated by Fig. 9(*a*) resulting in the *unequal bearing pressures* pictured in (*b*). Since this distribution of bearing pressure is far from uniform, the probability of bearing failure is increased. Recent specifications have taken this fact into consideration by allowing a considerably higher *unit* bearing stress for butt joints (double or symmetrical bearing) than for lap joints (single or unsymmetrical bearing).

Rivet Size for Equal Shear and Bearing Values. For the usual thicknesses of plates and sizes of rivets, we expect the design of a lap joint to be controlled by shear and the design of a butt joint to be controlled by bear-

ing. This is because the butt joint is *twice* as strong in shear as the lap joint. It is possible from the ratio of allowable unit bearing stress to allowable unit shearing stress to determine the economical rivet size for the

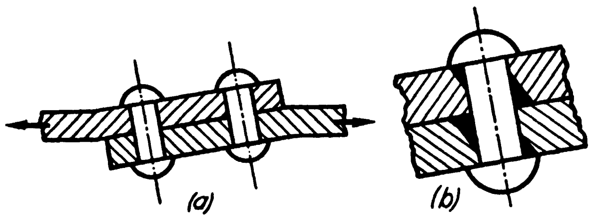


FIG. 9. NONUNIFORM BEARING.

balanced condition of equal strengths in shear and bearing. Thus, by equating the rivet resistance in shear to the rivet resistance in bearing, we obtain for the lap joint

$$s_s \frac{\pi d^2}{4} = s_b t d,$$

or

(1) $$d = \frac{4t}{\pi} \frac{s_b}{s_s} = 2.7t;$$

and for the butt joint

(2) $$d = \frac{2t}{\pi} \frac{s_b}{s_s} = 1.7t.$$

The final values are based upon *AISC* specifications § 215: $s_s = 15{,}000, s_b = 32{,}000$ for unsymmetrical bearing and 40,000 for symmetrical bearing. The usual size of rivet is from 1.5 to 3.0 times the thickness of the thinnest plate, which is within reasonable limits as indicated by equations (1) and (2).

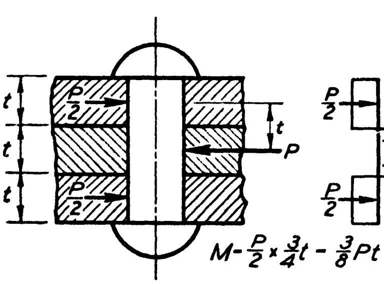


FIG. 10. RIVET FLEXURE.

15. Bending of Rivets. Since the forces acting on a rivet can never be in line (the plates must be side by side in order for the rivet to pass through them), it is inevitable as soon as friction is overcome that the rivet must resist flexure as well as shear and bearing. In Fig. 10 the flexural moment will be $\frac{3}{8}$ *Pt* if the plates are of the same thickness. Bending in such standard cases is *not considered in design.* The allowable shearing and bearing values for rivets have been based upon tests of riveted joints in which the rivets tested must have had to resist flexure. Therefore, any weakening of

the rivet from such bending moment is allowed for automatically by the standard working stresses in shear and bearing.

Filler Plates. A really serious condition of flexure may develop if filler plates are used improperly as at the left of Fig. 11. Some specifications require that the number of rivets shall be increased as much as 25 to 33 per cent for the effect of each filler as a rule of thumb for allowing for rivet flexure. The extra rivets should be placed through an extension of the filler plate as shown at the right of Fig. 11. This arrangement will effectively eliminate excessive bending.

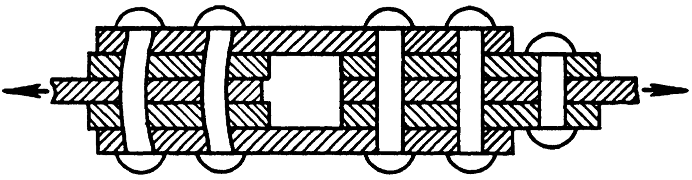


FIG. 11. RIVET ACTION WITH FILLER PLATES.

Long Rivets. Bending is more serious for long than for short rivets. A structural rivet (⅞ in.) is considered normal up to a length of 3½ or 4 in. Beyond that length its strength is reduced. For railway bridges it has been usual to increase the number of long rivets by 1 per cent for each 1/16 in. of grip above 4.5 diameters. Thus for ⅞-in. rivets, a 4-in. length forms the standard and the number of 5-in. rivets would be increased 16 per cent for railway bridges. Similar specifications govern other types of structures.

Tapered Rivets. It has been found by experiment that tapered rivets are desirable for lengths beyond 6 diameters. The driving force heads up the small end of the rivet and causes it to swell laterally to fill the hole. Untapered long rivets are found to fill the hole near the driven head but not at some distance along the shank. A tapered rivet can be designed for driving under a given pressure and at a definite temperature with assurance that it will fill the entire hole. Such long rivets are only needed in massive structures.

RIVETED DETAILS

16. Riveted Column Details. Since compression members and details acting in compression need not have the rivet holes deducted, their design is usually simpler than that of tension member details. Columns must be furnished with a base at the bottom and also with a cap at the top if the load or a part of the load rests directly on the top of the column. A column splice is a common detail wherever the column is more than two stories high. Brackets and beam seats are attached to the sides of the columns but these

details usually involve flexure of rivet groups and hence must be studied separately.

COLUMN BASE, *DP*1*a*. The base detail designed here is the simplest possible type for direct load without flexure. This detail would resist a small flexural moment about the major axis of the column section but almost no moment at 90 degrees to this direction because of the positions of the *anchor bolts*. It is common to design the rivets in such a base detail for from 25 per cent to 75 per cent of the column load. The fact is that the entire load is probably transferred through the *milled end* of the column. However, unless a definite load is specified for the design of the rivets, they may be entirely inadequate to hold the column in place against an accidental blow or against a jacking force applied during an alteration of the structure. A base detail designed for 25 per cent of the load may be adequate for a heavy column while a 75 per cent detail may be needed for a light column. If the column end and the base plate are not milled, the base detail must be designed to transfer the entire column load.

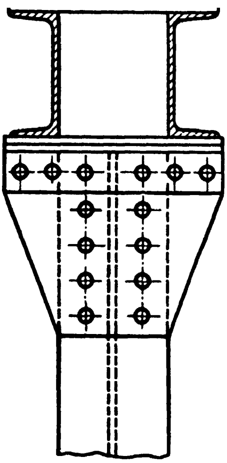


FIG. 12. HEAVY CAP.

COLUMN CAP, *DP*1*b*. The cap shown is adequate for the transfer of relatively light loads. Since the number of rivets cannot be increased without changing the type of detail, its capacity is limited to 8 times the value of a rivet in single shear. For narrow columns, the inside angle may be reversed so that its short leg extends further along the beam span. Heavier loads require a cap detail similar to Fig. 12.

COLUMN SPLICE, *DP*2. This splice is designed to resist direct load, shear, and moment. Some column splices are designed for direct load only. When the faces of the abutting sections of such columns are *milled to bear* over the entire area, the splice plates serve only the function of aligning the sections for erection and of resisting an accidental force or a blow. Such a splice often has riveted splice plates designed to transfer only 25 per cent of the direct column load.

In the problem analyzed as *DP*2, the sections do not bear over the entire column area. The design is arranged to provide fill plates which enlarge the area of the upper column section and thus provide a larger area in direct bearing. These fill plates are " feathered out " to provide end rivets that are of shorter length than the rivets passing through all plates. This arrangement reduces the tendency for the rivets to bend as illustrated by Fig. 11.

The *transverse shear splice* is analyzed to illustrate the real action of such rivets. Since the rivets are spaced 2 in. away from the splice line, a pure shear on this line produces both shear and moment on the line of the rivets. The total moment of rotation is equal to the shear times the 4-in. vertical distance between rivet lines. This moment may be divided by the horizontal distance between the two rivet lines to obtain the vertical component of the rivet shear. This vertical force is combined with the horizontal component of rivet shear to obtain the resultant which controls the size of the rivet. The splice plate must be and is in equilibrium.

The *moment splice* is designed very simply. The number of field rivets shown in the splice plate is 33 per cent greater above than below the splice. This tends to allow for the fact that the upper rivets pass through fill plates. The fill plates also have *extra* rivets (shop) beyond the 8 (field) computed as necessary. Stressed fill plates should always be carried beyond the connection and riveted to the main part of the section with *extra*

DP1a. *Design a column base for a 10WF49 column where the total load is 144,000#. One half of the load is transferred through the rivets, the remainder by bearing on a steel plate resting on the concrete footing. AISC spec.*

Base Plate:

Area reqd. $= 144{,}000 \div 600 = 240\square''$.
Try a base plate $16'' \times 16''$. *Bearing* $= 563\#/\square''$.
Cantilever span beyond col. sect. $= 3''$.
$M = 563 \times 3 \times 1.5 = 2530''\#$ *per in. of width.*
$t = \sqrt{6M/bf} = \sqrt{6 \times 2530/20{,}000} = 0.87 = \tfrac{7}{8}''$.
Angles strengthen the plate where the overhang exceeds $3''$.

Side Angles:

Thickness chosen to make single shear rather than bearing control.
$32{,}000 \times t \times 0.87 = 0.6 \times 15{,}000$ ($\tfrac{7}{8}''$ *rivets*).
$t = 0.33''$; *use* $\tfrac{3}{8}''$ *angles.*
Rivet value $= 0.6 \times 15{,}000 = 9000\#$.
$n = 144{,}000 \times 0.5 \div 9000 = 8$ *rivets.*
Anchorage. Provide 2 holes $1\tfrac{1}{4}''$ *diam. for* $1\tfrac{1}{8}''$ *anchor bolts.*

DP1b. *Design a cap for this column to carry two 12″-31.8# I-beams, each having a reaction of 36,500#. Keep the size small.*

Rivets:

Value of $\tfrac{7}{8}''$ *rivet* $= 0.6 \times 15{,}000 = 9000\#$.
$n = 2 \times 36{,}500 \div 9000 = 8.1$; *use 8 rivets.*

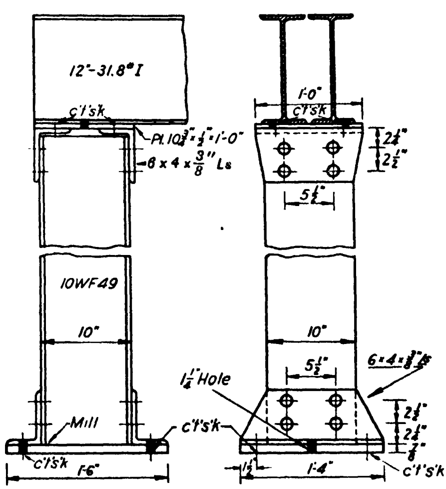
Angles:

Use $6 \times 4 \times \tfrac{3}{8}''$ *size.*

Cap Plate:

The function of this plate is similar to that of a bearing plate. For this load a $10\tfrac{3}{4}'' \times \tfrac{1}{2}'' \times 1'\text{-}0''$ *pl. will be adequate. For a light load the plate might be omitted.*

Remarks: *The maximum load for a cap of this type (2 angles) cannot be increased beyond the value of 8 rivets since it is usually impossible to arrange such a detail for a greater number of rivets.*



DP2. *Design a bearing splice between a 10WF72 column section and a 12WF99 column section. The direct load is 297,000# and in addition there is a shear of 25,000# and a wind moment of 700,000″#, both in the plane of the webs. Use AISC spec.*

Direct Bearing:

$A_b = 297{,}000/30{,}000$
$= 9.9\square''$.

A_b for web =
$0.51 \times 10.5 = 5.3$.

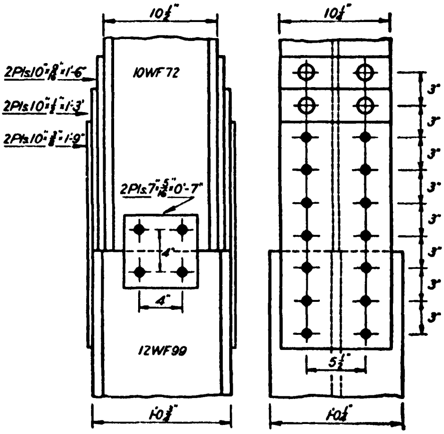
A_b on fill pls. = diff.
$= 4.6\square''$.

Load through rivets =
$4.6 \times 30{,}000$.

Single shear value for ⅞″ rivet
= 9000#.

$$n = \frac{2.3 \times 30{,}000}{9000} = 8$$ *(field rivets above splice).*

Fill pls.; use two ½″ and two $\frac{9}{16}$″ pls.



Transverse Shear Splice:

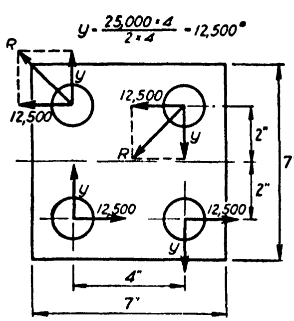
$$A_s = \frac{25{,}000}{13{,}000} = 1.92\square''.$$

2 pls. 7 × $\frac{5}{16}$″ furnish 2.2□″ each.

Value of ⅞″ rivets in bearing on web = 40,000 × 0.87 × 0.51 = 17,800#.

$$n = \frac{25{,}000}{17{,}800} = 2 \text{ rivets approx.}$$

$$R = \sqrt{12{,}500^2 + 12{,}500^2} = 17{,}700\#.$$



Moment Splice:

$$\text{Tension in plate} = \frac{700{,}000}{12.75} = 55{,}000\#.$$

$$t = \frac{55{,}000}{20{,}000(10.2 - 2)} = 0.34; \text{ use } \tfrac{3}{8}'' \text{ pls.}$$

Value of ⅞″ rivet in single shear = 9000#.

$$n = \frac{55{,}000}{9000} = 6.1; \text{ use 6 rivets below splice.}$$

Extra Rivets: *Eight field rivets are shown above the splice in contrast to 6 below. Extra rivets are also placed through the ends of the fill plates. This is in accordance with specifications for fill plates which carry stress. Actually, if the direct stress and bending moment occur simultaneously, 2 more rivets should be added through each pair of fill plates. Since 6 rivets were needed for flexure and 8 for direct stress transfer, 14 would be needed in total. Only 12 are shown on the sketch above the splice.*

rivets. Some specifications require a large increase in the number of rivets. Since 8 rivets were required, this design provides 25 per cent excess rivets placed in the end of each fill plate. This provision seems adequate in this particular connection since the outside plates have not been considered to be available for transferring direct stress. Where the splice plates must transfer a part of the direct load, the proper design will be to develop the fill plates beyond the limits of the splice plates for their full calculated stress.

Rivet Values

17. Working Stresses in Rivets. There is considerable variation in the unit stresses permitted by different specifications. The allowable stresses of importance for consideration here are the unit stresses in shear, bearing, and tension. The current values of allowable unit stresses for shop rivets are compared in Table 1. These values are taken from the specifications given in Chapter 17.

TABLE 1

Working Stresses for Power Driven Rivets

	Shear	Tension	Bearing (Dissymmetry)	Bearing (Symmetry)
AISC	15,000	15,000	32,000	40,000
AASHO	13,500	7,500	27,000	27,000
AREA	13,500	—	27,000 (single shear)	27,000 (double shear)

It will be observed in Table 1 that the unit stresses of the American Institute of Steel Construction are more liberal than those of the American Association of State Highway Officials or those of the American Railway Engineering Association. It is true that the attitude of the building designer has always been more *liberal* than the attitude of the bridge designer. An explanation of this difference of opinion may be found in the severe conditions of impact, fatigue, reversal, corrosion and even collision to which bridge members are subjected. Design specifications attempt to provide for each of these conditions, but their possible individual or combined severity in bridges as contrasted to their negligible importance in building construction justifies a difference in allowable unit stresses.

Working stresses in rivets seem to have been increased each decade since 1900. It appears from tests of structural steel shapes and plates that increased working stresses have been justified by improved material of higher elastic limit. On the other hand, the driving qualities of rivets require a rather low elastic limit so that further improvement of rivet steel may not be of great significance. Hence, it may well be that unit rivet stresses, particularly for building rivets, have approached an upper limit. Another possible reason for this will be found in § 38 where *fatigue of riveted joints* is discussed.

18. Tension Resistance of Rivets. Tension rivets have been used for a generation to furnish wind resistance in building construction. During this time nearly all bridge design specifications outlawed tension rivets and some still do. However, by 1930 two sets of tests had been reported* that eliminated most of the argument against the use of rivets in tension. These tests in reality justified the use of rivets at a tensile working stress fully equal to the working stress *in single shear* since the rivets were shown to have an initial tensile stress equal, on the average, to about 90 per cent of their elastic limit in tension. Since externally applied tension does not increase initial tension until the external force is greater than the initial tension force,† no possible harm could come to the rivet from substituting a given amount of externally applied tension for an equal amount of initial tension.

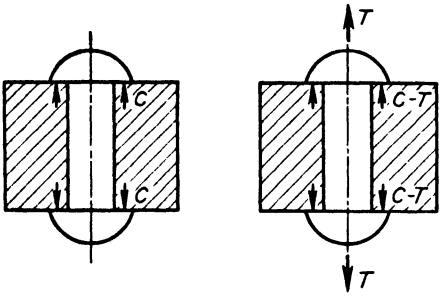


FIG. 13. INITIAL TENSION.

Combined Tension and Shear. It has been suggested in the *AISC* specifications that rivets in tension should not be permitted their full working values in shear or vice versa. This specification comes from the fact that tension and shear combine into a resultant shear or tension on a diagonal plane and this theoretical picture leads to the thought that such a resultant should be used to limit the value of the rivet. The other point of view is that every hot driven rivet has inevitably been resisting tension nearly equal to its elastic limit. Therefore, those rivets which were tested to furnish a basis for the selection of working stresses in shear and bearing actually were subjected to and perhaps weakened by such initial tension. Hence, any reasonable tension applied externally would simply take the place of an equal amount of initial tension and would not weaken the rivet further.

It is difficult to reconcile these opposite points of view and we must simply decide between them. As failure is approached, initial tension disappears because of rivet distortion. Then the rivet stress is dependent upon applied shear and applied tension which do combine into a resultant shear and a resultant tension. It is therefore safe to design upon the basis of this resultant stress and to allow *equal working stresses in tension and*

* C. R. Young and W. B. Dunbar, *Permissible Stresses on Rivets in Tension*, Bulletin No. 8, University of Toronto School of Engineering Research, 1928.

W. M. Wilson and W. A. Oliver, *Tension Tests of Rivets*, Bulletin No. 210, University of Illinois Engineering Experiment Station, 1930.

† The illustration, Fig. 13, shows how initial tension may be looked upon as produced by a compression C in the block. An applied tension T (smaller than C) reduces these compressive forces by the value of T so that the final rivet tension is again $(C - T) + T = C$.

shear. If, however, the allowable stress in tension is set at considerably less than the allowable stress in shear, it would seem that combined tension stress had been allowed for adequately and there would be no need to combine tension with shear to obtain a resultant stress.

19. Shear Distribution to Rivets. In a lap joint with 2 rivets, symmetry demands that the load be divided equally between the rivets. The division of load will never be uniform, however, when there are more than 2 rivets in line. For the triple riveted lap joint of Fig. 14 the end rivets must be deformed more than the center rivet *because the plate stresses between the rivets are not equal.* The upper plate between the rivets *a* and *b*

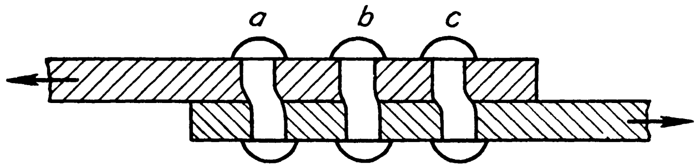


FIG. 14. VARIATION OF RIVET DEFORMATION.

has nearly twice the stress of the lower plate and therefore stretches nearly twice as much. Similarly, between the rivets *b* and *c*, the lower plate has the larger stress and the greater deformation.

The actual theoretical determination of the variation of rivet shears in a lap or butt joint has been made upon the assumption that there is no friction between the plates. (See Table 2.) Actually, of course, this condition cannot occur until the plates and rivets have passed their respective yield points, at which time deformations will be so large that all rivets will be deformed more or less equally, as has been shown by tests. Nevertheless, there is always the possibility of failure by fatigue which occurs in moving structures and even in railroad bridges. *Failure by fatigue ordinarily takes place without appreciable deformation* so that stress inequalities either in plates or rivets are not ironed out by plastic deformation before fatigue failure occurs. Hence, the data given in Table 2 may be particularly useful for the design of moving structures.

It will be observed from Table 2 that the end rivet for a lap joint of 6 rivets in a row may be overstressed 100 per cent if a uniform division of shear is assumed in design. Fortunately, the percentage increase is much less for most joints. Even though we may feel that tests have justified the entire neglect of such stress concentrations for structures acting only under *static loads with relatively few repetitions* (less than 2,000,000 in the life of the structure), we cannot help but be impressed by the desirability of keeping rows of rivets that are in line with the stress as short as possible.

TABLE 2

THEORETICAL DISTRIBUTION OF LOADS TO RIVETS*

(Spacing between rivet lines = 3 in. Diameter of rivets = ⅞ in.)

Lap Joints

No. Rivets	Pitch	Plate Thickness	Avg. Load per Rivet	Load Rivet 1	Load Rivet 2	Load Rivet 3	Percentage Overstress
3	2.5	0.5	0.33	0.37	0.26		11
	5.0	0.5	0.33	0.39	0.21		19
	2.5	1.0	0.33	0.35	0.30		5
	5.0	1.0	0.33	0.36	0.27		10
4	2.5	0.5	0.25	0.33	0.17		31
	5.0	0.5	0.25	0.37	0.13		47
	2.5	1.0	0.25	0.29	0.21		15
	5.0	1.0	0.25	0.32	0.18		26
5	2.5	0.5	0.20	0.31	0.14	0.10	56
	5.0	0.5	0.20	0.36	0.11	0.06	81
	2.5	1.0	0.20	0.26	0.17	0.15	28
	5.0	1.0	0.20	0.29	0.15	0.11	48
6	2.5	0.5	0.167	0.30	0.13	0.07	83
	5.0	0.5	0.167	0.36	0.10	0.04	116
	2.5	1.0	0.167	0.24	0.15	0.11	44
	5.0	1.0	0.167	0.29	0.13	0.08	71

Butt Joints

No. Rivets	Pitch	Thicknesses Plate	Thicknesses Straps	Avg. Load per Rivet	Load Rivet 1	Load Rivet 2	Load Rivet 3	Percentage Overstress
2	2.5	0.5	0.37	0.25	0.27	0.23		7
	5.0	0.5	0.37	0.25	0.28	0.22		11
	2.5	0.75	0.50	0.25	0.26	0.24		5
	5.0	0.75	0.50	0.25	0.27	0.23		7
3	2.5	0.75	0.50	0.167	0.21	0.13	0.17	23
	5.0	0.75	0.50	0.167	0.22	0.10	0.18	34
	2.5	1.0	0.50	0.167	0.19	0.13	0.19	12
	5.0	1.0	0.50	0.167	0.20	0.10	0.20	19
	2.5	1.0	0.75	0.167	0.20	0.14	0.16	21
	5.0	1.0	0.75	0.167	0.22	0.11	0.16	33

SPECIAL FEATURES OF DESIGN

20. Assumptions for the Design of Riveted Joints. The action of riveted joints from first load to failure has been discussed and many complications have been mentioned. The design of a riveted joint is a procedure that must be performed literally hundreds of times in the design of any

* A. Hrennikoff, *Work of rivets in riveted joints*, Transactions, *ASCE*, 1934, p. 447.

major structure. Therefore, it is desirable to simplify this procedure although the safety of the structure must not be endangered.

1. Friction Is Neglected. In other words, joints are designed for the kind of action that occurs near failure rather than for elastic conditions. Of course, an adequate factor of safety is introduced through the choice of working stresses for shear and bearing on rivets. Riveted joints might be designed on the basis of a *friction theory* but this is never done.

2. Rivets Fill the Holes. The gross section can therefore be used for a structural member acting in compression (column) or shear (web of a beam or girder) but holes must be deducted to obtain the net section that will resist the pull in a tension member.

3. All Rivets Are Stressed Equally. Each rivet of a riveted joint (centric load) is assumed to resist the same shear. Analysis shows that this cannot be true, even when friction does not exist, until after plastic flow above the yield point in rivets and plates has equalized the rivet deformations. At failure, test joints seem to have acted in this manner (static loading). Nevertheless, good design evidently opposes the use of long rows of rivets in line with the plate stress.

4. Plate Stress Is Distributed Uniformly. It is assumed that a plate reduced 25 per cent in cross-sectional area by rivet holes will still retain 75 per cent of its full tensile strength. This assumption neglects high concentrated stresses around rivet holes that are known to exist but which are of minor importance after the average plate stress passes the elastic limit.

5. Fatigue Is Not Considered. Assumptions (3) and (4) may be summarized by saying that fatigue failure is not considered to be probable in the standard design of riveted joints. The most effective method of design of riveted joints for fatigue would seem to be to reduce the maximum load on the joint to less than the minimum friction value. Allowance should also be made for fatigue failure on the net section through the plate.

6. Rivet Flexure Is Negligible. For ordinary joints the flexure of the rivet caused by the fact that plate forces cannot be in direct line is allowed for in the permissible working stresses in shear and bearing. Extraordinary flexure should either be eliminated (as by use of extra rivets through filler plates) or considered in the design. Long rivets are increased in number for resistance to flexure (Spec. 30).

7. Rivets Will Resist Tension. Since tests have shown that rivets act under an initial tension of about 90 per cent of the elastic limit, tension resistance equal to the rivet resistance in single shear seems permissible in design. The tension stress in the rivet is not increased beyond its initial tension stress by an externally applied tensile force of less than the initial tension.

8. Reversal Is Serious. Stress reversal tends to break down the frictional resistance of a joint and to wear the rivets loose in their holes. The proper design procedure is the one recommended for fatigue, that is, reduce the allowable load well below the friction value of the joint. This is often accomplished by designing the joint for the sum of the tension and compression forces acting on the member.

21. Net Section Through Rivet Holes. The design of the rivets for the end connection of a tension member usually is made for the net value of the member, which is taken as the minimum cross-sectional area (with rivet holes deducted) times the allowable working stress in tension. The deduction for a rivet hole is a *rectangular area* equal to the thickness of the plate times the diameter of the hole, which is taken as $\frac{1}{8}$ in. greater than the

nominal diameter of the rivet; for example, we deduct $\frac{7}{8}$-in. holes for $\frac{3}{4}$-in. rivets.

Line of Failure. The number of holes to be deducted is determined by an empirical formula set up to agree with observations and tests on the actual manner of failure of riveted members. It is reasonably evident in Fig. 15 that the joint (*a*) would fail on a net section through 1 rivet hole while the joint (*b*) would fail through 2 rivet holes. (It is assumed that the rivets are stronger than the angle.) There is some question whether the joint (*c*) would fail on a net section perpendicular to the member through 1 rivet or on a net section as indicated through 2 rivets. If the rivet spacing *s* is small, the failure will be along the diagonal section. Likewise, in (*d*) there is a possible choice of paths, either perpendicular to the member through 2 rivet holes or along the diagonal line through 3 rivet holes.

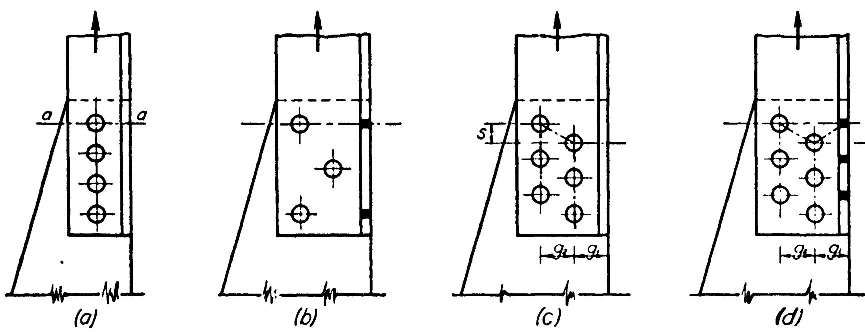


FIG. 15. NET SECTION OF TENSION MEMBERS.

22. Formulas for Deduction of Rivet Holes. *Formula Based upon Pitch.* The simplest possible formula was in use for many years and is still found in some specifications. It is based upon the assumption that failure will occur along a diagonal line in (*c*) or (*d*) of Fig. 15 if the pitch of the rivets is less than 4 in. Expressed as a formula, this linear relationship becomes

$$A_{\text{deduct}} = A_{\text{hole}}\left(1 - \frac{s}{4}\right). \tag{3}$$

To apply this formula we draw any line perpendicular to the member and deduct the areas of all holes along this line and fractional areas of all holes within 4 in. of the line as controlled by the formula. The factor *s* is usually the pitch, but it would be the gage if the lines of rivets were perpendicular to the direction of stress, as is common in tank work.

Formula Based upon Pitch and Gage. Another relationship based upon an analysis of tests * is as follows:

$$(4) \qquad A_{\text{deduct}} = A_{\text{hole}}\left(1.5 - \frac{s}{g}\right) \qquad \left(\frac{s}{g} < 1.5 \text{ and } > 0.5\right).$$

In this formula, s is the rivet spacing in successive lines of rivets and g is the gage distance between these adjacent rivet lines. This formula is applied somewhat differently than formula (3). That is, we deduct the first hole cut by a transverse zigzag section and the fractional part of each succeeding hole as controlled by equation (4) for the *successive values of* s/g. The calculations must be made in sequence and, therefore, s will only have the value of zero when *successive holes are on the same transverse section* of the member.

Formula Based upon Pitch, Gage and Size of Hole. The most comprehensive empirical formula and the one now used most generally involves three variables, the pitch s, the gage g and the diameter of the rivet hole h which is taken as ⅛ in. larger than the nominal size of the rivet.† (Spec. 19, 106, 171.)

$$(5) \qquad A_{\text{deduct}} = A_{\text{hole}}\left(1 - \frac{s^2}{4gh}\right) \qquad \left(\frac{s^2}{4gh} < 1\right).$$

This formula is sometimes modified to

$$(6) \qquad A_{\text{deduct}} = A_{\text{hole}}\left(1 - \frac{s^2}{4g}\right) \qquad \left(\frac{s^2}{4g} < 1\right).$$

Again, we are supposed to deduct the full value of the first hole through which a zigzag section is drawn across the member and fractional parts of succeeding holes as controlled by the formula. Note that the value of s in Fig. 15(d) would be used twice, that is for the fractional deductions of the second and third rivet holes.

23. Net Section of an Angle. The formulas (4), (5), and (6) involve the gage of the angle. In Fig. 15(c) the gage that controls rivet deduction evidently is the gage g_2 along with the pitch or spacing s. The same statement applies to deduction for the second rivet hole in Fig. 15(d) (starting from the left with a deduction of the full value of the first rivet hole on the transverse section). However, the third rivet hole is in the outstanding leg of the angle. Obviously, the spacing back to the transverse section is still s which will now be used along with the gage g_3 (Fig. 16) to obtain the fractional deduction for the

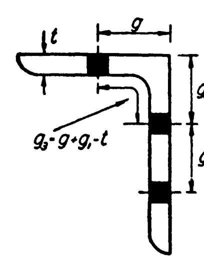


FIG. 16. ANGLE GAGES.

* V. H. Cochrane, Engineering News-Record, Nov. 16, 1922 (1.5 − s/g).
C. R. Young and T. R. Loudon, Bulletins 6 and 9, School of Engineering Research, University of Toronto (1.5 − s/g).
† D. B. Steinman, Proceedings *ASCE*, April, 1922 (1 − $s^2/4gh$).

third rivet hole. Several points may be repeated: first, the gage and the pitch are always measured from the rivet just deducted to the next rivet on the assumed line of failure; second, the gage is along the center line of the metal; third, the pitch may be either away from or back toward the transverse section; and fourth, the rivets must be treated successively like the links of a chain.

NET SECTION BY AASHO FORMULA, *DP3a*. This example illustrates the use of a standard formula for rivet hole deduction to determine the rivet spacing consistent with *a permissible deduction* of a certain number of rivet holes. This is a common problem in design. As the design of a tension member proceeds, it is necessary to estimate the net section that will be furnished by a given angle. Since the rivet details have not been made as yet, the designer must use his best judgment as to the proper deduction. Then, later, when the rivet details are drawn, it is necessary for the detailer to arrange the rivets so that the required deduction will be no greater than the deduction assumed by the designer.

24. Rational Procedure of Rivet Hole Deduction. The use of simple formulas as given in the previous section seems entirely justified as a practical design procedure. They will undoubtedly be retained in standard specifications although perhaps in somewhat modified form as new tests become available. However, these formulas do not account directly for the fact that the *line of failure* is often a diagonal one that is longer than a right section. Indirectly, of course, the fact that less than a full rivet hole is deducted for a staggered rivet is an allowance for an increased length of the diagonal line of failure.

Another attack upon the problem of net section and perhaps a more rational one is to deduct all holes on *any possible line of failure* and to credit diagonal distances with some reduced length. The factor of 10 per cent reduction for diagonal distances was introduced into the Specifications for Steel Highway Bridges (*ASCE*, 1924) and the General Specifications for Steel Railway Bridges (*ASCE–AREA*, 1929). If this factor is accepted as reasonable, the proper statement of the rule becomes:

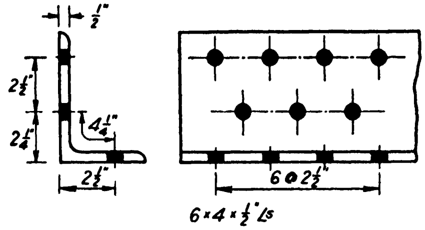
1. Deduct all rivet holes lying on any right section from the true gross area to obtain the net area, or
2. Deduct all rivet holes lying on any zigzag line from the gross area along that zigzag line of failure when diagonal distances are reduced 10 per cent.

It will be evident that a single factor, such as 10 per cent, could not rationally be used as the proper deduction for diagonal lengths of all slopes and it must therefore be looked upon as an average value. One is not impressed with the advantages of increased accuracy of this procedure over the use of formula (5) which is more convenient in that diagonal distances are not involved.

DP3a. *Arrange the riveting in a 6 × 4 × ½" angle for 3 gage lines so that only 2 holes for ¾" rivets need be deducted. AASHO spec.*

The AASHO formula for rivet hole deduction is

$$A_{deduct} = A_{hole}\left(1 - \frac{s^2}{4gh}\right). \quad (\textit{Spec. 106.})$$



$g = 2\frac{1}{2}"$ *or* $4\frac{1}{4}"$ (*where* $2\frac{1}{4} + 2\frac{1}{2} - \frac{1}{2} = 4\frac{1}{4}"$).

$$\textit{Hence};\ 1 + \left[1 - \frac{s^2}{4 \times 2\frac{1}{2} \times \frac{7}{8}}\right] + \left[1 - \frac{s^2}{4 \times 4\frac{1}{4} \times \frac{7}{8}}\right] = 2,$$

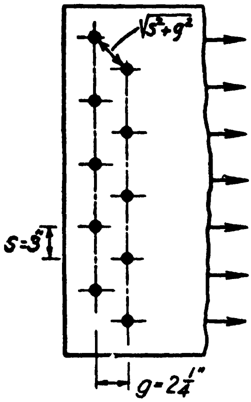
or $\quad s^2(0.114 + 0.067) = 3 - 2 = 1;$

hence, $\quad s = \sqrt{\frac{1}{0.18}} = 2.4".$

For an allowable deduction of 2¼ holes,

$$s = \sqrt{\frac{0.75}{0.18}} = 2".$$

DP3b. *Arrange the riveting for a wide plate in 2 gage lines so that the gross section is reduced by only 33% more than by the rivet holes on a single line. Use ⅞" rivets at 3" centers.*



The controlling specification reduces the net section by all holes on a zigzag line but permits use of 90 per cent of the zigzag length.

$$\textit{Zigzag length} = \sqrt{s^2 + g^2}.$$

Holes are 1" diam. for ⅞" rivets.

Hence, $0.9\sqrt{s^2 + g^2} - 1.0 = (2s - 1.33) \div 2.$

For $s = 3"$; $\sqrt{9 + g^2} = 3.33 \div 0.9;$

$\quad g = \sqrt{13.7 - 9} = 2.2".$

For $s = 2\frac{1}{2}"$; $\sqrt{6.25 + g^2} = 2.83 \div 0.9;$

$\quad g = \sqrt{9.9 - 6.25} = 1.9".$

NET SECTION OF A PLATE, *DP3b*. A problem is solved by use of the rational procedure to illustrate how the gage distance between rivet lines in a plate might be determined for a given rivet spacing in order that the deduction of rivet holes should not exceed a specified allowance (33 per cent in *DP3b*). The use of the rational procedure always makes a study of the *zigzag length* between staggered rivets necessary. This length in *DP3b* can be expressed as $\sqrt{s^2 + g^2}$. Then, if either s or g is fixed, the value of the other can be computed.

For comparison, the problem *DP3b* will be solved by use of the simple straight-line formula

$$A_{\text{deduct}} = A_{\text{hole}}\left(1 - \frac{s}{4}\right).$$

Thus, when s becomes the gage, more commonly designated as g, we obtain

$$1 - \frac{g}{4} = 0.33,$$

or

$$g = \frac{8}{3} = 2.66 \text{ in.}$$

This value of 2.66 in. compares conservatively with the value of 2.2 in. obtained by the rational procedure.

25. Tension Member Splices. The function of a tension member splice is to replace the *net effective area* of the member. This is the requirement of all specifications and it results in the fact that the connection must develop any *excess area* put into the member over and above that needed to carry the design stress. The reason is economic. If money is expended on excess area, it is certainly good judgment from an economic point of view to expend the small additional cost of extra rivets to develop this excess area which at some time in the life of the structure may prove useful.

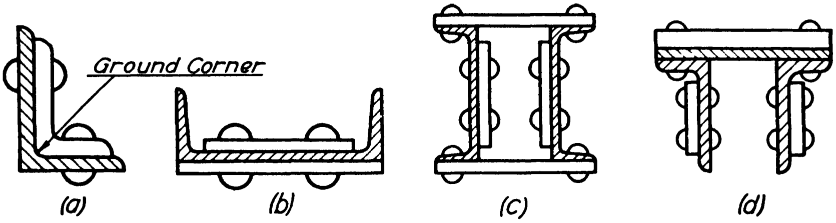


FIG. 17. SPLICES OF RIVETED TENSION MEMBERS.

Some details of riveted splices for tension members are illustrated in Fig. 17. A single angle can be spliced by a slightly thicker angle with shorter or shortened legs. This detail shown in (*a*) is the most satisfactory type of splice. Each part of the section is in contact with splice material

and rivets pass through each part of the section. This cannot be true in (*b*) where plates are attached only to the web of the channel. In (*c*) and (*d*) the splice rivets pass through each part of the section. Note that the splices (*a*), (*c*) and (*d*) have rivets in single shear while the channel splice (*b*) has rivets in double shear. This device of double splice plates can be used in other instances to reduce the number of rivets needed.

Indirect Splices. The top splice plate of Fig. 17(*d*) is a direct splice for the horizontal plate of the tension member but it is an indirect splice for any part of the angle area that it may be used to splice. Specifications commonly require the use of *excess rivets* where the splice is indirect (intermediate plate). The requirement may be an increase of as much as 25 or 33 per cent in the number of rivets for the effect of *each intermediate plate.*

Chord Member Spliced at a Chord Joint, *DP*4. Most specifications warn the designer against arranging a chord member splice that involves the use of the gusset plates as splice plates. However, it is sometimes desirable to use the gusset plates in this manner and there can be no objection raised if the gusset plates are analyzed properly as splice plates. The trouble has been that gusset plates were simply chosen for their function as gussets and were then called upon to withstand the *extra duty* of splice plates for which they had not been designed.

In the design problem *DP*4, the angles of the chord are spliced for the value of their outstanding legs by the horizontal splice plates. The vertical legs are simply attached to the vertical gusset plates. On a vertical line through the center of the joint between the angles to be spliced, the cross-section resisting stress consists of the two 14 × ⅜-in. gussets and the 20 × ½-in. horizontal splice plate. These plates, joined together through the medium of the chord angles, form a U-section. Since the pull in the angles to the right is eccentric by 1.5 in. from the neutral axis of the U-section, there is not only a direct stress but also a *flexural moment* resisted by this section. The fiber stresses are computed and a proper design is suggested.

Although in this design problem a solution is reached, it is not always feasible to obtain such a satisfactory design for a joint splice. For instance, in *DP*4 the problem will be found to be seriously complicated by an extension of the height of the gussets to 15 in., 16 in., or more, which will increase the eccentricity. A similar effect is obtained by the use of thicker gussets or a thinner or narrower splice plate. Naturally, the problem will be more difficult to solve where the chord angles are thicker. A *splice between joints* is usually the better solution.

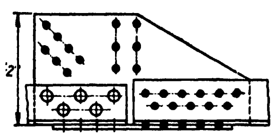
Eccentricity in Riveted Connections

26. Eccentrically Riveted Connections. The ideal riveted connection for a tension or compression member has the center of gravity of the rivet group comprising the connection lined up exactly with the center of gravity of the member or with the line of the applied load. This ideal is seldom attained and it is not uncommon to find considerable eccentricity even in *standard connections.* For example, the double angle tension member of Fig. 18(*a*) has two lines indicated on the figure, the gage line and the gravity axis, which happen to be eccentric by 1⅜ in. The standard beam connec-

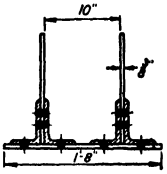
DP4. *Arrange a splice for a tension member that consists of four 6 × 4 × ½″ angles forming the lower chord of a highway bridge truss. The splice is at a joint. AASHO spec.*

Horizontal Splice Plate:

Area of angles with two 1″ holes deducted from each = 4(4.75 − 1.0) = 15□″.

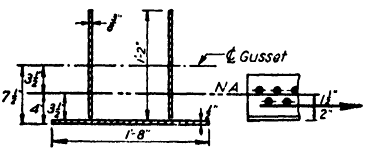


Side



End

Value = 18,000 × 15 = 270,000#. (Spec. 99.)
Value of 4″ legs = 270,000 × 0.4 = 108,000#.
Area of splice pl. = $\dfrac{108{,}000}{18{,}000} = 6\square''$.



Section

Pl. 20″ × ½″ gives 8□″ net.

Value of ⅞″ field rivet in single shear = 0.6 × 11,000 = 6600#.

$$n = \frac{108{,}000}{6600} = 16.4 \text{ rivets. Use 18 rivets.}$$

Rivets Through the Gusset Plate:

Value of a ⅞″ rivet in bearing on a ⅜″ pl. = 22,500 × 0.37 × 0.87 = 7400#.

Number of rivets to splice vert. legs; $n = \dfrac{0.6 \times 270{,}000}{7400} = 22$ *rivets.*

Stresses Due to Gusset Flexure:

Height to N.A. of gusset-splice pl. section $= \dfrac{2 \times 14 \times 0.37 \times 7.25}{2 \times 14 \times 0.37 + 20 \times 0.5} = 3\tfrac{3}{4}''.$

Moment = 270,000 × 1.50 = 405,000″#.

$$\text{Moment of Inertia} = 20.0 \times \tfrac{1}{2} \times 3.75^2 + 2\left[\frac{1}{12} \times \frac{3}{8} \times 14^3 + \frac{3}{8} \times 14 \times 3.5^2\right]$$
$$= 141 + 172 + 129 = 442.$$

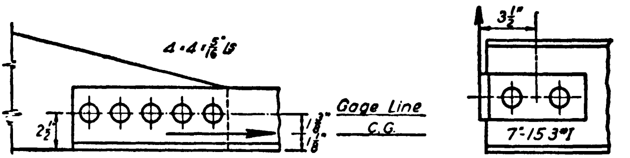
Fiber stress (increased 25% for net section)

$$= 1.25\left[\frac{270{,}000}{20.5} + \frac{405{,}000 \times 4.0}{442}\right] = 16{,}400 + 4600 = 21{,}000\#/\square''\ T;$$

$$= 1.25\left[\frac{270{,}000}{20.5} - \frac{405{,}000 \times 10.5}{442}\right] = 16{,}400 - 12{,}000 = 4400\#/\square''\ T.$$

Redesign: *In order to reduce the tension fiber stress from 21,000 to less than 18,000#/□″, the splice plate will be increased to ⅝″ in thickness. Then the P/A term will reduce to 14,600 and the Mc/I term will reduce also. It is evident that the design will then be adequate and a recalculation of fiber stress is not considered necessary.*

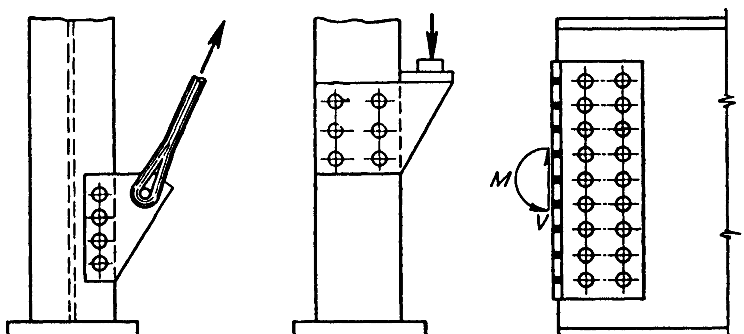
tion shown in Fig. 18(*b*) has an eccentricity of 3½ in. which produces a large moment to be resisted by the two rivets. With this eccentricity, the value of this riveted connection is relatively small, but its resistance need not be very great since it is only used for 6-in. and 7-in. beams. For deeper beams with rivets in a vertical line, the eccentricity is reduced.



(a) Tension Angles (b) Beam Connection

FIG. 18. ECCENTRICITY IN STANDARD CONNECTIONS.

Some connections are actually designed for moment resistance. The tie-rod connection of Fig. 19(*a*) is intended to intersect with the center line of the post at the base plate. The force does not pass through the center of the rivet group and, therefore, it produces both a direct shear and a twisting moment to be resisted by the rivets. The side bracket shown on the column in (*b*) acts in about the same manner as the connection (*a*), but



(a) Tie-Rod Connection (b) Side Bracket on Column (c) Moment Resistant Floor-Beam Connection

FIG. 19. CONNECTIONS DESIGNED FOR MOMENT RESISTANCE.

the eccentricity is greater and two rows of rivets are desirable for this reason. The heavy end connection shown in (*c*) is for a floor beam of a low-truss bridge where lateral stability is dependent upon a moment resistant connection from the floor beam to the vertical post. The connection is designed to resist both the end shear of the beam and a moment caused by a lateral wind force.

27. Analysis of Eccentric Riveted Connections. The extent of the printed matter on the analysis of eccentric riveted connections would lead

one to believe that the subject is very complex. Actually, it is rather simple. The theory in one of its special applications may be compared to the formula for the fiber stress on any section carrying a normal load P eccentric from the center of gravity of the cross-section by the distance e.

$$f = \frac{P}{A} \pm \frac{Pec}{I}. \tag{7}$$

Rivets in a Single Line. The revised formula for a *line of rivets* (n rivets at distances y from the center of gravity of the group, c being the maximum value of y) becomes

$$S_s = \frac{P}{n} + \frac{Pec}{\Sigma y^2}. \tag{8}$$

In this formula, S_s is the maximum shear per rivet. The use of this equation assumes that the direct shear on the most highly stressed rivet and the shear caused by the moment of eccentricity are in line, as in Fig. 18(b). In case they are 90 degrees apart, as in Fig. 18(a), the expression for the maximum shear per rivet becomes

$$S_s = \sqrt{\left(\frac{P}{n}\right)^2 + \left(\frac{Pec}{\Sigma y^2}\right)^2}. \tag{9}$$

The direction of the shear represented by the term P/n may be oblique to the direction of the shear represented by the term $Pec/\Sigma y^2$ as in Fig. 19(a) and Fig. 20. Here, the term P/n represents a shear acting in line with the load while the term $Pec/\Sigma y^2$ represents a horizontal shear caused by the moment of eccentricity. The design shear for the most highly stressed rivet (the lower one of the group in Fig. 20) is the resultant of these two shears, and it may be obtained graphically as illustrated. The same general procedure may be followed irrespective of the directions of the two components of shear.

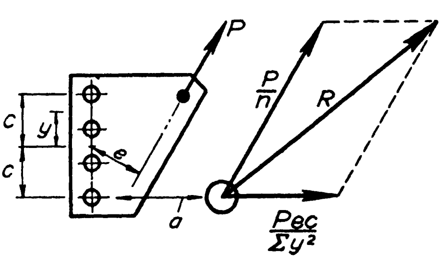


FIG. 20. RESULTANT RIVET SHEAR.

ECCENTRICITY IN ANGLE CONNECTIONS, *DP*5. Where two lines of rivets are used in a 5-in. or 6-in. angle leg, there is always considerable eccentricity. This eccentricity for the example *DP*5 is 1½ in. The resultant moment produces a cross shear perpendicular to the direct shear on the rivets. The result obtained by equation (9) is shown to be an increased rivet stress that will be resisted adequately by the addition of an extra rivet to the 12 required for direct load. It is significant that this increase is rather nominal, because it is not common to consider the moment of eccentricity in practical design. However, the moment of eccentricity will prove more serious for a short connection. The careful designer will add a rivet even though he may not complete a check analysis as in *DP*5. It is to be noted that the moment of eccentricity stresses the angles (in flexure) as well as the rivets.

DP5. *Design the connection for two $6 \times 4 \times \frac{1}{2}''$ angles attached back to back to a $\frac{1}{2}''$ plate. Take account of eccentricity in the connection. AASHO spec.*

Standard Design Procedure:

Holes to deduct (1″ holes) $= 1 + \left[1 - \dfrac{s^2}{4gh}\right]$

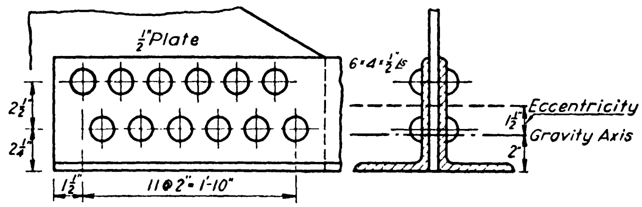
$$= 1 + \left[1 - \frac{4}{4 \times 2\frac{1}{2}}\right] = 1.6.$$

Net area of 2 angles $= 2(4.75 - 1.6 \times 1 \times \frac{1}{2}) = 7.9\square''$.

Value of angles $= 7.9 \times 18{,}000 = 142{,}000\#$.

Value of $\frac{7}{8}''$ *rivet for bearing on* $\frac{1}{2}''$ *plate;* $0.875 \times 0.5 \times 27{,}000 = 11{,}800\#$.

Number of rivets (neglecting eccentricity) $= 142{,}000 \div 11{,}800 = 12$.



Allowance for Eccentricity:

Moment of eccentricity $= 142{,}000 \times 1.5 = 213{,}000''\#$.

I of rivet group $= 2\,[1^2 + 3^2 + 5^2 + 7^2 + 9^2 + 11^2] = 572$.

Flexural shear $= \dfrac{213{,}000 \times 11}{572} = 4100\#/\text{rivet}$.

Resultant rivet shear $= \sqrt{4100^2 + 11{,}800^2} = 12{,}500\#/\text{rivet}$.

Try a connection with 1 extra rivet, 13 in all.

$I = 2\,[2^2 + 4^2 + 6^2 + 8^2 + 10^2 + 12^2] = 728$.

Resultant rivet shear

$$= \sqrt{\left[\frac{213{,}000 \times 12}{728}\right]^2 + \left[\frac{142{,}000}{13}\right]^2} = 11{,}500\#/\text{rivet}.$$

Design: *Use 13 rivets in place of the 12 shown in the illustration. Actually this allowance for eccentricity is seldom made in design although it is fully justified. It will be more serious in connections with fewer rivets.*

28. Torsion Formula for Rivet Groups. The flexure formula applies only to rivets in a single line or for two or more lines that are close enough together so that the *length of the group is several times the width.* Actually, the action of a rivet group in resisting the moment of an eccentric load is comparable to the action of a shaft in resisting a torque moment. If it is assumed that the rivets are forced by the plate to deform so that rivet shears are proportional to radii from the center of rotation, the most important requirement of the torsion formula is met. Hence, we may write

$$s_s = \frac{Tr}{J}, \tag{10}$$

or, by substitution of Σr^2 for J and Pe for T, we obtain

$$S_s = \frac{(Pe)r}{\Sigma r^2} = \frac{Per}{\Sigma(x^2 + y^2)} = \frac{Per}{\Sigma x^2 + \Sigma y^2}. \tag{11}$$

The factors x and y are the coordinates of the rivets measured from the center of gravity of the rivet group from which point the radii (r) also are measured.* S_s is the shear per rivet.

* It hardly seems necessary to derive equation (11) from basic principles since it amounts merely to a rederivation of the torsion formula. However, this relationship may be derived quite simply.

Let the rivet shear at unit distance from the c. g. be z_0.

Then the rivet shear at a radius r from the c. g. is rz_0.

The moment of this rivet shear about the c. g. becomes r^2z_0.

The total moment resistance of the entire rivet group is

$$\Sigma r^2 z_0 = \Sigma(x^2 + y^2)z_0 = z_0\,(\Sigma x^2 + \Sigma y^2).$$

Hence, we may equate this resisting moment to the moment of eccentricity

$$Pe = z_0\,(\Sigma x^2 + \Sigma y^2),$$

or

$$z_0 = \frac{Pe}{\Sigma x^2 + \Sigma y^2}.$$

The stress on any rivet at a radius r from the c. g. becomes

$$S_s = rz_0 = \frac{Per}{\Sigma x^2 + \Sigma y^2}. \tag{11}$$

When the rivets are in a single vertical line, $\Sigma x^2 = 0$, $r = y$ and equation (11) becomes

$$S_s = \frac{Pey}{\Sigma y^2}.$$

Hence, for the shear on any rivet in a group where the direct shear P/n is in line with the shear caused by rotation, we may write

$$S_s = \frac{P}{n} \pm \frac{Pey}{\Sigma y^2}.$$

The maximum shear becomes

$$S_s = \frac{P}{n} + \frac{Pec}{\Sigma y^2}. \tag{8}$$

This equation is identical with equation (8) obtained as a revision of the formula for fiber stress in an eccentrically loaded column.

The reader should familiarize himself with the details of this simpler derivation so thoroughly that in future studies he can revise the beam flexure formula or the torsion formula without a complete rederivation and with full assurance that his revision is applicable to the case in hand.

An analysis of the side bracket connection of Fig. 19(*b*) is indicated in Fig. 21. There is a vertical shear on each rivet equal to P/n and the rivet a (upper right-hand rivet) also resists a shear acting downward and to the right caused by rotation about the center of gravity of the rivet group. The *resultant shear* is obtained graphically as the vector R. The other rivet shears may be found likewise.

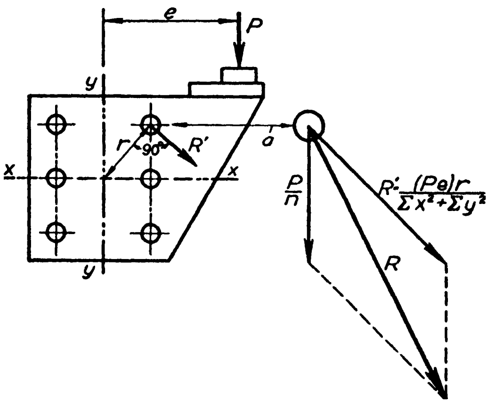


FIG. 21. ACTION OF A RIVET GROUP.

ASSUMPTIONS. There were several assumptions involved in the theory presented in this section for the analysis of riveted connections acting under eccentric loading. It will be well to review them here in order that the methods presented shall not be extended beyond their legitimate fields of use.

1. The direct load produces an equal shear P/n on each rivet. This assumption is open to the same criticism that was discussed in § 19. The end rivets of rivet lines such as the one shown in Fig. 18(*a*) (even without eccentricity) are more heavily stressed than the inside rivets because of *stretch in the metal* between the rivets. On the other hand, the center rivets of the floor-beam connection of Fig. 19(*c*) (but acting under vertical shear alone) are probably more heavily stressed than the end rivets as would be indicated by the beam shear formula, $S_s = VQ/I$. The assumption of uniform distribution seems, however, to be fulfilled before final failure takes place.

2. The rotation about the center of gravity of the rivet group produces a shear on each rivet proportional to its radius from the c.g. and *normal* to that radius. This involves the assumption that the plates are so heavy and stiff that *all deformation is thrown into the rivets*. This will not be true for thin plates, split plates, long narrow plates, or plates of irregular shapes as, for example, where a large reentrant corner exists.

3. Long rivet lines resisting moment may be treated by a simplification based upon the flexure formula. (See *DP*5.) This assumes that the term x^2 in the expression for polar moment of inertia $\Sigma x^2 + \Sigma y^2$ is negligible. Hence, the polar moment of inertia becomes the same as the ordinary moment of inertia, Σy^2, and the radius to the farthest rivet

becomes the extreme fiber distance y or c. This procedure is proper for a single line of rivets, but it is an *approximation for two or more parallel lines.*

4. Friction is neglected. As in other riveted joints it seems best to base our study upon the resistance of the rivets after failure of elastic action. Actually, initial tension produces a friction force that resists all plate slip under normal working conditions. For reversed loading it is desirable to reduce stresses so that friction will always be effective.

29. Instantaneous Center of Rotation. A convenient picture of the resultant shear on any rivet is presented in Fig. 22 by reference to an instantaneous center of rotation.* If the final resultant shear on the upper right-hand rivet is plotted as the vector R, a radius r_i to this rivet may be drawn perpendicular to this vector. This radius crosses the x-x axis (drawn through the center of gravity of the rivet group) at i, the center of rotation for the load shown in Fig. 22. If the load P had been acting horizontally, the instantaneous center of rotation would have been located at the intersection of the radius r_i with the vertical gravity axis y-y. The gravity axis to be used for a diagonal load is perpendicular to the load because the direction of shear for a rivet on this axis (equations (8) and (11)) is parallel to the load. Any rivet that happens to lie on this gravity axis will have a resultant stress perpendicular to the axis. Thus the gravity axis chosen is in reality a second radius, and the instantaneous center of rotation naturally lies at the *intersection of any two known radii.*

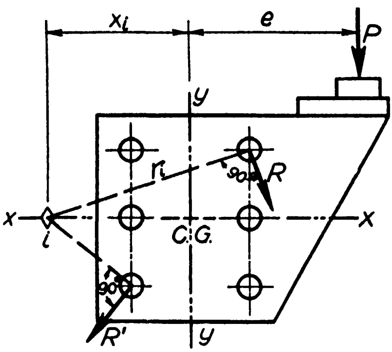


FIG. 22. INSTANTANEOUS CENTER OF ROTATION.

Once the instantaneous center of rotation has been located, each resultant rivet shear is known to be proportional to its radius from the center as, for example, R' in Fig. 22 is proportional to the value of its radius r. Thus the location of the center of rotation has a unique *pictorial* value. It is possible to locate the instantaneous center of rotation by direct analysis†

* The word " instantaneous " refers to the fact that the center of rotation shown in Fig. 22 applies only for the load in the position shown. A change in position or direction of the load would change the center of rotation.

† An imaginary rivet located on a vertical line through i in Fig. 22 would have a horizontal resultant shear. Its vertical downward shear P/n must be equalized by the upward vertical component of its shear caused by the eccentric moment Pe. Hence, we may write

(12) $$\frac{P}{n} = \frac{Pex_i}{\Sigma x^2 + \Sigma y^2},$$

or,

$$x_i = \frac{\Sigma x^2 + \Sigma y^2}{en}.$$

(*Continued on next page.*)

and to make its location serve to evaluate the resultant shears on all rivets (graphically) but its main service is as indicated above. Evidently, another use is to locate the *rivet of greatest stress* which is the one farthest away from the instantaneous center of rotation.

30. Design of Rivet Lines to Resist Moment. The use of equations such as (8) and (9) which involve the term Σy^2 are convenient for the analysis but not for the design of a riveted joint for moment resistance. If we consider a line of rivets $(n - 1)s$ in length (n in number at spacing s) as comparable to a long narrow rectangular cross-section ns in length, as indicated in Fig. 23, the total force resisted by the upper rivet, S_s, can be obtained approximately as the upper fiber stress of the rectangular section times the rivet spacing. For a section modulus of $(ns)^2/6$, this expression becomes

$$S_s = \frac{6Ms}{(ns)^2} = \frac{6M}{n^2 s}. \tag{14}$$

This equation may be solved for the number of rivets n which gives the relation

$$n = \sqrt{\frac{6M}{S_s s}}. \tag{15}$$

The use of equation (15) is convenient as a *design procedure.* First, a choice of rivet spacing s is made to agree with standard rivet spacing for other parts of the structure. Then, the moment M is computed and S_s is taken as the limiting shear on the rivet. The computed value of n is the number of rivets (not the number of rivet spaces) needed on the line.

If precision is desired, the fiber stress at the level of the rivet could have been used in place of the extreme fiber stress in Fig. 23. This correction results in the factor $(n - 1)/n$ entering under the radical of equation (15), but, for more than 5 or 6 rivets in a line, this correction factor is

---

This relationship offers a direct method of locating the distance from the center of gravity to the center of rotation measured along the gravity axis perpendicular to the load. After the center of rotation has been located, each rivet shear can be computed by use of a revision of equation (11), as follows.

$$S_s = \frac{Per_i}{\Sigma x^2 + \Sigma y^2}. \tag{13}$$

In this equation, r_i is the radius from the instantaneous center of rotation to any rivet, Pe is the moment of the load about the center of gravity of the rivet group, and $\Sigma x^2 + \Sigma y^2$ is the polar moment of inertia (J) of the rivets about the center of gravity of the rivet group. This formula would seem self-evident if we substituted $P(e + x_i)$ for Pe and used in the denominator the polar moment of inertia of the rivets about the point i to give the equation

$$S_s = \frac{P(e + x_i)r_i}{(\Sigma x^2 + \Sigma y^2) + n(x_i)^2}.$$

But a substitution of the value of x_i from equation (12) will show that this expression is the same as the simpler equation (13) above.

very nearly unity and its omission is on the side of safety. The precise formula is given in Shedd's *Structural Design in Steel*, 1934 edition.

$$n = \sqrt{\frac{6M}{S_s s}\left(\frac{n-1}{n}\right)}. \tag{16}$$

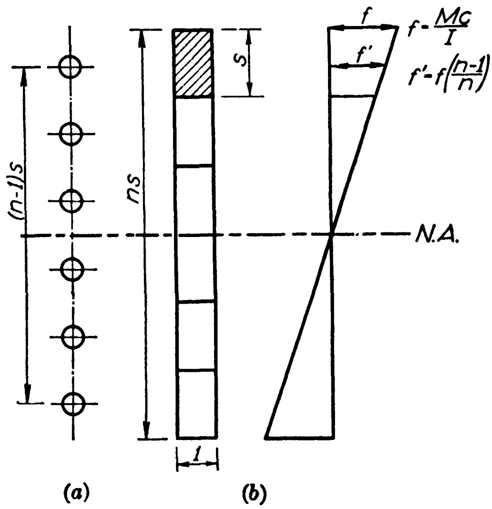


FIG. 23. DESIGN FOR MOMENT RESISTANCE.

31. Design to Resist Moment and Direct Shear. If there is a direct shear perpendicular to the rivet line in addition to the moment of eccentricity (Fig. 24), we may apply equation (15) directly to the design by reducing the limiting value S_s of the rivet by some estimated value of the average direct shear P/n. Of course, this usually involves an *initial guess* and a succeeding *revision.* Where the load is parallel to the rivet line, the shear P/n is at 90 degrees to the shear caused by flexure. The two combine into a resultant, but the effect of P/n is less serious. Its effect must be estimated and later rechecked, a process that becomes more tedious whenever the load is oblique to the line of rivets. However, since the number of rivets must be increased or decreased by the value of *one full rivet,* a solution can always be reached without great difficulty.

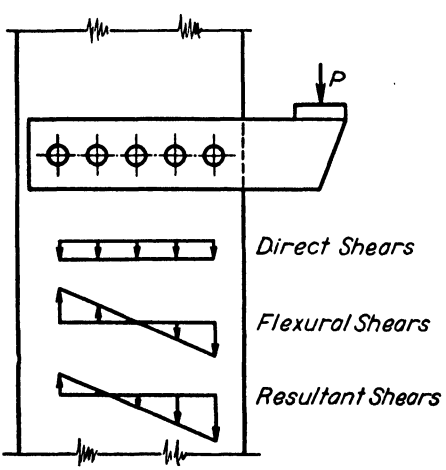


FIG. 24. DIRECT ADDITION OF RIVET SHEARS.

Design of Heavy Column Bracket, *DP*6. The rivets are shown arranged in 4 lines, the outside rows spaced 12 in. apart. This group of rivets will possess considerable moment of inertia about each axis or, in other words, its polar moment of inertia will be considerably greater than the moment of inertia about its horizontal axis. As an initial estimate, the use of equation (15) will aid us in obtaining the approximate number of rivets needed in each line.

The important point to consider in applying equation (15) to any such problem is the estimated value of S_s. For a single row of rivets resisting pure flexure, S_s is the value of one rivet. In case there is a direct load as well, S_s must be *reduced* to compensate for this effect. On the other hand, the value of S_s should be *increased* in order to allow for the moment of inertia about the second principal axis. These two influences tend to cancel each other. Therefore, in the example *DP*6, S_s is estimated as the actual value of the rivet. The check analysis shows that the resultant rivet stress is almost exactly the allowable shear. Hence, our guess at the proper value of S_s to use in equation (15) was a good one.

Instantaneous Center of Rotation. The center of rotation is located graphically on the design sheet *DS*6. We may obtain the distance x_i from the center of gravity to the center of rotation by use of equation (12).

$$x_i = \frac{\Sigma x^2 + \Sigma y^2}{en} = \frac{9900}{20.3 \times 88} = 5.5 \text{ in.}$$

Then, by geometry

$$r_i = \sqrt{11.5^2 + 15^2} = 18.9 \text{ in.}$$

The maximum rivet shear can be computed by equation (13)

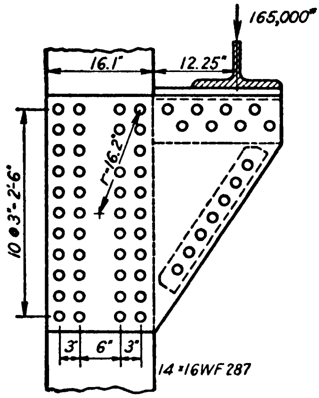
$$S_s = \frac{Per_i}{\Sigma x^2 + \Sigma y^2} = \frac{165{,}000 \times 20.3 \times 18.9}{9900} = 6400 \text{ lb.}$$

This value checks with the rivet shear of 6450 lb. obtained graphically.

32. Moment Resistance with Tension Rivets. Beam connections and brackets attached to the face of a column offer moment resistance because of the tension values of the rivets. A typical example is the beam-to-column connection by use of clip angles for the full depth of the web as shown in Fig. 25. There is a difference of opinion among designers as to which of the two following methods of analysis should be used.

1. *Assume that initial rivet tension is never exceeded* so that the flexural moment merely releases some of the bearing pressure under the tops of the clip angles (Fig. 25, Case 1) and increases the bearing pressure under the lower ends of the clip angles. The neutral axis for flexure is at the mid-height as shown in Fig. 25. This assumption implies elastic action and is not in agreement with the assumed action of either direct connections or eccentric connections as explained in § 20 and § 28. Upon this assumption, however, we already have developed all of the theory needed for the analysis and design of these connections. They may be analyzed by the use of equation (14) and designed by the use of equation (15) or equation

DP6. *Design a heavy riveted bracket connection to the faces of a 14×16WF287 column to resist a girder reaction of 165k. as shown. Use 4 lines of rivets in each face of the column. AISC spec.*



Standard Design Method:

Value of a ¾" rivet in single shear = 15,000 × 0.44 = 6600#.

Approx. number of rivets per row can be estimated by use of equation (15). In making this estimate, S_s should be decreased to allow for direct shear and increased because the rivet lines are spread laterally. Hence, it will be used as 6600.

$$n = \sqrt{\frac{6M}{S_s s}} = \sqrt{\frac{6(165{,}000 \times 20.3)/8}{6600 \times 3}} = 11.2;\ \text{try 11 rivets per row.}$$

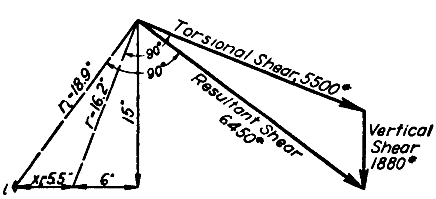
Check Analysis:

Polar moment of inertia $= \Sigma y^2 + \Sigma x^2;$

$$16[3^2 + 6^2 + 9^2 + 12^2 + 15^2] + 44[3^2 + 6^2] = 7920 + 1980 = 9900.$$

$$\text{Vertical rivet shear} = P/n = \frac{165{,}000}{88} = 1880\#.$$

$$\text{Torsional rivet shear} = \frac{Tr}{J} = \frac{(165{,}000 \times 20.3)16.2}{9900} = 5500\#.$$



Graphical Determination of Resultant Rivet Shear:

Value of resultant shear = 6450# per rivet.
The estimate of 11 rivets per row for 8 rows was satisfactory.
The instantaneous center of rotation can be located at i as indicated.

(16). The value of S_t is the tension value of the rivet in pounds, frequently taken as equal to its value in single shear.

2. *Neglect consideration of initial rivet tension* and assume that the neutral axis is at the center of gravity of the effective cross-section which consists of the circular rivet areas above the neutral axis and the rectangular bearing area of clip angles against the column face below the neutral axis.

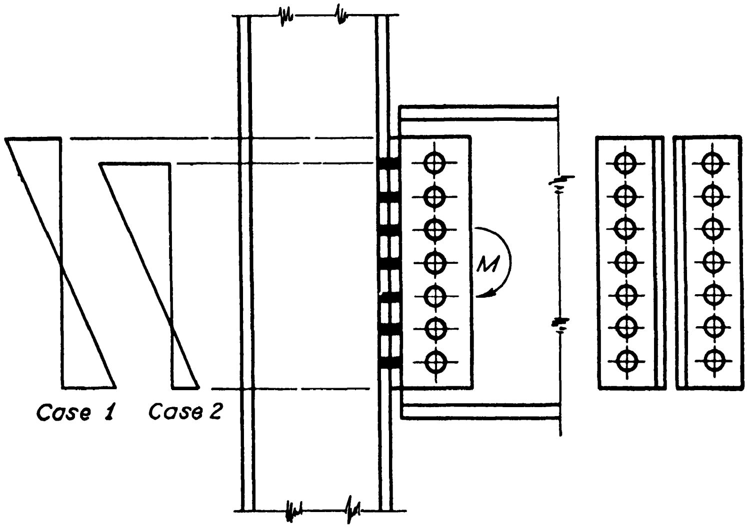


FIG. 25. MOMENT RESISTANT CONNECTION.

This procedure agrees with the method of analyzing direct riveted connections and eccentric riveted connections as discussed in § 20 and § 28. It is therefore a consistent procedure and is recommended by the author.

The method of analysis for such a connection is reasonably direct. The neutral axis will lie somewhere near *⅙ of the length of the clip angles above their lower ends.* It may be taken at this position and the statical moment of the rivet areas above this line can be compared with the statical moment of the bearing area below. An adjustment in the neutral axis should then be made to obtain an approximate balance of these statical moments. An exact location is not needed; an adjustment to the nearest half inch is common. Then the moment of inertia is determined and the tension stress in the upper rivets is computed from the flexure formula. Since actual cross-sectional areas of rivets have been used, the stress will be obtained in pounds per square inch.

SEAT-ANGLE CONNECTION, *DP7*. The design of this seat angle or unstiffened bracket involves the design of the angle leg for flexure and the design of the rivets for tension and shear. A triangular bearing pressure is assumed on the outstanding leg to account for

the fact that this leg deflects downward and therefore resists less bearing at the toe than at the heel of the angle. The length and thickness of the leg are then selected to keep the flexural fiber stress under 20,000 lb. per sq. in. *at the net section of the vertical leg.*

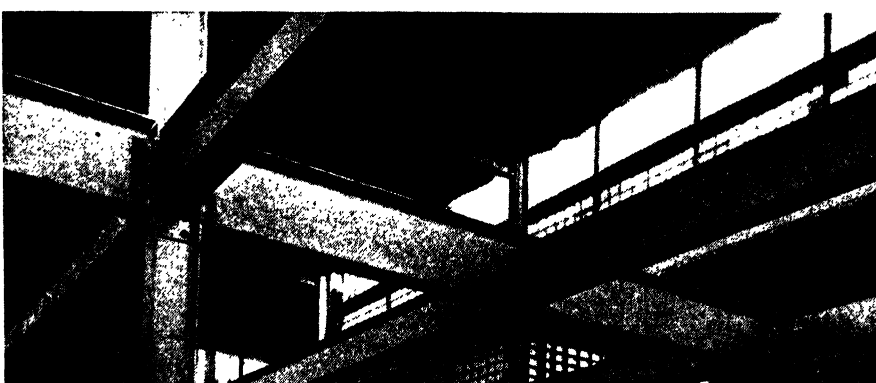
The neutral axis for flexure is assumed to be ⅞ in. above the bottom of the angle. A comparison of statical moments of rivet areas (circular areas) above the axis with bearing area (rectangular area) below the axis shows a statical moment of 3.1 against one of 2.9, a satisfactory balance for locating the neutral axis. Then, the moment of inertia is determined for this effective section and the rivet tension is computed by an application of the Mc/I formula. The rivet tension of 4800 lb. per sq. in. is very low and the vertical shear of 5700 lb. per sq. in. is not critical. However, fewer rivets would not ordinarily be used because of the danger of loosening the seat angle in shipment.

Combined Stress in Rivets. If this bracket (*DP7*) is to be designed strictly in accordance with *AISC* specifications, we should compute a maximum rivet stress by combining shear and tension. Thus by equation (19) from § 152 we obtain these results.

$$\text{Max. } s_t = \sqrt{\left(\frac{s}{2}\right)^2 + s_s^2} = \sqrt{\left(\frac{4800}{2}\right)^2 + 5700^2} = 6200 \text{ lb. per sq. in. for shear.}$$

$$\text{Max. } s_n = \frac{s}{2} + \text{max. } s_t = \frac{4800}{2} + 6200 = 8600 \text{ lb. per sq. in. for tension.}$$

Here, s is the axial rivet stress and s_s is the average unit shearing stress. The maximum unit stress s_n and the maximum unit shear s_t occur on sloping planes.



Courtesy Eng. News-Record

FIG. 26. BEAM AND GIRDER FRAMING.

33. Design of Tension Rivets for Moment Resistant Connections. The problem of design is complicated here by the fact that the widths of clip angle legs are almost unlimited. A convenient procedure will be obtained by use of standard clip angles with the rivets spaced either at 3 in. as in standard beam connections or at the minimum spacing of 3 diameters. First, however, it seems desirable to develop a general expression for section modulus of the discontinuous cross-section by representing each clip angle and its line of rivets as equivalent to the inverted *T-section* of Fig. 27. The

DP7. *Design a seat angle for a column to support an 8″ I-beam with an end reaction of 10,000#. The angle legs are 6″ and 3½″. AISC spec.*

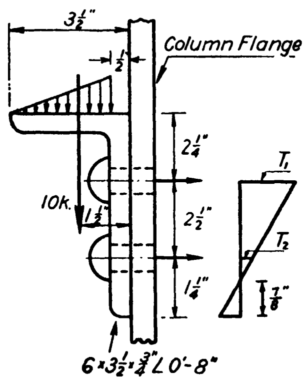
Flexure of Vertical Angle Leg at Net Section on Upper Rivet Line:

Assume thickness of angle leg to be ¾″.
Assume triangular variation of bearing pressure.

$M = 10{,}000 \times (1.5 - 0.375) = 11{,}250''\#.$

$$\text{Length of angle} = \frac{6M}{t^2 f} = \frac{6 \times 11{,}250}{0.75^2 \times 20{,}000} = 6''.$$

Set gross length at 8″ to allow for 2 rivet holes on the net section.



Shear and Tension in Rivets: *(assume 4 rivets, ¾″ diam.)*

$s_s = 10{,}000 \div (4 \times 0.44) = 5700\#/\square''.$

Guess location of N.A. at ⅞″ above bottom. (This approaches ⅑ of depth.)

Stat. mom. of comp. area $= 8.0 \times 0.87 \times 0.44 = 3.1.$

Stat. mom. of rivet areas $= 2 \times 0.44(0.37 + 2.87) = 2.9.$

This balance of statical moments is satisfactory for locating the N. A.

$$\text{Moment of inertia} = 2 \times 0.44(0.37^2 + 2.87^2) + \frac{1}{3} \times 8.0 \times 0.87^3 = 9.1.$$

$$\text{Rivet tension} = \frac{10{,}000 \times 1.5 \times 2.87}{9.1} = 4800\#/\square''.$$

Remarks: *Two rivets would have been adequate for stress, but a connection with 2 rivets is likely to be loosened in shipment. Four rivets are common in seat angles.*

breadth b may be taken as the bearing width of the clip angle. The breadth a is such that the area for a length equal to the rivet spacing will be the same as the cross-sectional area of a rivet.

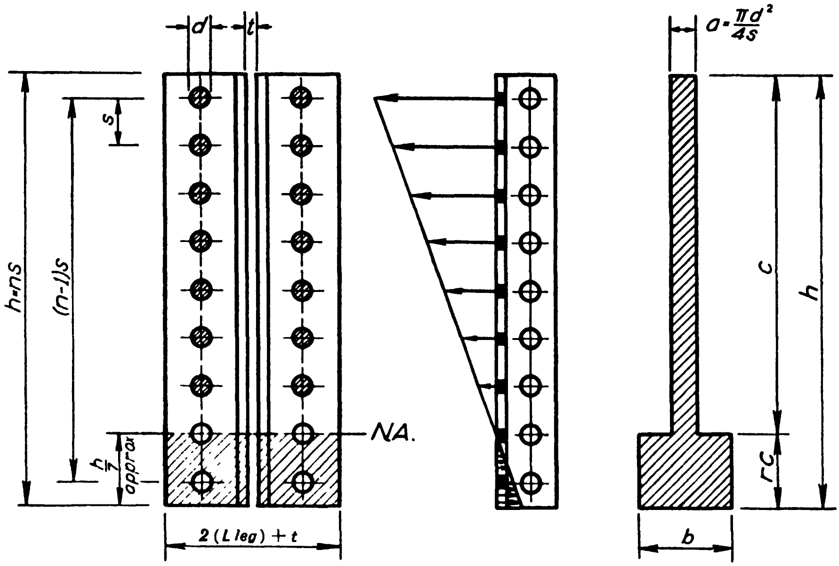


FIG. 27. ACTION OF A BEAM CONNECTION.

Formula for Modulus. For a zero statical moment about the neutral axis of the T-section, we may write

$$\frac{ac^2}{2} = \frac{b(rc)^2}{2},$$

or

(17) $$r = \sqrt{\frac{a}{b}}.$$

Then the section modulus will be

$$S = \frac{I}{c} = \frac{ac^3}{3c} + \frac{b(rc)^3}{3c} = \frac{c^2}{3}(a + br^3).$$

Substitute $\frac{a}{b}$ for r^2 to obtain

(18) $$S = \frac{I}{c} = \frac{ac^2}{3}(1 + r).$$

But, $h = c + rc = c(1 + r)$, or $c = h/(1 + r)$.

Hence, by substitution for c we get

(19) $$S = \frac{I}{c} = \frac{ah^2}{3(1 + r)}.$$

Design Procedure. In design we determine the required section modulus as the flexural moment *per rivet line* divided by the allowable unit tensile stress for rivets. Then we choose a size of rivet d and a rivet spacing s. These values control the factor a which is $\pi d^2/4s$ and the ratio r which is $\sqrt{a/b}$. The width in bearing b is the width of one angle leg. Equation (19) can then be solved for the value of h which is the height of the clip angles.

$$h = \sqrt{\frac{3S(1+r)}{a}}. \tag{20}$$

The value of h can be expressed as ns to give the required number of rivets.

$$n = \sqrt{\frac{3S(1+r)}{as^2}}. \tag{21}$$

Standard Beam Connections. Equation (21) will be applied to the "standard" and "heavy standard" beam-connection angles for which the following data apply:

(1) STANDARD CONNECTION: $d = \frac{3}{4}$ in., $s = 3$ in., $b = 3.5$ in.

$$a = \frac{\pi d^2}{4s} = \frac{\pi \times 0.75^2}{12} = 0.15,$$

$$r = \sqrt{\frac{a}{b}} = \sqrt{\frac{0.15}{3.5}} = 0.21,$$

$$n = \sqrt{\frac{3 \times 1.21S}{0.15 \times 9}} = 1.64\sqrt{S}. \tag{21a}$$

$$\text{For } \tfrac{7}{8}\text{-in. rivets, } n = 1.44\sqrt{S} \tag{21b}$$

(2) HEAVY STANDARD CONNECTION: $d = \frac{7}{8}$ in., $s = 3$ in., $b = 4$ in.

$$a = \frac{\pi d^2}{4s} = \frac{\pi \times 0.87^2}{12} = 0.20,$$

$$r = \sqrt{\frac{0.20}{4}} = 0.22,$$

$$n = \sqrt{\frac{3 \times 1.22S}{0.20 \times 9}} = 1.43\sqrt{S}. \tag{21c}$$

Effective Bearing Width of Connection Angles. It is seen from cases (1) and (2) above that the change of b from 4.0 to 3.5 in. changed the coefficient of $\sqrt{S}$ only from 1.43 to 1.44. A further reduction of b to 2.0 in. increases the coefficient only to 1.48. This fact rather effectively answers any criticism directed toward our lack of knowledge of the effective width of bearing. The width of the angle leg or any desired distance such as three or

four times the rivet diameter may be used for b with assurance that the influence upon the design will be negligible.

34. Simplified Relations for Design with Tension Rivets. The relation expressed by equation (21) can be simplified without introducing serious approximations. Where the width of angle leg b and the spacing of rivets s are equal to 4.0 times the rivet diameter, the value of r is 0.22. Its common range is from 0.20 to 0.25. We may therefore introduce its value as 0.25 in equation (21) and obtain a conservative value of n. It will also be more convenient to replace the factor a with its value of $\pi d^2/4s$. Thus we obtain

$$n = \sqrt{\frac{3 \times 1.25S}{\frac{\pi d^2 s}{4}}},$$

or

$$n = \frac{2.2}{d}\sqrt{\frac{S}{s}}. \tag{22}$$

For the usual case where the rivets are spaced 3 in. apart, we may substitute this value for s and introduce appropriate rivet sizes to obtain

$$n = 2.0\sqrt{S} \quad (\tfrac{5}{8}\text{-in. rivets}), \tag{22a}$$

$$n = 1.7\sqrt{S} \quad (\tfrac{3}{4}\text{-in. rivets}), \tag{22b}$$

$$n = 1.45\sqrt{S} \quad (\tfrac{7}{8}\text{-in. rivets}), \tag{22c}$$

$$n = 1.3\sqrt{S} \quad (1\text{-in. rivets}). \tag{22d}$$

For the closest permissible spacing of rivets (3 diameters) we obtain

$$n = 2.6\sqrt{S} \quad (\tfrac{5}{8}\text{-in. rivets}), \tag{22e}$$

$$n = 2.0\sqrt{S} \quad (\tfrac{3}{4}\text{-in. rivets}), \tag{22f}$$

$$n = 1.55\sqrt{S} \quad (\tfrac{7}{8}\text{-in. rivets}), \tag{22g}$$

$$n = 1.3\sqrt{S} \quad (1\text{-in. rivets}). \tag{22h}$$

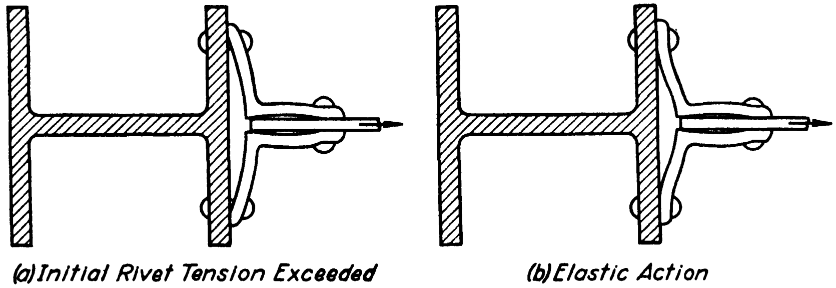


FIG. 28. ASSUMPTIONS AS TO THE ACTION OF CONNECTION ANGLES.

In all of these equations S represents the required section modulus obtained by dividing the flexural moment per rivet line in inch pounds by the allowable tension stress on a rivet in pounds per square inch.

Design of a Moment Resistant Connection to a Beam Web, *DP8a*. This connection is essentially the standard end connection for a rolled beam. Its moment resistance is in no sense equal to that of the beam, but it is of importance in several usages as, for example, to form a moment resistant joint between a floor beam and a vertical post of a low-truss highway bridge. Use is made of equation (22*c*) which applies to standard beam connections (⅞-in. rivets at 3-in. spacing). It should be noted that no attempt is made in this problem to *combine* tension and shearing unit stresses for the tension rivets. Since the working stress in tension is reduced to only one half of the single shear value, and the rivets are but lightly stressed in shear, this seems unnecessary. However, as a matter of interest, the *combined stresses* would be as follows.

$$\text{Max. } s_t = \sqrt{\left(\frac{s}{2}\right)^2 + s_s^2} = \sqrt{\left(\frac{7500}{2}\right)^2 + 7750^2} = 8600 \text{ lb. per sq. in.}$$

$$\text{Max. } s_n = \frac{s}{2} + \text{max. } s_t = \frac{7500}{2} + 8600 = 12{,}350 \text{ lb. per sq. in.}$$

These stresses are considerably under the allowable shear and tension values permitted by *AISC* specifications.

Design of Tees and Connection Angles

35. Design of Connection Angles with Tension Rivets. Contrasting assumptions regarding the structural action of connection angles are illustrated by Fig. 28. These assumptions form the bases of common design methods. In (*a*) the rivets are assumed to have elongated due to tension stress above the yield point, and the outstanding legs of the angles curve in *simple flexure*. In (*b*) the angle legs are held flat against the column by the initial tension in the rivets, and each outstanding leg bends into a *reversed curve*. The split-beam connection in Fig. 29 is shown undergoing double flexure comparable to Fig. 28(*b*). The analysis of these two types of flexure is indicated in Fig. 30.

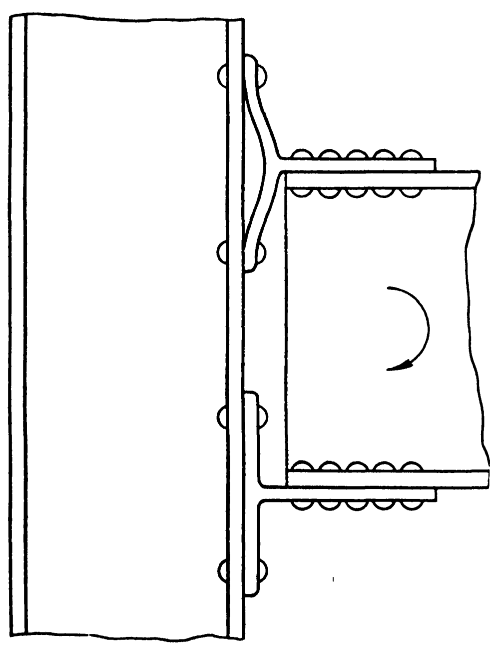


Fig. 29. Split-Beam Connection.

Simple Cantilever Flexure. In (*a*) of Fig. 30 the outstanding angle leg acts as a simple cantilever; the pull on the rivet is the total applied tension load P on a length of angle equal to the spacing or pitch of

DP8a. *Design a clip-angle connection to the face of a column for a 24WF94 beam to develop a moment resistance of 30,000′# and an end reaction of 65,000#. Use AISC spec. except reduce direct rivet tension to one half of single shear value for local code requirement.*

Tension Rivet Connection to Column Face: *For a standard rivet spacing of 3″ and for ⅞″ rivets, we may use equation (22c) to determine n.*

$$\text{Sect. mod., } S = \frac{30{,}000 \times 12}{2 \times 7500} = 24 \quad \text{(per rivet line)}.$$

$$n = 1.45\sqrt{24} = 7.1 \text{ rivets.}$$

Use a standard A7 connection (7 — ⅞″ rivets @ 3″ spacing).

$$\text{Rivet shear} = \frac{65{,}000}{14 \times 0.6} = 7750\#/\square''.$$

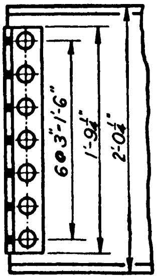
Check on Rivets through Beam Web:

Value of a rivet in bearing on web $= 0.87 \times 0.52 \times 40{,}000 = 18{,}100\#.$

I of rivet group $= 2(3^2 + 6^2 + 9^2) = 252.$

$$\text{Resultant rivet shear} = \sqrt{\left(\frac{65{,}000}{7}\right)^2 + \left(\frac{360{,}000 \times 9}{252}\right)^2} = 15{,}900\#/\text{rivet}.$$

This is satisfactory for double shear.



DP8b. *Design the connection angles (assumed as ½″ thick).*

Double Flexure of Angle Leg:

Pull on upper rivet $= 7500 \times 0.6 = 4500\#.$

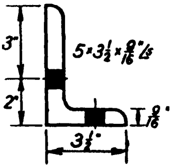
Length of angle resisting this pull $= 3\tfrac{1}{8}''.$

$$\text{Lever arm of tension rivet} = \frac{g - t}{2} = \frac{2 - 0.5}{2} = 0.75.$$

Bending moment on one angle leg $= 4500 \times 0.75 = 3400''\#.$

Thickness of angle leg, $t = \sqrt{6M/bf}.$

$$t = \sqrt{\frac{6 \times 3400}{3.12 \times 20{,}000}} = 0.57''; \text{ use } \tfrac{9}{16}'' \text{ angles.}$$



Extra rivet stress (equation 27) in tension;

$$1 + \frac{3}{4}\left(\frac{g - t}{g}\right) = 1 + \frac{3}{4}\left(\frac{1.44}{3}\right) = 1.36 \text{ or } 36\% \text{ increase.}$$

Remarks: *A satisfactory solution would be to increase the tension rivets to 1″ diameter, a 31% increase in area. Note that no allowance was made for the weakening of the angle leg by holes. It is assumed that the rivet head clamps down on the plate with sufficient force to replace this loss. The use of net width would be conservatively correct.*

the rivets. The bending moment to be resisted by the angle leg is

$$M = P(g - t). \tag{23}$$

The moment represented by this expression, where g is the gage and t is the thickness of the angle, must be resisted by a length of angle leg equal to the rivet spacing.

Double Cantilever Flexure. The moment to be resisted by the angle in Fig. 30(*b*) is computed as the bending moment of a double cantilever. There is a point of *contraflexure* midway between the rivet and the face of the angle. This bending moment is

$$M = P\left(\frac{g - t}{2}\right). \tag{24}$$

This is but one half of the moment that had to be resisted by simple cantilever action as expressed by equation (23). It has been assumed that

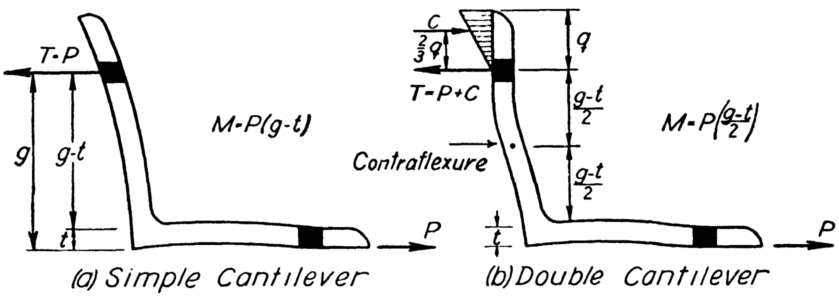


FIG. 30. ANALYSIS OF ANGLE FLEXURE.

any angle change at the heel of the angle in Fig. 30(*b*) will be balanced by an equal angle change at the outer rivet and that the point of contraflexure will therefore be as indicated by Fig. 30(*b*). For safety, some shift in the point of contraflexure should be assumed; and, therefore, equation (25) is recommended for reasonably conservative design of clip angles and split-beam connections.

$$M = 0.6P(g - t). \tag{25}$$

Increase of Rivet Stress. The 50 per cent reduction of moment shown in equation (24) or the 40 per cent reduction in equation (25) is obtained at the expense of an increased rivet stress. The tension pull in the rivet is increased from P to $(P + C)$ where C is the compression under the outstanding toe of the angle required to fix the far end of the double cantilever leg. We will call the distance from the tension rivet to the toe of the angle

leg q. Then, upon the assumption of triangular variation of toe pressure C, the arm of the couple formed by the two forces C (Fig. 30(b)) is $\frac{2}{3}q$. Hence, we may equate the expression $C(\frac{2}{3}q)$ and the moment expressed by equation (24).

$$C\left(\frac{2}{3}q\right) = P\left(\frac{g-t}{2}\right),$$

or

$$C = \frac{3}{4}P\left(\frac{g-t}{q}\right). \tag{26}$$

The tension pull on the rivet becomes

$$T = P + C = P\left[1 + \frac{3}{4}\left(\frac{g-t}{q}\right)\right]. \tag{27}$$

This is the tensile force for which the rivet in Fig. 30(b) should be designed.

36. Choice of a Design Method. For consistency with design methods used for other types of riveted joints, where initial rivet tension is always neglected, we would design connection angles and split-beam connections for simple cantilever flexure as shown in Fig. 28(a) and Fig. 30(a). The design of large beam or girder connections by these equations results in angles or split beams that are too thick to punch. They add materially to the weight and cost of the structure. It is permissible to reduce the bending moment according to equation (24) or preferably equation (25), but there is the *absolute necessity then of increasing the number or size of rivets* to meet the requirements of equation (27). This has often been overlooked and the result is a serious error.

For 4-in. clip angles, g is $2\frac{1}{2}$ in. and we will take t at $\frac{3}{8}$ in. Then

$$C = \frac{3}{4}P\left(\frac{2.5 - 0.37}{1.5}\right) = 1.06P.$$

Thus the rivet tension is increased from P to $2.06P$. Either the number or the cross-sectional areas of the rivets must be doubled.

For 6-in. clip angles, g is $3\frac{1}{2}$ in. and we will take t at $\frac{1}{2}$ in. Then

$$C = \frac{3}{4}P\left(\frac{3.5 - 0.5}{2.5}\right) = 0.90P.$$

The rivet tension has been increased from P to $1.90P$.

Rivet Tension for Split-Beam Connection. For a split-beam connection made from a $30WF180$ section, where the flange width is 15 in., the web is 0.67 in., and the gage is $5\frac{1}{2}$ in., we may let P represent the tension load on a

pair of offset rivets in the flange and write without special derivation

$$C = \frac{1}{2}\left[\frac{3}{4}P\left(\frac{2.75 - 0.33}{7.5 - 2.75}\right)\right] = 0.19P.*$$

The rivet tension has been increased from $0.5P$ to $0.69P$.

37. Design Problems in Clip-Angle and Split-Beam Connections. The two common types of riveted connections, for use where moment resistance is needed, are (1) the use of clip angles to the web and (2) split-beam tees attached to the flanges of the beam or girder. The mistake that has been made most frequently has been to design the rivet arrangement carefully but to specify the thickness of the connection material as ⅜ in. or 7⁄16 in., without study, where double these thicknesses would have been more nearly correct.

Clip Angles to the Web of a Beam, *DP8b*. These connection angles have been estimated as ½ in. thick. It is assumed that double cantilever flexure can be depended upon. The angle leg outstanding is set at 5 in. in order to increase the quantity q (projection beyond the rivet) which helps to hold down the increase in rivet stress that accompanies *double cantilever flexure*. It is found that the required thickness of the angle leg is 9⁄16 in. (it would be 13⁄16 in. for simple cantilever flexure). However, it is important to note that there is also an increase in rivet tension ($\frac{3}{4}(g - t)/q$) which amounts to 36 per cent. The proper solution would be to increase the tension rivets to 1 in. diameter (a 31 per cent increase of area) but some designers would prefer to add extra rivets on a second gage line in the 5-in. leg of the angle. The latter solution is of doubtful value since the second line of rivets can hardly be stressed appreciably before the first line is rather seriously distorted. See Fig. 31 As a minimum precaution, the second line of rivets should be discounted at least 50 per cent in tension value.

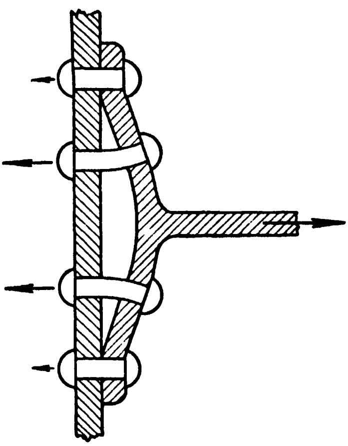


Fig. 31. Questionable Use of Tension Rivets on Four Gage Lines of a Split-Beam Section.

Clip Angles as Tension-Plate Connection, *DP9*. Here the connection angles are first designed as simple cantilevers and found to be 1⅛ in. thick. Such thickness of metal requires drilling. Hence, the use of extra rivets to permit the assumption of double cantilever flexure may be more economical. It is found that the angles can then be reduced to a thickness of ¾ in. but that they must be increased in length from 6 to 9 in. to hold 3 tension rivets in each angle leg instead of 2 rivets. Probably the cost is not actually reduced, but the design with ¾-in. metal seems a more practical solution to the problem. This is true because there are no angles rolled of 1⅛-in. thickness with 4-in. legs. It would be necessary to reduce 8-in. angles to 4 in.

* For a special derivation of the formula for rivet tension in a split-beam connection, see Vol. 1, *Theory of Modern Steel Structures*, p. 275; also see Transactions, *ASCE*, 1933, p. 734 for a published discussion by the author.

Split-Beam Tees for Beam Connection, *DP*10. The use of split-beam tees is common as a means of providing wind-moment resistance for bracing tall buildings. Sometimes the end-moment resistance desired is only a fraction of the full moment resistance of the beam. Under such circumstances the split-beam connection is ideal. In this design problem the calculations are made for simple cantilever flexure. This is necessary because the increased number of 1⅛-in. rivets required to produce double cantilever flexure could not be accommodated in this connection, and larger rivets than 1⅛ in. could not be driven without special equipment. The result is that a flange thickness of 1.49 in. for the split-beam tee is needed for simple cantilever flexure and this requires a 36*WF*280 section. Thus the weight of the connection at each end of the beam is found to be 385 lb., a 40 per cent addition to the weight and cost of the beam if the span is assumed to be around 20 ft.

It is shown in this design problem that the rivet tension before the elastic limit of the rivet is exceeded is controlled by double cantilever flexure and is 15 per cent greater than the allowable rivet tension. This overstress is not objectionable since the design is consistent for simple cantilever flexure and the rivet stress will therefore reduce when *rivet stretch* permits simple curvature of the split-beam flanges as illustrated on the design sheet. The rivets are not designed to resist vertical shear or combined shear and tension because a web connection will be provided to resist the end shear of the beam.

Repeated Stresses

38. Fatigue Tests of Riveted Joints. Results of fatigue tests of actual riveted joints were reported in 1938.* These tests were the first to be

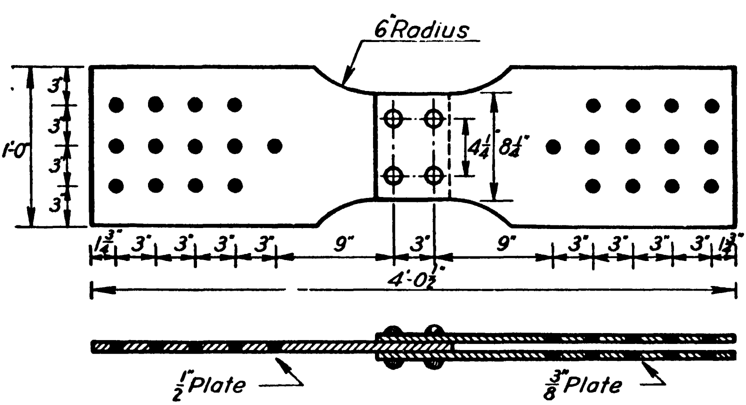


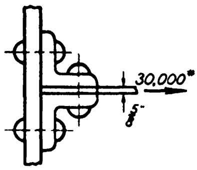
Fig. 32. Fatigue Test Specimen.

completed on other than miniature specimens. A detail of one reasonably typical test specimen is shown in Fig. 32. Failure was produced through the center section joined by 4 rivets for the particular case illustrated.

The fatigue limit was defined arbitrarily to be the greatest stress to which a joint can be subjected 2,000,000 times before failure. Naturally,

* W. M. Wilson, *Fatigue tests of riveted joints*, Civil Engineering, Aug. 1938, pp. 513–516; also Bulletin 302, University of Illinois Engineering Experiment Station.

DP9. *Design a pair of clip angles not over 9″ long to resist a pull of 30,000# in a plate 9″ × 5/8″.*



Shear Rivets: ***On 5/8″ metal, double shear controls.***

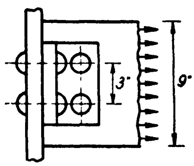
Allowable shear = 15,000#/□″. (AISC).

$$n = \frac{30,000}{2 \times 0.6 \times 15,000} = 2 \text{ rivets } (7/8'').$$

Tension Rivets: *For single cantilever flexure;*

Value in tension = value in single shear. (Spec. 10.)

Use 4 tension rivets.



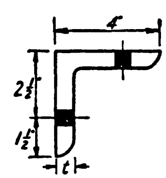
Thickness of Connection Angles: *(Use 4″ × 4″ angles.)*

Length = 6″ to hold rivets at 3″ spacing.
(Assume t = 3/4″.) g = 2½″.

$$M = \frac{30,000}{2} \times (2.5 - 0.75) = 26,200''\#.$$

$S = 26,200 \div 20,000 = 1.31''^3.$

Hence, $\frac{1}{6} \times 6 \times t^2 = 1.31$, *or* $t = \sqrt{1.31} = 1\frac{1}{8}''.$



Metal 1⅛″ thick must be drilled. Design will be revised for double cantilever flexure. Extra rivets will require longer angles. Assume angles to be 9″ long and 3/4″ thick.

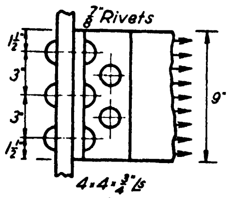
Double Flexure:

$$M = 0.6 \times \frac{30,000}{2} (2.5 - 0.75) = 15,700''\#.$$

(Equation 25)

$$S = \frac{15,700}{20,000} = 0.78''^3, \text{ hence, } \frac{1}{6} \times 9 \times t^2$$

$$= 0.78, \text{ or } t = \sqrt{0.52} = 0.72''.$$



Use angles 4 × 4 × 3/4″ × 0′ − 9″ long.

Rivet Tension: *Equation (27).*

$$T = P\left(1 + \frac{3}{4}\frac{g - t}{q}\right) = \frac{30,000}{2}\left[1.0 + 0.75\left(\frac{2.5 - 0.75}{1.5}\right)\right] = 28,000\# \text{ for}$$

each line of rivets.

Tension value of a 7/8″ rivet = 0.6 × 15,000 = 9000#.

$n = 28,000 \div 9000 = 3.1;$ *use 3 rivets 7/8″.*

DP10. *Design a split-beam connection to develop a bending moment of one half of the resisting moment of a 24WF100 beam. AISC spec.*

Rivets:

Sect. mod. of a 24WF100 = 248.9.

Resisting moment = 248.9 × 20,000 = 4980″k.

Pull on tee $= 0.5 \times \dfrac{4{,}980{,}000}{24}$

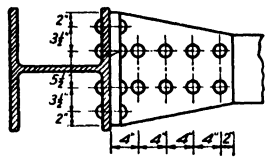
$= 104{,}000\#.$

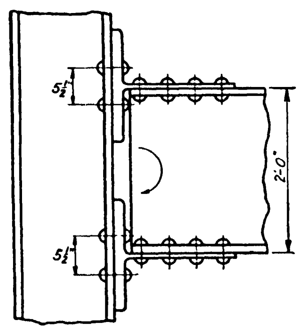
For 2 lines, the max. number of rivets is 8.

Tension per rivet $= \dfrac{104{,}000}{8} = 13{,}000\#.$

Value of a 1⅛″ rivet in tension = 1.0 × 15,000 = 15,000#.

Shear rivets to beam flange have same value as tension rivets. (Spec. 10.) Use 8 1⅛″ rivets to each flange.





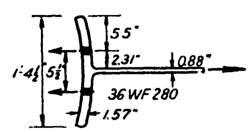
Thickness of Tee Flange:

Length of tee = 16½″.

Gage of tee = 5½″.

Assume web to be ¾″ thick,

$M_{approx.} = \dfrac{104{,}000}{2} \times \dfrac{4.75}{2} = 123''k.$



Sect. mod. $= \dfrac{123{,}000}{20{,}000} = 6.1 \text{ in.}^3$

$\therefore \tfrac{1}{6} \times 16.5 \times t^2 = 6.1, \quad or \quad t = \sqrt{\dfrac{6.1 \times 6}{16.5}} = 1.49''.$

Use a 36WF280, 1′-4½″ long, for 2 tees.

Double Flexure: *This connection cannot be lightened by designing for double cantilever flexure because the rivets would be overstressed and they cannot be increased in number. Before elastic failure, the rivet stress will be controlled by equation (27).*

$$T = P\left[1 + \frac{3}{4}\left(\frac{g-t}{g}\right)\right] = 104{,}000\left(1.0 + 0.75\,\frac{2.31}{5.5}\right) = 138{,}000\#.$$

Tension per rivet = 138,000 ÷ 8 = 17,200# (15% overstressed).

most specimens would not fail exactly at this number of applications and, therefore, an empirical formula (justified by previous tests with miniature polished metal specimens) was used to determine the fatigue limit (F) from the number of applications (N) of stress (S) that produced failure. This equation is

$$F = S\left(\frac{N}{2,000,000}\right)^{0.10} \tag{28}$$

In this equation, F and S refer to tension or compression stresses without reversal.

Failure in the Rivets. Those specimens designed to fail by rivet shear indicate that the *fatigue limit of ordinary rivets* is 30,000 lb. per sq. in. This figure signifies that a fatigue shear failure can be expected at 2,000,000 repetitions whenever the rivet shear varies from 0 to 30,000 lb. per sq. in. for a non-reversing load. Results with a reversing load (direction of shear on rivet reversed) are not very consistent since they show a variation of fatigue limit from as little as 15,000 lb. per sq. in. to as great as 30,000 lb. per sq. in. So many variables seemed to be involved in the case of full reversal, that further tests were considered necessary.

Failure Through the Plates. Plate failure always started at a rivet hole and developed as indicated in Fig. 33. These tests show quite conclusively that the average stress on the net section which will produce failure at 2,000,000 repetitions is approximately 26,000 lb. per sq. in. Strangely enough, this value seems to be independent of the kind of steel used or of its ultimate strength since it held constant for carbon, silicon, and nickel steels (ultimate strengths varying from 63,000 to 99,000 lb. per sq. in.). (See Table 3.) Furthermore, the method of making rivet holes — punching, sub-punching and reaming, or drilling — had no measurable effect upon the fatigue strength of the plates.

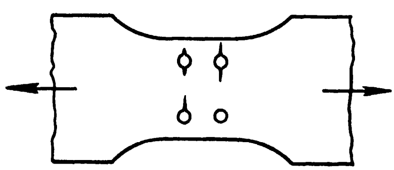


FIG. 33. FATIGUE CRACKS.

TABLE 3

FATIGUE STRENGTH AND ULTIMATE STRENGTH

MATERIALS	FATIGUE STRENGTH	STATIC STRENGTH
Carbon-steel rivets and plates	25,900 lb. per sq. in.	63,600
Carbon-steel rivets and silicon-steel plates	25,600 " " " "	80,200
Carbon-steel rivets and nickel-steel plates	26,700 " " " "	99,000
Manganese-steel rivets and silicon-steel plates	27,800 " " " "	80,200

Correction for Ratio of Minimum to Maximum Stress. Tests on small polished solid specimens have indicated that the following expression defines the fatigue strength (F) for any ratio (r) of minimum to maximum stress. The fatigue strength (F') for a value of $r = -1$ (complete reversal) is assumed to be known. Table 4 shows how the actual tests on structural joints agreed with the expression

$$F = \frac{3F'}{2 - r}. \tag{29}$$

It is highly significant that complete reversal of a normal working stress of 20,000 lb. per sq. in. from tension to compression will produce failure at 2,000,000 cycles of stress. This number of reversals could readily occur in industrial buildings and is even a possibility in bridge structures.

TABLE 4

FATIGUE LIMIT FOR VARIATIONS OF STRESS CYCLE

Stress Cycle	Ratio r	No. of Tests	Average Fatigue Strength (lb. per sq. in.)	
			(Observed)	(Eq. 29)
Full reversal	-1	4	19,700	19,700
Zero to maximum	0	5	28,600	29,500
One half of maximum to maximum in same direction	$+\frac{1}{2}$	3	39,000	39,400

PROBLEMS

1. Determine the edge distance by eighths for rivets from ⅝ to 1 in. in terms of the plate thickness so that a shearing failure as indicated in Fig. 5(*c*) would be less likely than (*e*) shear failure of the rivet or (*f*) bearing failure against the plate. *AISC* working stresses. *Ans.* Compare with $d + \frac{1}{2}$ in.

2. Repeat Problem 1 for *AASHO* working stresses.

3. Repeat Problem 1 for 1½-in. rivets and *AREA* working stresses.

4. Determine the initial tension that would be set up by the cooling of a rivet from below the softening temperature (600° F.) to room temperature (70° F.) if the rivet can stretch under stress but the plates gripped cannot be compressed. Use grips of 1 in., 2 in., and 3 in. *Ans.* Stress exceeds yield point.

5. Revise the calculations of Problem 4 upon the assumption that 1-in. rivets are spaced 3 in. apart in both directions or the net plate area resisting compressive deformation is 10.5 times the rivet area resisting tension. *Ans.* Stress still exceeds yield point.

6. Connect a 6 × ⅜-in. plate directly to the face of a heavy column to resist 30,000 lb. of plate tension. Select size and number of rivets for *AISC* spec. *Ans.* One solution is four ⅞-in. rivets.

7. A lap joint in a tank carries a stress of 2000 lb. per lineal in. The plates are ¼ in. thick. Find the spacing for ¾-in. rivets. Use *AISC* spec. *Ans.* Rivets at 3-in. spacing.

8. Revise Problem 7 to allow for the changed conditions of a butt strap joint with ¼-in. straps. *Ans.* Rivets at 3¾-in. spacing.

9. A truss member consists of a 12*WF*50 beam section placed between two 5⁄16-in. gussets. (*a*) Find the number of ¾-in. field rivets through flanges and gussets that will develop a tension stress of 196,000 lb. *AASHO* spec. (*b*) Suggest revision by studying Table 2. *Ans.* (*a*) Four rows of 10 rivets per row.

10. Revise the example *DP*1*a* for a load of 120,000 lb. Transfer 40 per cent through the rivets. Use *AASHO* spec. *Ans.* Eight ¾-in. rivets.

11. Revise the example *DP*1*b* for a column cap load of 24,000 lb. from each I-beam. Use *AASHO* spec. *Ans.* Eight ¾-in. rivets.

12. Revise the example *DP*2 for a 20 per cent reduction of load, shear, and moment. Use *AASHO* spec.

13. Design a column base for an 8*WF*48 section carrying a 160,000-lb. central load. Transfer 50 per cent of the load through the riveted detail with ¾-in. rivets. *AASHO* spec. *Ans.* 14 rivets required; 16 rivets for practical arrangement.

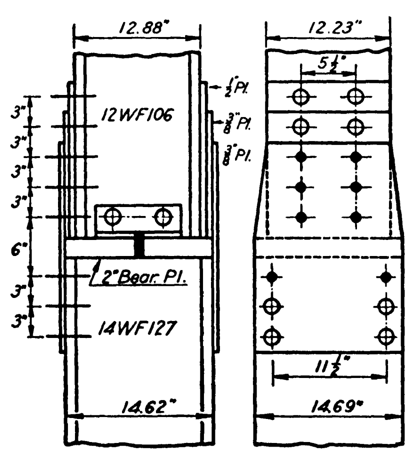
14. Design a column cap for the column of Problem 13 to seat a 20-in., 65.4-lb. I-beam whose end reaction is 47,000 lb. Turn the web of the column parallel to the web of the beam. Use ¾-in. rivets. *Ans.* Use 6 × 3½ × ½-in. angles and 8 rivets.

15. Design a column cap for the column of Problem 13 to seat four 20-in., 65.4-lb. I-beams in two lines spaced at 10 in. on centers. Each beam has an end reaction of 44,000 lb. This is an inside column carrying a line of double I-beams. Let the column web be parallel to the webs of the double beams. Use *AISC* spec.

Ans. Twenty ⅞-in. rivets with both plates and angles.

16. Design a column splice between a 10*WF*66 section and a 12*WF*92 section to transfer 250,000 lb. Splice plates must transfer 25 per cent of the load. Use *AISC* spec. *Ans.* Eight ⅞-in. rivets to one side of splice.

17. Revise Problem 16 to allow for a shear of 30,000 lb. and a bending moment of 500,000 in-lb. at the splice.

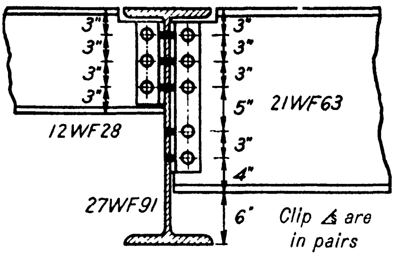


PROBLEM 18.

18. Check the column splice detail illustrated to find what percentage of the total column stress could be transferred by the riveted detail. This is a typical column splice arranged with a 2-in. bearing plate which actually transfers nearly all of the load. Use ⅞-in. rivets and *AISC* spec. Neglect *L*/*r* factor.

Ans. 20 per cent across and 34 per cent through bearing of fill plates.

19. Design a splice between a 12*WF*99 column section and a 14*WF*127 column section to transfer 70 per cent of the value in direct compression (short column), 35 per cent of the value in flexure, and 25 per cent of the shear value (web) of the smaller section. *AISC* spec.



PROBLEM 20.

20. Determine the allowable end reaction of each beam connection shown. Consider the fact that these reactions may occur either separately or together. Use ⅞-in. rivets, *AISC* spec. Do not allow for eccentricity of the rivets through the beam webs.

Ans. 25 k. and 72 k.

21. A 1½-in. square bar is welded between two ⅝-in. plates which are field riveted through the 1½-in. flange of a heavy beam. (*a*) Design the riveted connection to develop the bar according to *AISC* spec. Consider net section. (*b*) How would the design be affected if the beam flange is only 15/16 in. thick and you must use two ¼-in. fill plates?

Ans. (*a*) Place three ⅞-in. rivets through plate in form of an isosceles triangle 3 in. on a side. Plate must be 2.8 in. wide at first rivet hole.

22. Revise the examples *DP*3*a* and *DP*3*b* by interchanging the specifications used in the two cases for the calculation of the net section.

23. Design a riveted connection to attach two 5 × 3 × ⅜-in. angles by the long legs on opposite sides of a 5/16-in. gusset plate for the net value of the angles in tension. Revise the design to allow for eccentricity. Use ¾-in. rivets and *AREA* spec.

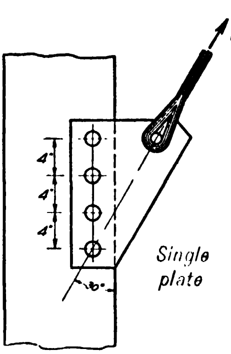
24. Design a riveted connection to attach two 4 × 4 × 7/16-in. angles on opposite sides of a ⅜-in. gusset plate for the net value of the angles in tension. Revise the design to allow for eccentricity. Use ⅞-in. rivets and *AISC* spec.

Ans. 9 rivets at 3-in. spacing are acceptable.

25. Revise the example *DP*4 for 5 × 3 × ½-in. angles and ⅜-in. gussets.

26. Revise the example *DP*4 for 6 × 4 × 9/16-in. angles, 7/16-in. gussets and *AREA* spec.

27. Find the size of rivet necessary to resist a pull of 11,000 lb. in the tie rod for the detail illustrated. Allow 15,000 lb. per sq. in. for rivet shear which controls the design. *Ans.* ⅝-in. rivets.



PROBLEM 27.

28. Extend the rivet line of Problem 27 upward to hold a total of 6 rivets at 4-in. spacing. Find the allowable rod stress for ¾-in. rivets at 13,500 lb. per sq. in. for single shear.

Ans. 20 k.

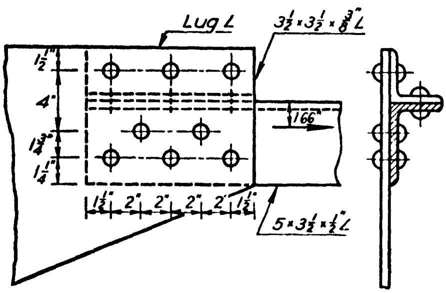
29. Revise the example *DP*5 for 5 × 3 × ⅜-in. angles and *AISC* spec.

30. Revise the example *DP*5 for 4 × 4 × ⅜-in. angles.

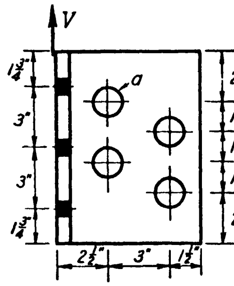
31. Design the lug-angle connection illustrated for a 5 × 3½ × ½-in. angle to develop 52,000 lb. of tension stress with ⅞-in. rivets at 13,500 lb. per sq. in. in shear. Analyze the connection for eccentricity to determine the resultant rivet shear. Lug-angle rivets may be considered 100 per cent effective in resisting moment but only 50 per cent effective in resisting direct load. This latter factor influences the line of action of the resisting force. The need for the lug angle is due to the fact that the main angle cannot project more than 11 in. onto the gusset plate. Note that many specifications

prohibit any allowance for lug angles in resisting direct load although all permit their use to resist moment of eccentricity.

Ans. Connection shown is adequate for direct load and eccentricity.



PROBLEM 31.



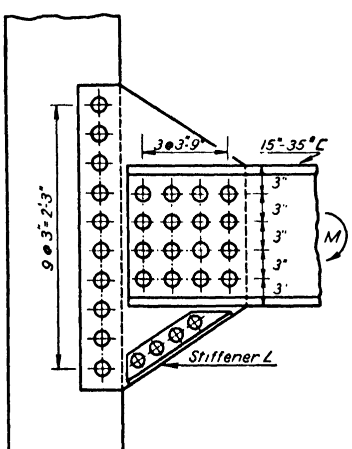
PROBLEM 32.

32. Find the allowable shear V for the connection shown in order to stress the extreme rivet a to its value of 15,000 lb. per sq. in. (1-in. rivets) in double shear. This stress is assumed to control the design. *Ans.* $V = 33$ k.

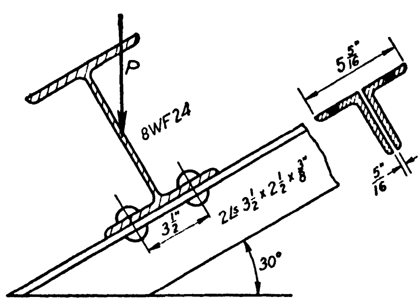
33. Revise the example *DP*6 for a load of 150,000 lb. at 10 in. from the edge of the column.

34. Revise the example *DP*6 for a load of 238,000 lb. and ⅞-in. rivets.

35. Design a connection of the general type illustrated for attaching a 15-in., 35-lb. channel to a heavy column flange for 60 per cent of the full value of the channel in flexure at 16,000 lb. per sq. in. fiber stress. Unit rivet shear = 12,000 and unit bearing = 24,000 lb. per sq. in. Use ⅞-in. rivets. The number of rivets shown is not necessarily correct.



PROBLEM 35.

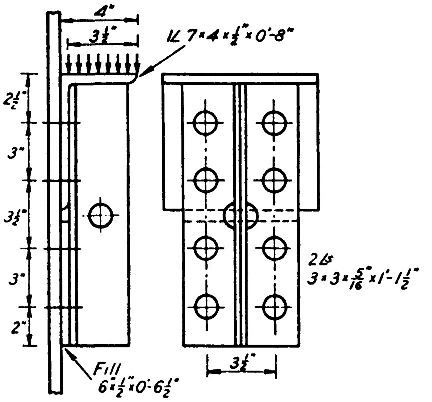


PROBLEM 37.

36. Revise example *DP*7 for a 6 × 4-in. angle and a load of 12,000 lb. *AASHO* spec.

37. Determine the vertical load P, acting through the center of gravity of the purlin section, that will stress the two upper ¾-in. rivets to 15,000 lb. per sq. in. in resultant stress. Combine shear and tension as specified in the *AISC* code. (Refer to the final paragraph of § 32.) *Ans.* 24 k.

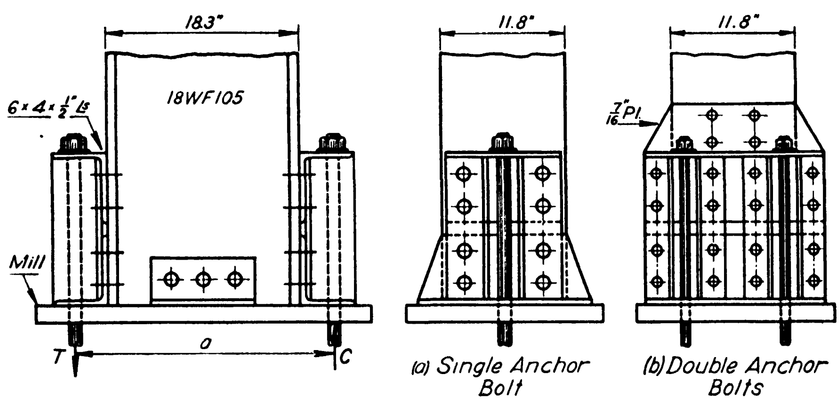
38. Determine the size of rivets that will make the stiffened bracket shown carry a 30,000-lb. end reaction of a beam, distributed uniformly over 3½ in. Combine shear and tension as specified in the *AISC* code. (Refer to the final paragraph of § 32.) Allow the combined rivet stress to reach 15,000 lb. per sq. in.



PROBLEM 38.

39. Revise Problem 32 to allow for the moment resistance of the tension rivets. (1-in. diameter) through the outstanding legs of the angles. This moment resistance reduces the twisting moment acting on the four staggered rivets. Allowable stress in tension is the same as in shear.

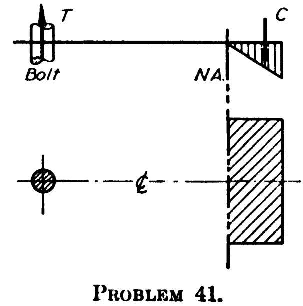
40. Design a riveted base connection to resist 1,600,000 in-lb. of moment for the 18*WF*105 section shown in the illustration. The details will be similar to those shown. Assume that the flexural moment will be resisted by a couple having an arm equal to the distance between anchor bolts. Select all material for the base detail according to *AISC* spec. You may choose between (*a*) single anchor bolt and (*b*) double anchor bolts.



PROBLEM 40.

41. Repeat Problem 40 but select the base plate so that the pressure on the concrete below the plate does not exceed 600 lb. per sq. in. The effective section is the tension area of the anchor bolt on one side of the neutral axis and the bearing area of the plate on the other side of the neutral axis as illustrated. The statical moments of these areas are balanced about the neutral axis.

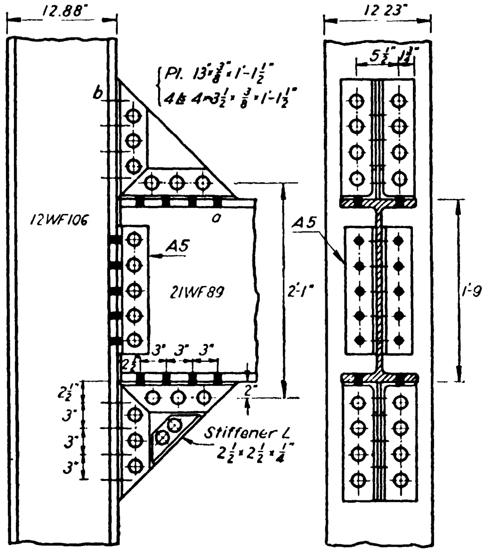
42. How would your design for Problem 41 be revised if the vertical load is of such magnitude that the stress under one edge of the base plate is zero when the stress at the opposite edge is 600 lb. per sq. in.?



PROBLEM 41.

43. Revise the example $DP9$ for a pull of 40,000 lb. in a plate 12 × ⅝ in.

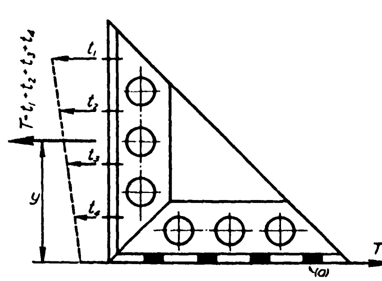
44. Design a 45° bracket connection similar to the one shown in the illustration. The end wind moment in the beam is 2,000,000 in-lb. By *AISC* specifications, rivets for wind resistance may be stressed 33 per cent in excess of normal working stresses. The



PROBLEM 44.

*A*5 web connection resists the vertical shear. Assume that the rivets through the beam flanges resist only horizontal shear and that the tension rivets to the column resist stress in proportion to their vertical distances from the center line of the beam. Maintain rivet tension at 75 per cent of the single shear (*AISC*) value to meet requirements of a local building code.

45. Restudy the design of Problem 44 on the basis that the bracket must be in equilibrium. The couple Ty must be resisted by the rivets along the horizontal leg of the bracket *if the rivet stresses along the vertical leg are as computed.* Determine the maximum tensile rivet stress at (*a*) on the basis of the theory used in the final paragraph of § 32. Observe that there is a reduced shear on the horizontal row of rivets equal to the force T.



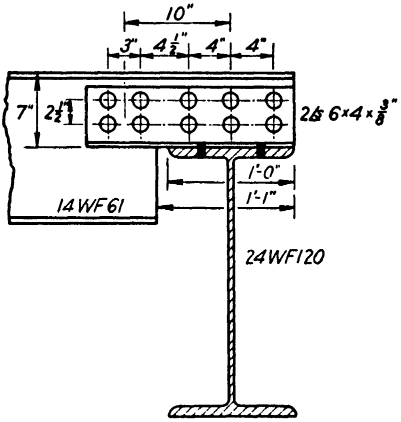
PROBLEM 45.

46. Revise the example $DP10$ for a $27WF114$ beam.

47. Revise the example $DP10$ for 60 per cent of the moment resistance of a $16WF78$ beam.

48. Analyze the seat-angle connection shown for an end reaction of 55,000 lb. Assume that all reaction passes through the 6 rivets directly above the girder and that as much shear is transferred by the seat angles across the cut as is necessary to hold the web shear to the allowable value. Assume that the transferred shear produces a flexural moment because of the effective arm of 10 in. between the rivet groups, and that 67 per

cent of this moment is resisted by the group of 6 rivets above the girder, the remaining 33 per cent being resisted by the 4 rivets in the beam. Choose a rivet size according to *AISC* specifications and increase the number of rivets if needed.



PROBLEM 48.

49. Find the instantaneous center of rotation for the detail of Problem 27. Use its location to check the required size of rivet. *Ans.* ⅝-in. rivets.

39. Review of Riveting Theory. The point has been made repeatedly through the various sections of this chapter that the design of riveted joints might be based either upon *elastic conditions* (dependent upon initial rivet tension) or upon conditions existing *near failure* (rivets permanently deformed). Actually, initial rivet tension produces sufficient joint friction to resist the applied load whether centric or eccentric so that the rivets in most joints probably never act in shear and bearing, according to their design, at all. Allowable tension in rivets is always limited to about one half of the initial tension that tests have proved to exist; therefore, applied tension does not destroy joint elasticity or even joint friction. Finally, cross shear and applied tension have been combined to obtain either a maximum shear or a maximum tension. In each case discussed it would seem, at least superficially, that elastic conditions were governing the design.

To obtain a true insight into the action of a simple tension joint, however, we must consider the distribution of shear among the rivets of a row that resists a longitudinal load. The high shears initially resisted by the end rivets of the row can only be reduced by an overload that strains the rivets and plates beyond the yield point. Furthermore, for a moment resistant connection (with tension rivets) it will be recalled that we located the neutral axis at the *center of gravity of the effective section* of tension rivet areas and bearing area, a position of the neutral axis that will not occur until

after the tension rivets have passed the yield point. It should also be recalled that the member, and the plates to which it is attached, are considered to be fully effective (net section in tension) despite the high concentrations of stress that are known to exist around rivet holes (particularly in tension members) which can only be ironed out by straining the material well beyond the yield point. Rivets resisting cross shear should not be weakened by a small applied tension (below the initial tension) since the applied tension only cancels an equal amount of initial tension, but, near failure, initial tension disappears and applied tension does weaken a rivet in ultimate shear resistance.

Failure as a Basis for Design. The structural engineer cannot set a fixed rule as to whether all structural parts should be designed upon the basis of elastic conditions or for conditions near failure. It seems reasonably clear that the proper criterion is whether the structural part adjusts its action to become *relatively stronger* or *relatively weaker* as the yield point is passed. If the structural part becomes relatively weaker, failure conditions *should* be considered; if it becomes relatively stronger, failure conditions *may* be used with a proper factor of safety, and a conservative design for static loads will be obtained. This criterion of design may be illustrated by a comparison of tension and compression members. Tension members become relatively stronger beyond the yield point (stress concentrations disappear and the tensile strength increases); compression members become relatively weaker and buckle. We are always willing to stress tension members more heavily than compression members, and, as working stresses have been raised, allowable tension stresses have been increased more rapidly than allowable compression stresses. Another example may be found in the rapid increase that has occurred in allowable bearing stresses. Metal in bearing is *confined* and will become relatively stronger as the yield point is passed.

Riveted joints seem in general to fall into the category of structural parts that become relatively stronger as the yield point is passed. Shear variations between rivets reduce or disappear whether caused by plate distortion, member eccentricity, or merely by irregularities in fabrication. A bracket connection undergoes a shift in the neutral axis that increases its moment resistance. Rivets highly stressed in tension stretch and pass their loads on to rivets of lower stress. Applied tension, which weakens a shear rivet near failure, may be taken into consideration by use of a combined stress. With these points in review, we reach the conclusion that conditions at failure may properly and consistently be used to direct the design of riveted joints *acting under static loading.*

The study of plasticity as a basis for design must be limited to members and details that do not undergo reversal or fatigue. Reversal is a very

severe test upon a riveted joint in that a slight slip repeated in two directions may eventually wear the joint out and loosen all the rivets. Reversal is covered in most specifications by a reduction of working stresses or by an increase of the effective design loading. Fatigue is important where the number of repetitions approaches two million. Both in the design of members and in the design of joints, stress concentrations as found by the theory of elasticity should govern the design of parts subjected to fatigue loading.

CHAPTER 3

WELDED CONNECTIONS

40. Arc Welding Process. Structural welding may be either gas welding or electric arc welding. However, the electric arc process is now so widely used that its name has become almost synonymous with the term *structural welding.* The basic procedure in electric arc welding is very

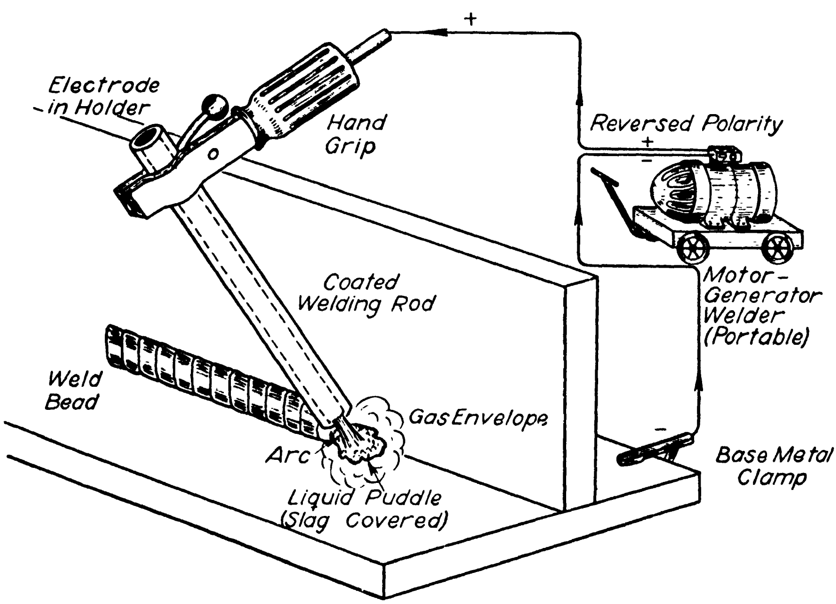


FIG. 34. WELDING PROCEDURE WITH HEAVILY COATED ELECTRODE.

simple. Electric current, usually direct current, provides the welding heat through the medium of an electric arc. One terminal of the direct current generator is connected to the base metal and the other terminal is connected to the electrode or welding rod through an insulated electrode clamp or holder which the welder grips in his hand. See Fig. 34. Depending upon the choice of type of electrode and other factors, the positive terminal of the generator may be attached either to the base metal or to the electrode, producing respectively *straight* or *reversed polarity.*

The type of work to be done, the required welding speed, the penetration of the weld metal into the base metal, and the physical characteristics of the weld determine the choice of such factors as size of electrode, type of electrode covering, voltage, and current. These matters are usually left to the welder or to the welding superintendent. The designer is interested primarily in the results obtained rather than in the devices used to obtain them.

The Electrode. The designer must realize, however, that many important properties of the weld such as strength, ductility, and resistance to corrosion are changed by the choice of welding rod. Two different types of rods are available and of course there are unlimited commercial variations of these two types, (1) washed or *lightly coated electrodes* which produce rather brittle welds and (2) *heavily coated electrodes* that produce tough ductile welds which are, in general, also stronger and more resistant

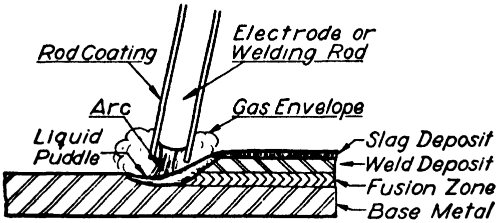


FIG. 35. THE WELDING PROCESS.

to corrosion than those produced by washed electrodes. Structural welding of the best quality is produced by heavily coated electrodes. No other type should be permitted for important structural work unless tests are used to determine that the qualities of the weld obtained are satisfactory. Unquestionably, some of the early failures of structural welding were due to the use of bare electrodes which produced very brittle welds with little resistance to impact.

There are three desirable actions of a proper rod coating: (1) The coating burns away slowly and extends below the rod, thus directing and shielding the upper part of the arc. (2) The fused coating gives off an inert gas that envelops the lower part of the arc and keeps away atmospheric oxygen and nitrogen which always act to embrittle the weld. (3) The slag remaining after the burning of the rod coating floats on top of the weld and protects it from the atmosphere while it is cooling.

As indicated by Fig. 34 and Fig. 35 the weld rod must be fed into the weld, but the weld is really composed of a mixture of the base metal and the electrode metal. Since the temperature of the arc is 6500° F., it is evident that the weld can be made to penetrate a considerable distance into the

base metal — this *penetration* being controlled by the current consumed. Incidentally, the electrode metal is carried across the arc in minute droplets or as a metallic vapor. This metal acquires a velocity that is used by the welder to stick it in position when he is doing *overhead welding.* Such welds are slower to produce and more expensive, but they are as strong as ordinary welds.

Arrangement of Structural Welds

41. Kinds of Welds. Besides overhead welds mentioned in the paragraph above, there are *flat, horizontal,* and *vertical welds.* (A group of

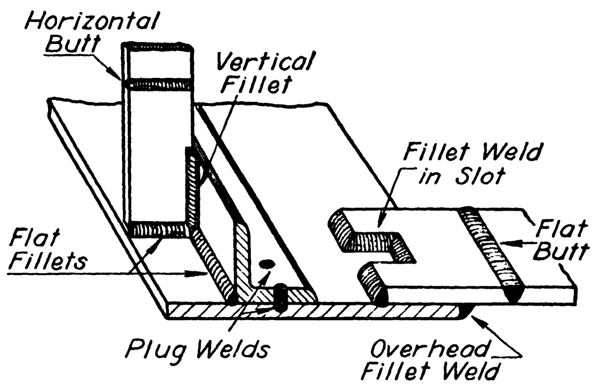


Fig. 36. Types and Positions of Welds.

important definitions is given in Spec. 122.) These are all illustrated in the composite picture, Fig. 36. For economy we would choose the flat weld first, the horizontal weld second, the vertical weld third, and the overhead weld last. Good design will avoid overhead welding in almost all instances.

Butt and Fillet Welds. Another distinction between types of welds refers to fillet welds and butt welds. Specifications 129 and 130 define these terms. Several of each are shown in Fig. 36. A butt weld may be acting only in direct tension or compression while a fillet weld, being placed on the side or edge of the base metal, is undergoing shear as well as tension or compression and usually flexure besides. Fillet welds are nearly all of one type or shape since they are usually placed in a right angle formed by two plates or two structural members. The plain tee shown in Fig. 37 is actually formed by two fillet welds. The other welds shown in Fig. 37 are butt welds. The single vee is produced by burning the edge of the plates away with a torch after which the weld bead is placed in one or more passes. For thicker plates the volume of weld metal becomes too great for the use of a single vee and it is progressively re-

duced by changing to the single U, the double vee, and the double U. Each of these joints can now be prepared for welding largely by use of

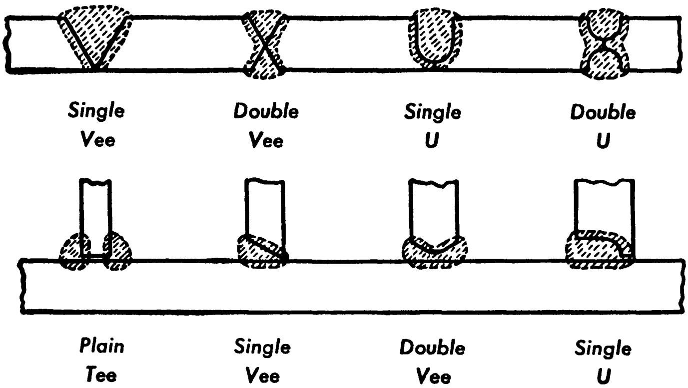
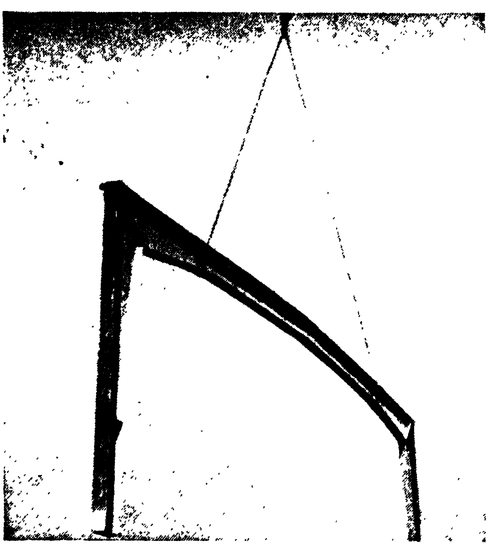


Fig. 37. Kinds of Butt Welds.

an automatic torch, but, of course, the single vee is prepared most simply and cheaply.



Courtesy Eng. News-Record

Fig. 38. Erection of Welded Frame.

42. Direct Structural Connections. Although the butt weld shows consistently greater strength than the fillet weld, structural connections are produced largely by fillet welding. Even direct connections of structural shapes as illustrated by Fig. 39 are most likely to be produced by the use of fillets since this saves the operation of scarfing or vee-ing the ends of the members. Such direct connections are not particularly common in structural frames. The butting of sections directly together as in Fig. 39 necessitates the cutting of the members to *exact length* and presupposes that the other parts of the frame will be fitted together so perfectly that the length of each member will be found to be exact in the field. Field engineers know that this is seldom true even in riveted structures where it is common practice to pull the structure together with *drift pins*. Hence, direct connections are usually reserved for small jobs where only a few

members are being joined or where members can be shipped long and burned to the necessary length in the field. A direct connection can also be used at one end of a beam and the necessary " play " can be taken

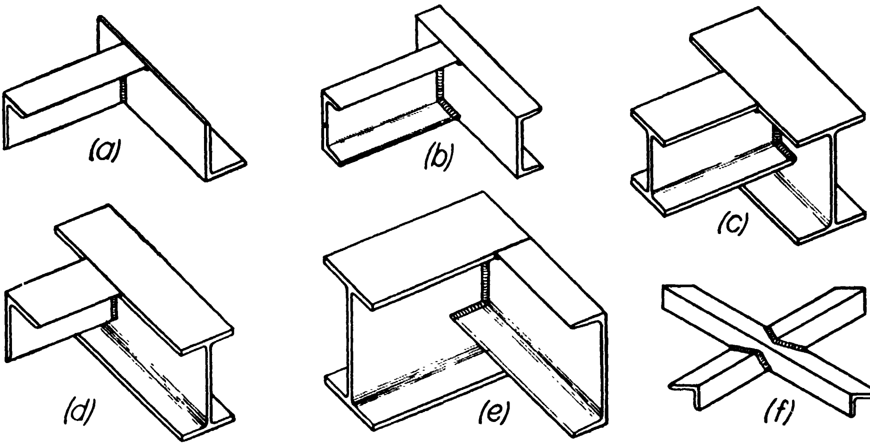


FIG. 39. DIRECT CONNECTION OF STRUCTURAL SECTIONS BY WELDING.

(*a*) Angle to Angle; (*b*) Channel to Channel; (*c*) Coped Beam to Beam; (*d*) Coped Angle to Beam; (*e*) Coped Channel to Beam; (*f*) Crossing Angle Diagonals. Direct connections are not commonly used for main members, but they are used for stair-well connections and other less important details. They are also useful in reconstruction work.

up at the other end by the use of extension plates or other devices. Since the direct connection is always the cheapest, field practices should be devised to make this method of construction more common.

43. Beam Connections Permitting Adjustment of Length. The connections of Fig. 40 are for attaching beams to columns by resting the beams on welded seats. The actual connection of the beam to the seat is by two location bolts, but the seat is structurally welded to the column to resist the end reaction of the beam. If desired, the bolt holes can be slotted in one member for easy alignment of the columns, and the beams can be field welded to the brackets after alignment. This would increase the stiffness of these connections greatly.

If there is moment resistance needed at the end of the beam (designed as a continuous frame) then a connection must also be made to the top flange of the beam of the type shown in Fig. 41. A point to be noted is the use of plate stiffeners between the flanges of the column section to prevent flexure of the column flange to which the beam connection is made. The *tie plate* at the top is butt welded to the column and fillet welded to the beam. Its odd shape is such that groups of these plates can be cut from a single large plate without wastage. The beam may or

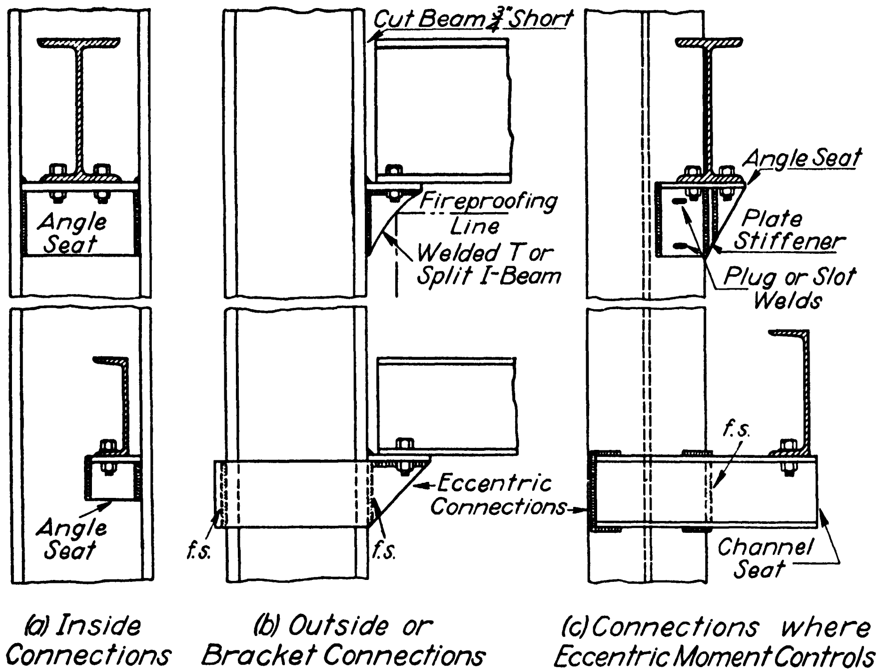


FIG. 40. WELDED BEAM-TO-COLUMN CONNECTIONS.

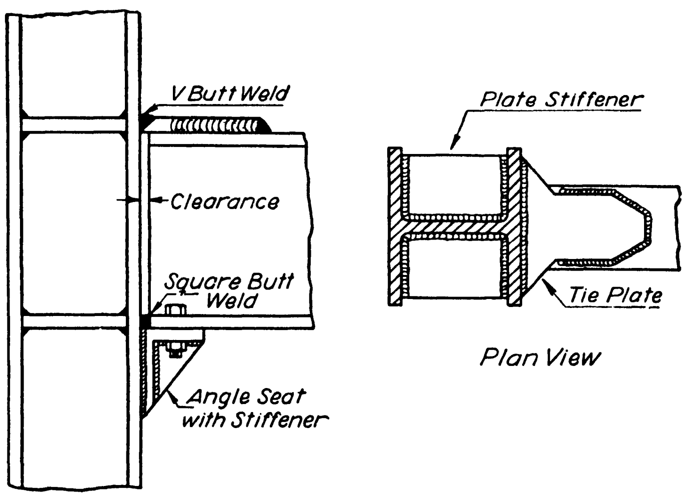


FIG. 41. MOMENT RESISTANT CONNECTION.

may not be fillet welded to the angle seat depending upon whether the square butt weld shown is adequate to transfer the compression caused by negative moment in the beam.

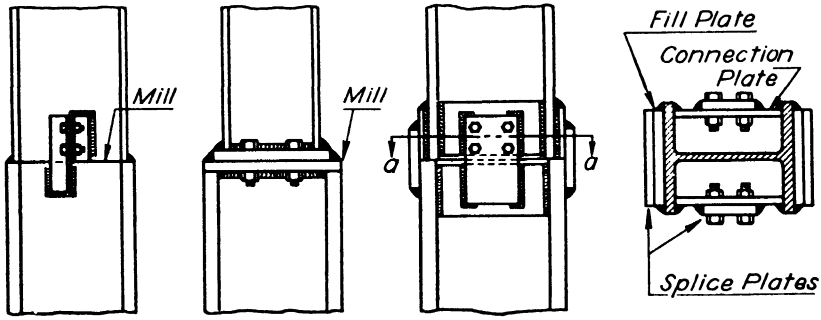
Courtesy Eng. News-Record

FIG. 42. WELDED BEAM-TO-COLUMN WIND CONNECTION WITH RIVETED CLIP ANGLES TO BEAM WEB.

44. Column Splices and Bases. A welded column splice like a riveted splice is not expected to be designed for the full compressive value of the member. A large part of the compression should be transferred by *direct bearing*. (Spec. 128.) Where all of the stress could be transferred by direct bearing of milled ends, the requirements of a satisfactory splice are a pair of connection angles for aligning the columns during erection and sufficient weld to produce a stiff connection. As a rule of thumb for producing adequate stiffness, the welds may be designed to transfer 25 per cent of the direct column load. This type of connection is illustrated in Fig. 43(*a*).

In many splices the entire load cannot be transferred by bearing of the upper section upon the lower one. In such instances there is use for horizontal bearing plates as shown in Fig. 43(*b*). A column that undergoes severe flexure must be spliced to resist tension as well as compression. Either of the splices of Fig. 43(*a*) or (*b*) could be designed to resist some flexure, but Fig. 43(*c*) shows a splice that will resist heavy flexural moments about *either axis* of the column. By care in choosing the welds that are

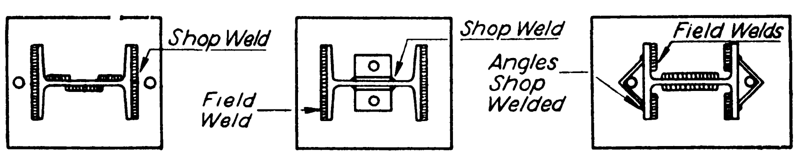
to be produced in the field, it is possible to attach each splice plate to one member or the other in the shop. Loose plates are always to be avoided by preference.



(a) Light Sections (b) Heavy Splice (c) Flexure Splice (d) Section a-a of (c)

FIG. 43. WELDED COLUMN SPLICES.

Column Bases. The cheapest column base for light construction is a welded plate of the type shown in Fig. 44(*a*). Such rolled plates may become damaged in shipment and would not then present a satisfactory flat bearing surface for heavy loads. If *loose base plates* (usually milled for bearing) are desired for this reason, a connection such as Fig. 44(*b*) may be used where clip angles are shop welded to the column but where



(a) Shop Welded Base Plate for Light Column (b) Field Welded Base with Shop Welded Angles (c) Welded Base where Heavy Anchorage is Required

FIG. 44. WELDED COLUMN BASES.

the column is welded to the base plate after erection in the field. The base plate may or may not have additional bolts anchoring it to the foundation.

The detail of Fig. 44(*c*) has been used successfully to provide real moment resistance at the column base. It is assumed that the flexural moment is about the vertical axis in the illustration. The heavy anchor bolts pass upward through the " hold-down " angles that are shop welded to the column

flanges. A thick washer plate rests on top of the angle and the anchor-bolt nut bears on this plate. Dual anchor bolts on each column face produce *moment resistance about both axes* of the column.

STRESS ANALYSIS FOR WELDS

45. Analysis of Stresses in Welds.* The actual analysis of internal stresses in unsymmetrical welds would be a very complicated procedure involving the mathematical theory of elasticity. However, for purposes of design, much simpler analyses based upon the usual direct stress formula $f = P/A$, the flexure formula $f = Mc/I$, the torsion formula $s_s = Tr/J$, and the shear formula $s_s = VQ/It$ are considered adequate. When we realize that design must be based upon theoretical working stresses that are reduced to allow for indeterminate fabrication and erection stresses (shrinkage after welding has a major influence), we conclude that such approximate design analyses are probably satisfactory.

If the structural connections shown in Figs. 39 to 44 inclusive are studied, it will soon appear that these connections form groups of welds that may be investigated individually. For example, the butt weld connecting the tie plate at the top of the beam to the column in Fig. 41 is acting in direct tension if the beam section is resisting negative end moment. The end shear of the beam passes down through the seat angle and must be resisted by its welds. Similarly, the end connection in (*b*) of Fig. 39, where the channel is welded " all around," would be assumed to have the same stress distribution that exists in the channel section itself. By such methods we will be able to analyze complicated structural connections as soon as we have investigated the action of the basic units of welding.

46. Use of the Direct Stress Formula. The assumption is made for all butt welds under direct tension or compression that the unit stress in the weld is equal to the total load divided by the net effective area, which is the length of the weld times the minimum or *throat* dimension. Although variation in the amount of heat applied, or variation in rate of cooling because of change of thickness of the metal may invalidate this assumption, it is useful in the analysis of a large majority of direct stress welds. *Ductility of the weld* will permit equalization of such stresses before failure. Heavily coated electrodes producing ductile welds are to be preferred for structural work where annealing is impossible after the welding is finished.

SPECIAL CASES. In Fig. 45(*a*) the two plate welds are placed in single-vee and double-vee notches. In either case the unit stress is the total load P divided by the net area, which is ab for the single-vee weld and

* Also see *Welding Design*, C. H. Jennings, Transactions *ASME*, Oct. 1936, pp. 497–509.

$(a_1 + a_2)b$ for the double-vee weld. No allowance is made for the bulge of the weld in producing excess throat area, and, in fact, it would be better to avoid such a bulge entirely since the result is a change of cross-section which produces an inevitable concentration of stress. In work of high grade, such welds are often *chipped* or *ground* flat after welding. The expense precludes this refinement in structural work, but good welding will show little bulge beyond the plate.

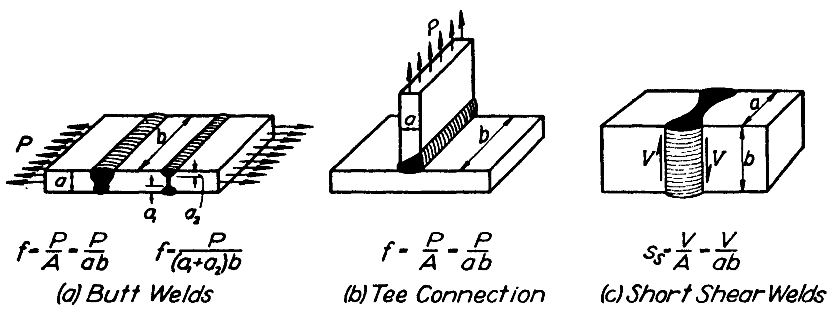


FIG. 45. USE OF DIRECT STRESS FORMULA.

There is no particular difference in the action of the weld of Fig. 45(*b*) from the similar weld in (*a*). Naturally, the weld is of a different shape since only one plate can be beveled. This is known as a *bevel groove* weld. Again the tension stress is taken as the load divided by the net area of the throat without allowance for any rounding of the weld surface.

The short shear welds of Fig. 45(*c*) are assumed to act under uniform shear. Hence, the unit shearing stress in this bar would be V/ab, or the shear divided by the net throat area. This is a double-vee weld where penetration joins the two parts. This arrangement is in contrast to the right-hand weld of Fig. 45(*a*) where the vees are not joined together. If the shear welds of Fig. 45(*c*) are considerably longer, the shear formula may need to be applied as will be seen in § 49.

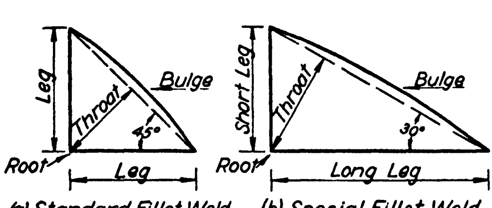
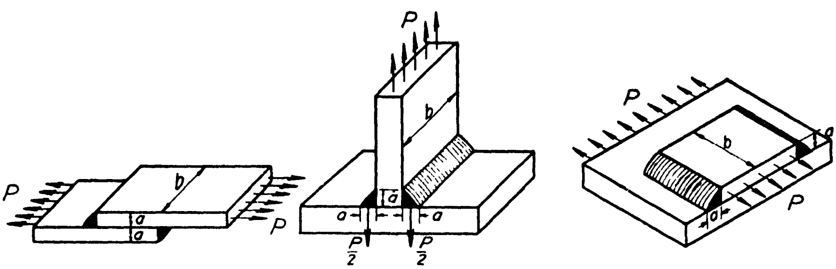


FIG. 46. THROAT ON CONTROLLING DIMENSION OF FILLET WELDS.

Direct Loads on Fillet Welds. The same relationship is used for computing the critical stresses in butt and fillet welds, that is, the total load is divided by the net area at the throat or smallest section of the weld. Evidently, for the standard 45-degree fillet of Fig. 46(*a*), the throat is 0.707 times the length of the side or leg of the weld. *The throat is the minimum dimension.* Parts (*a*), (*b*), and (*c*) of Fig. 47 illustrate the usual action of fillet welds.

Clearly, the lap welded plates in (*a*) undergo flexure and shear as well as direct stress since the two loads P are eccentric by the amount of the thickness a, but the welds are probably no more heavily stressed than those in (*b*) where the resisting forces $P/2$ are also eccentric by the distance $a/2$ on each side of the plate. Such eccentricity does not produce crossflexure in (*c*) where the welds have been turned through 90 degrees with respect to the load. However, tests as well as theory indicate that the shear distribution along welds in line with the load is



$f = S_s = \dfrac{P}{.707(2ab)} = \dfrac{.707P}{ab}$ $S_s = \dfrac{.707P}{ab}$ $S_s = \dfrac{.707P}{ab}$

(*a*) Fillet End Welds (*b*) Fillet Tee Welds (*c*) Fillet Side Welds

FIG. 47. DIRECT LOAD ON FILLET WELDS.

far from uniform so that the common practice is to treat all welds of Fig. 47 as if they were stressed equally. The controlling stress in a fillet weld is *shear* instead of tension or compression. The three stresses may occur in about equal intensities, but since the allowable stress in shear is always less, it controls the design. Failure seems to occur by shear at 45 degrees along the throat section. See § 52 for an analysis of internal stresses in fillet welds.

Unequal Thicknesses of Plates and Welds. When the plates in Fig. 47 (*a*) are of different thicknesses and each weld is of the same thickness as its corresponding plate, we assume that the *unit stresses* are the same in the welds. This is equivalent to an assumption that the unit stresses are the same in the two plates or that the load between the welds is divided between the plates in proportion to their relative thicknesses. See Fig. 48 where this relationship is indicated for the plate stresses irrespective of the thicknesses of the welds. The justification for this assumption is clearly that *the two plates must stretch equal amounts between the two welds;* hence, their unit deformations and unit stresses must be equal and their total stresses become proportional to their relative thicknesses. This evidently leads to the following relationship.

$$\text{Total stress in the plate of thickness } a_2 = P\frac{a_2}{a_1 + a_2}.$$

Unit throat stress in the fillet weld of thickness a_3 is obtained as follows:

(1) $$s_s = \left(P\frac{a_2}{a_1 + a_2}\right) \div 0.707a_3b = \frac{1.414Pa_2}{(a_1 + a_2)a_3b}.$$

For special cases this formula gives rise to the stress relationships shown on Fig. 48.

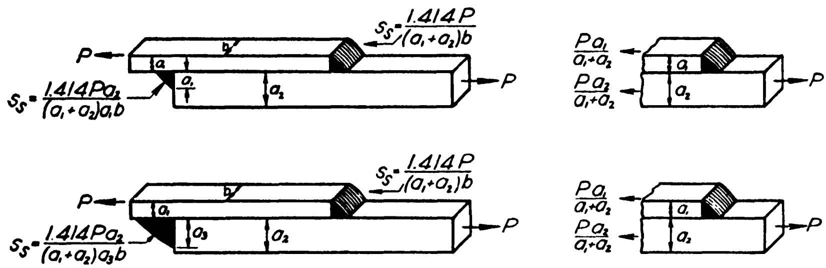


FIG. 48. DIVISION OF LOAD BETWEEN WELDS.

47. Use of the Flexure Formula. All calculations based upon the flexure formula partake of its *fundamental assumptions* which are: (1) that strain has a straight-line variation increasing from the neutral axis outward, and (2) that stress is proportional to strain. These assumptions give rise to the common relationship, $f = Mc/I$.

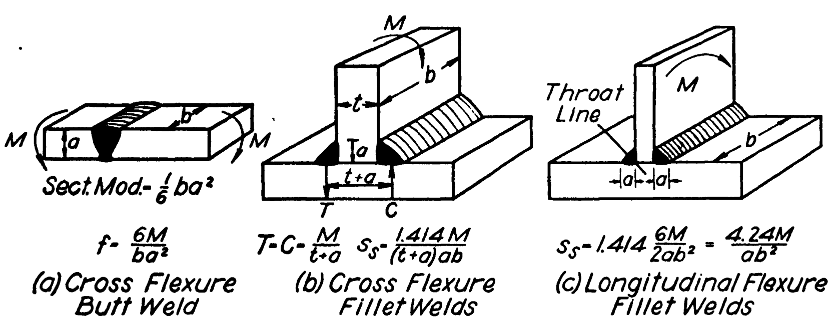


FIG. 49. FLEXURE RESISTED BY STRUCTURAL WELDS.

There should be no particular question as to the proper use of this formula for the determination of the stresses in the butt weld shown in Fig. 49(*a*). By discounting the influence of the weld reinforcement (surface bulge) we conclude that the stresses in the weld should be identical with the stresses in the plate, which are known to be represented quite closely (below the elastic limit) by the flexure formula.

Some little exercise of the imagination is necessary to visualize the proper application of the flexure formula to the fillet welded joint of Fig.

49(c). The actual failure and therefore the critical stresses exist on the throat of the weld or at about 45 degrees to the dimension a as shown on the sketch. However, since the throat of the weld has a length of 0.707 times the leg a, we may compute the stress on a section $2a$ in width and b in length and then multiply this stress by 1.414 to represent the maximum stress on the throat. Thus, we write

$$s_s = 1.414M \div \frac{2ab^2}{6} = \frac{4.24M}{ab^2}. \tag{2}$$

The throat stress is then treated in design as a shear since the 45-degree line of failure is indicative of a shear failure and since the throat is a line of high shearing stress as will be shown in § 52.

Discontinuous Cross-Sections. The analysis for the tee connection of Fig. 49(b) might be based upon the flexure formula by use of the section modulus for a discontinuous cross-section, but this would indicate a maximum stress at the toe of the weld (extreme fiber) while actually the maximum shearing stress as well as the maximum tensile stress seem to occur at the root of the weld. Accordingly, the approximate method indicated by Fig. 49(b) is more used. This procedure is to represent the weld stresses as a couple with an arm equal to the clear distance between the welds plus the length of one leg, or $t + a$. The applied moment M on the length b, divided by the arm $t + a$, is the force on one weld for the length b. This force divided by the nominal area ab of the weld may be multiplied by 1.414 to represent the design shear on the throat. Hence, we write

$$s_s = 1.414\frac{M}{t + a} \div ab = \frac{1.414M}{(t + a)ab}. \tag{3}$$

48. Use of the Torsion Formula. The flexure formula may be looked upon as a special case of the torsion formula in certain of its applications, that is, when the cross-section is long and narrow so that the polar moment of inertia reduces essentially to the moment of inertia. For instance, in Fig. 50(c), the torsional shearing stress is computed by the flexure formula, while, for the circular weld of Fig. 50(a), the torsion formula is used. It should be realized that only in these two cases, that is, (1) the circular cross-section, and (2) the long narrow cross-section, can torsional stresses be computed in a simple manner with reasonably accurate results. The torsion of square, rectangular, angle, channel, and I-beam cross-sections has undergone individual study, the results of which would have to be investigated for such special cases. The important thing to realize is that the ordinary torsion formula is exact *only for circular cross-sections* and that it becomes equivalent to the flexure formula for *long narrow cross-sections.*

Discontinuous Cross-Sections. Short fillet welds widely separated may be analyzed with reasonable accuracy as illustrated by Fig. 50(b). If there are more than two pieces of weld, or if those two approach in length the distance between them, the only tool of analysis reasonably available is the torsion formula. The use of the torsion formula in such a case may be a valuable *guide,* but it should not be thought that the procedure is in any sense exact. The weld at the greatest distance from the center of rotation (c. g. of weld area) may not be the most highly stressed. In particular, internal corners may be highly stressed by torsion. The only safe procedure in such a case is to double or triple the computed stress, depending upon the degree of resemblance of the discontinuous cross-section to a circle or a circular ring, for which the analysis would be correct. Another criterion may be found in the stiffness of the structure to which the welds are attached. For example, if this structure is a heavy plate, which will be little distorted by the action of the welds, it will be able to maintain the *shear distribution proportional to the radius* assumed by the torsion formula.

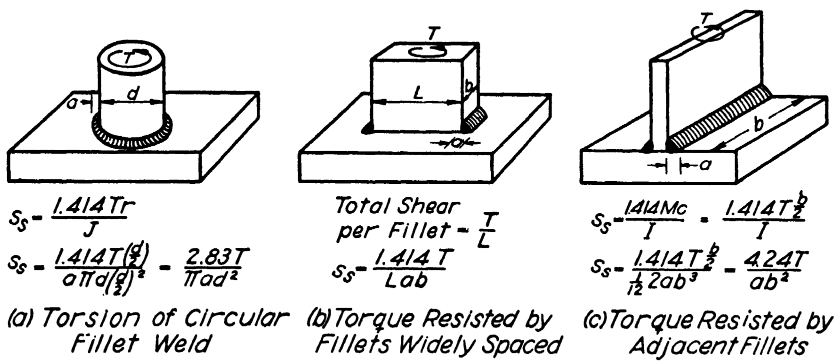


FIG. 50. TORSION RESISTED BY FILLET WELDS.

49. Use of the Beam-Shear Formula. In Fig. 45(c) it was assumed that the shear in the short welds of the butt welded bar would be uniformly distributed. This assumption is usual in design although it is not in agreement with the beam theory. According to the beam theory, fiber stresses have a straight-line variation from the neutral axis outward and this assumption requires that the distribution of horizontal (or vertical) shear be parabolic over a *rectangular cross-section.* The maximum shear (at the neutral axis) therefore, is 1½ times the average shear. It should be noted that this distribution is for shear that accompanies beam flexure.

For irregular cross-sections, as, for example, where the shape of the weld follows the outline of a structural section, such as a channel or an

I-beam, the procedure in shear computation (as in Fig. 51(*a*)) is to make use of the beam-shear formula

$$S_s = \frac{VQ}{I}. \tag{4}$$

In this formula, S_s is the shear per lineal inch of weld, V is the total shear at the cross-section, I is the principal moment of inertia (computed as in.³ for a weld of unit thickness), and Q is the statical moment of the area

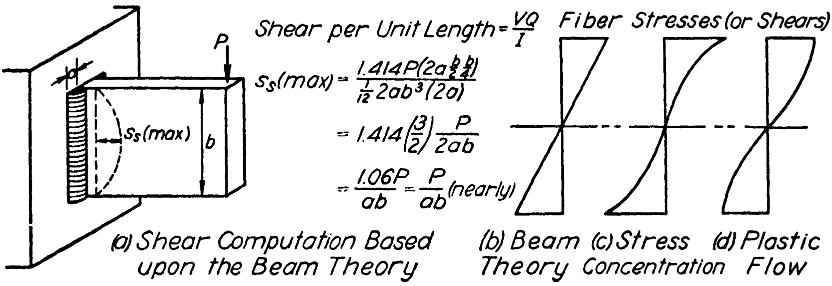


FIG. 51. VARIATION OF SHEAR IN BEAM WELDS.

The parts (*b*), (*c*), and (*d*) need explanation. The usual assumption of a straight-line variation of flexural stress is illustrated in (*b*). However, due to abrupt changes of cross-section at the ends, we might anticipate concentrations of stress as shown by (*c*). After plastic flow has taken place at the ends and extended inward, the picture of stress variation will approach the condition illustrated by (*d*). Since weld tension is accompanied by equal throat shear, the curves (*b*), (*c*), and (*d*) of this figure may also be interpreted as distribution curves for horizontal shear on the throats of the welds.

of weld section (again of unit thickness) outside of the line on which S_s is being computed, when taken about the neutral axis. Knowing the shear in the weld per lineal inch S_s, we can rapidly transfer this value into unit shear s_s on the throat section if it is desired.

50. Combined or Maximum Stresses in Butt Welds. It will be evident that all of the possibilities for combined stresses exist in butt welds that exist in materials of any kind, because the metal of a butt weld takes the place of an equivalent amount of base metal. Hence, we may have any of the stress combinations commonly studied in an advanced course in the strength of materials and summarized in § 152. Since the working stresses in butt welds are different for tension, compression, and shear, the maximum values of each must be determined to find the critical design stress for the weld.

From another point of view, however, the design of a structural butt weld has often been simplified by *dependence upon the design of the structure itself.* It is common knowledge that by special workmanship (annealing, X-raying for defects, etc.) welds can be obtained that will reproduce the strength of the structural steel joined by the welds. The use of such welds

would make a study of the weld stresses unnecessary if the structure itself had been designed properly. Combined stresses in the welds are significant, however, where the allowable stress in the weld either in tension, compression, or shear is less than the corresponding allowable stress in the base metal.

51. Combined Stresses in Fillet Welds. Two studies of combined stresses will be made:

1. Where two load systems produce collinear stresses that add directly.
2. Where two load systems produce stresses 90 degrees apart that combine into a resultant.

The classification (1) is always important and the addition of such stresses is always proper. A few cases are illustrated in Fig. 52. In (*a*) we find

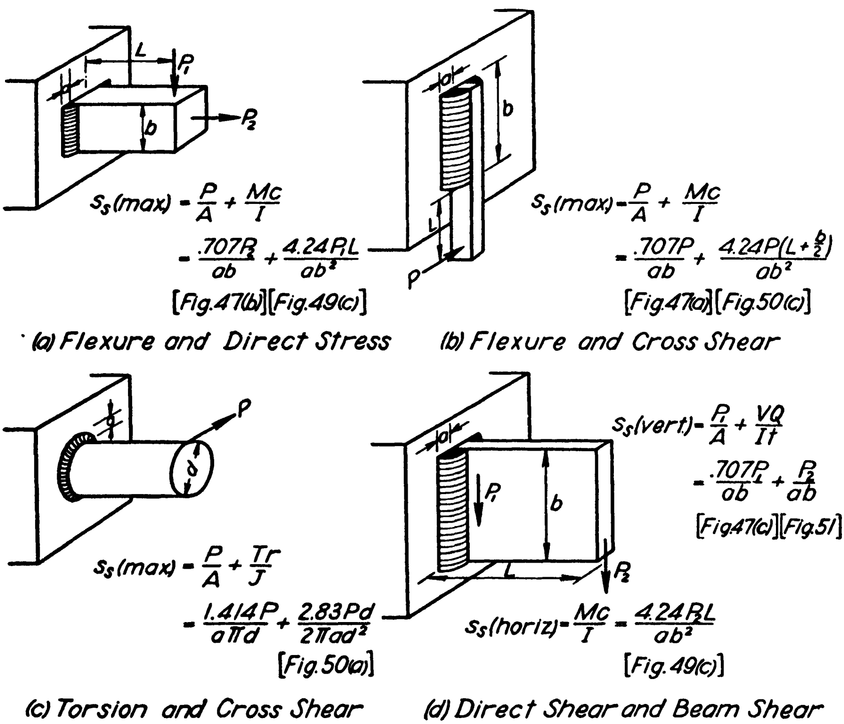


FIG. 52. COMBINED OR MAXIMUM SHEARS IN WELDS.

the common superposition of *direct stress and flexure.* The computed throat shears must add at one extreme fiber, the upper or tension fiber for the cantilever bar pictured. In (*b*) is illustrated a similar arrangement where *flexure and cross shear* combine. The flexure partakes of the nature of torsion, but since the cross-section consists of two long narrow welds

placed adjacent to each other, their end shears are computed properly by the flexure formula. The maximum shear occurs at the lower end of the weld. The torsion shaft shown in (*c*) of Fig. 52 is also subjected to a cross shear. There will be a combination of *torsional shear and cross shear*. The torsional shear is constant around the entire circumference. If the same assumption is made regarding the distribution of the horizontal cross shear, there will be one point, the upper end of the vertical diameter, where these two shears will add directly.

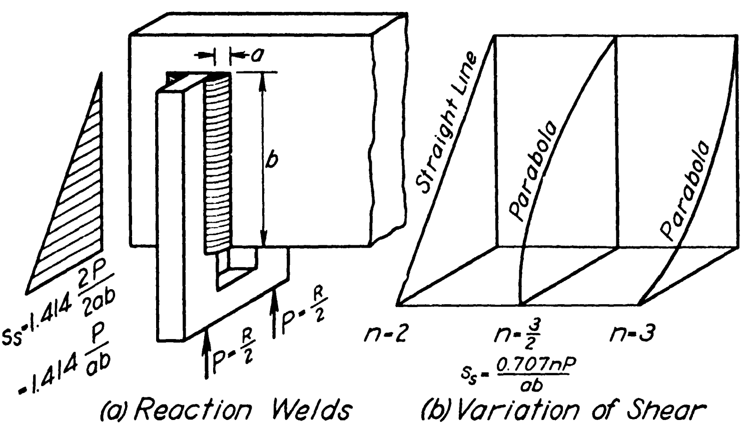


FIG. 53. REACTION TRANSFER.

Shear Distribution. The addition of direct end shear and the shear that accompanies beam flexure is indicated in Fig. 52(*d*). This is not a very important case since the existence of shear without flexure, which can be uniformly distributed, is not common. For instance, in the construction shown in Fig. 53, the shear along the weld is assumed to have *straight-line variation* such that the maximum shear is double the average. The reason for this assumption is found in the fact that the vertical reaction leg is under compression which reduces in intensity from bottom to top of the weld (the massive beam plate deforms inappreciably) and, therefore, the weld deformation (or shear) decreases from bottom to top. Some possible assumptions as to distribution are indicated in Fig. 53(*b*). The factors $n = 2$, $3/2$, or 3 should be selected according to the length of the weld. It would seem reasonable to take $n = 1$ for a weld only a few inches long, or, for a length up to $10a$ (where a is the nominal dimension or leg of the weld) $n = 3/2$ for welds up to $20a$ in length, $n = 2$ for welds up to $30a$ in length, and $n = 3$ to a limiting length of about $50a$. This is a crude approximation, but it is undoubtedly in the right direction and therefore better than the usual assumption of uniform distribution. Such approxi-

mations can be improved as soon as adequate tests have been made available.

Resultant of Cross Shears in Welds. The second classification of combined stresses in fillet welds relates to the computation of a resultant throat shear as caused by two load systems. This action is illustrated by Fig. 54. Each of the forces P_1 and P_2 produces a shear on the throat section, but one shear P_1 is across the weld, while the other shear P_2 is along the weld. The *diagonal resultant shear R* is also acting on the throat section and since it is larger than either P_1 or P_2, it must be taken as the critical shear in design. This action is not unusual in structural welding and should not fail to be considered properly in the design. In all respects this analysis is consistent with the usual procedure of computation of the resultant shear on a rivet when a rivet group resists shears in two or more directions. The use of a resultant has been common in rivet calculations for many years but it has been neglected in weld calculations.

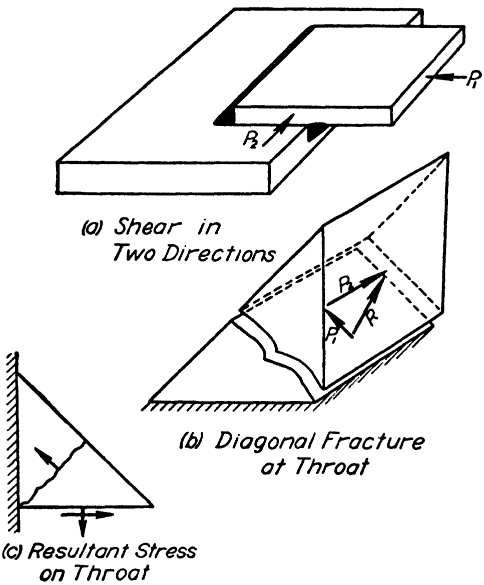


FIG. 54. RESULTANT THROAT STRESS ON WELD.

Resultant Stress on Throat. In (*c*) of Fig. 54 there is shown a common situation where shear and tension are applied to one face of a fillet weld. Evidently, each of the applied forces gives rise to a tension stress on the throat of the weld although, for the particular case shown, the two shearing stresses on the throat tend to cancel each other. If the direction of either applied force is reversed, the critical throat stress will become a shear. As a practical design method, we may always feel safe in using the *resultant applied force as a shearing force on the throat section.*

THROAT AND ROOT STRESSES IN FILLET WELDS

52. Internal Action of Fillet Welds. Even the inexperienced analyst will appreciate the fact that the design procedure just described for fillet welds is far from scientific. The action of a fillet weld as in Fig. 47(*b*)

is extremely complex. For the full length of the fillet there is tension along the contact surface with the horizontal plate and shear along the contact surface with the vertical plate. But this is merely the *simplest* picture of the structural action. Since the weld is eccentric to the pull, it is also under internal flexure which gives rise to both tension and compression forces along each leg or contact surface. This action is produced by the tendency of the fillet to rotate as pictured by the small sketch at the top of Fig. 55(a). This rotating tendency is measured by the moment of eccentricity, $Pa/2$.

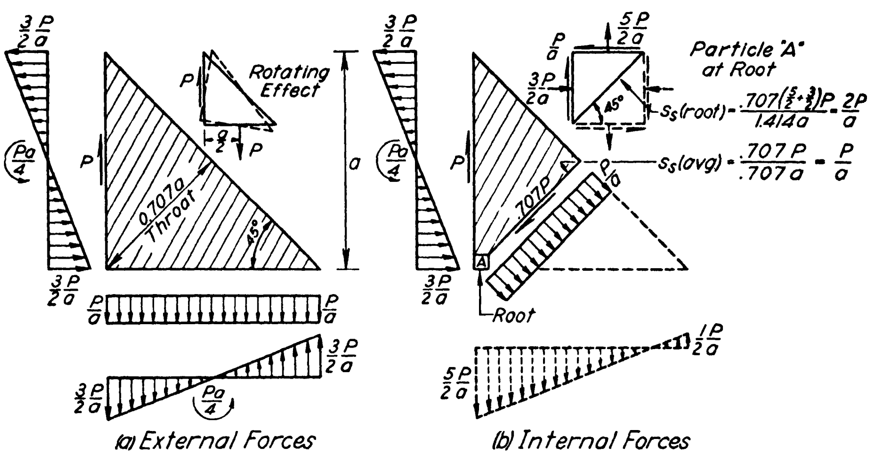


FIG. 55. INTERNAL STRESSES IN A FILLET WELD.

If it is assumed that the moment of eccentricity $Pa/2$ is resisted equally by flexural stresses along each leg, such stresses can be computed from the flexure formula for a cross-section one unit in breadth and with a depth equal to the weld leg a. Thus we find

$$f = \frac{M}{S} = \frac{Pa}{4} \div \frac{a^2}{6} = \frac{3}{2}\frac{P}{a}. \tag{5}$$

If this straight-line variation of stress from tension to compression is combined with the uniform tension P/a, the result is a variation from a tension of $\frac{5}{2}\frac{P}{a}$ to a compression of $\frac{1}{2}\frac{P}{a}$, as shown along the horizontal leg of the weld in Fig. 55(b). Again in (b) the analysis is made for the stresses along the 45-degree line of the throat. We will study the action upon the throat of the forces applied to the vertical face. The vertical shear P gives rise to tension and shear forces of $0.707P$ along the 45-degree line

of the throat. It follows that the throat stresses are

$$f(\text{avg.}) = \frac{0.707P}{0.707a} = \frac{P}{a}, \tag{6}$$

and

$$s_s(\text{avg.}) = \frac{0.707P}{0.707a} = \frac{P}{a}. \tag{7}$$

This is true because there is no flexural moment to be resisted along the line of the throat since the applied moment is exactly canceled by the moment of the force P. Thus

$$M_{\text{throat}} = \frac{Pa}{4} - 0.707P\left(\frac{0.707a}{2}\right) = 0. \tag{8}$$

53. Maximum Root Shear in a Fillet Weld. We conclude then that the most serious *average* stress along the throat section is the unit shear P/a. Actually, however, we have no assurance that this shear is uniformly distributed, and, in fact, the *high tension at the root* indicates that an investigation should be made of the shear on the 45-degree plane of failure for particle " A " shown at the root in Fig. 55(b). The view of this enlarged particle shows it to be acting under a vertical tension of $\frac{5}{2}\frac{P}{a}$ and a horizontal compression $\frac{3}{2}\frac{P}{a}$. There are also shears of indeterminate values along horizontal and vertical planes, but, since these equal shears do not give rise to shear along the 45-degree plane of failure, they may be neglected. The resultant shear along the 45-degree line of the root particle " A " is shown to be $2P/a$ obtained thus:

$$s_s(\text{max.}) = \frac{0.707\left(\frac{5}{2} + \frac{3}{2}\right)\frac{P}{a}}{1.414} = 2\frac{P}{a}. \tag{9}$$

If these crude analyses can be relied upon, they seem to indicate that the average shear along the 45-degree failure line of the throat is P/a while *the maximum shear along the same line occurs at the root and is equal to twice this value or* $2P/a$. The shear commonly used in design is the load divided by the throat area, or $P/0.707a = 1.414P/a$.

Study of Assumptions. There are several weaknesses in the assumptions made in these analyses. There undoubtedly is an induced shear along the horizontal leg of the weld. This shear forms one half of a couple (the other half being a net direct stress on the vertical face) that tends to resist the moment of eccentricity. This action reduces the flexural stresses and therefore the root shear $2P/a$. Another weakness is the fact that we cannot fully justify the assumption of the maintenance of plane

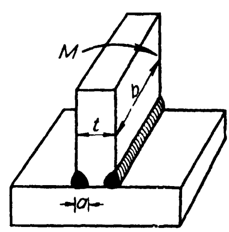
sections for a triangular body where the flexural stiffness varies radically from the root to the tip of the weld. Any change from straight-line variation of flexural stress should reduce the severe root stresses. The fact that no attempt has been made to determine the line on which the shear in Fig. 55 would reach a theoretical maximum value is because observed shear failure is always very near to the 45-degree line.

From these observations we may conclude that the usual design procedure may fail to account for the *maximum* root stress but that it more than allows for the *average* shear on the throat. If the weld is in any sense ductile, the first plastic flow will relieve the high root stress and distribute the throat shear more nearly in a uniform manner. The usual design procedure therefore seems to be adequate for ductile structural welds.

PROBLEMS

50. Derive an expression to represent the maximum tensile fiber stress in this double-vee butt weld caused by an applied moment. Assume that the stresses in the welds at the line of contact with the vertical plate are controlled by the beam-flexure formula. Consider the weld faces to be flush with the vertical faces of the plate.

Ans. $f = \dfrac{3Mt}{ab(3t^2 + 4a^2 - 6at)}$.



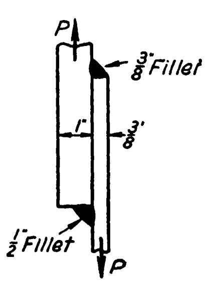
PROBLEM 50.

51. Compare the tensile stress computed by the formula derived in Problem 50 with the average weld stress that would be obtained upon the basis of the theory of Fig. 49(*b*). The welds have the following dimensions: $a = \frac{1}{4}$ in., $t = 1$ in., $b = 6$ in., and $M = 12{,}000$ in-lb. Note that the flexural stress in the 1-in. plate is 12,000 lb. per sq. in. These computations are for tensile stress rather than shearing stress.

Ans. $f = 13{,}700$, $f_{\text{avg.}} = 9050$ lb. per sq. in.

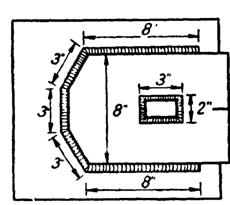
52. If the lap welded joint illustrated is a horizontal seam in a circular tank, compute the maximum weld shears on the throat sections that would occur when the thinner plate was stressed to 18,000 lb. per sq. in. in a vertical direction.

Ans. 6950 and 13,900 lb. per sq. in.



PROBLEM 52.

53. A plate 8 × ¾ in. as illustrated is to be developed by fillet welding for a tensile working stress of 20,000 lb. per sq. in. The weld along the outside of the plate is a ½-in. fillet and the weld placed around the slot is a ¼-in. fillet. Compute the average throat shear in each weld. *Ans.* 11,300 lb. per sq. in.



PROBLEM 53.

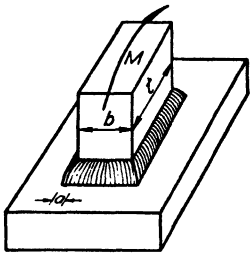
54. Develop a formula to express the throat shear caused by flexure of the bar shown welded all around. (1) Assume that the dimensions *b* and *t* are large as compared to *a*. (2) Revise the formula for the case where *a* may be considered to increase the dimension *b* appreciably. Otherwise treat the analy-

ses the same since the extreme fiber distance must be taken to the root of the weld where failure starts.

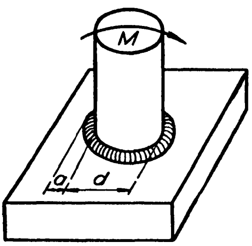
$$\textit{Ans.}\quad (1)\ s_s = \frac{4.24M}{3abt + at^2},\quad (2)\ s_s = \frac{4.24M}{3at(b + 2a) + at^2}.$$

55. Develop a formula for root shear caused by flexural moment applied to the fillet welded circular shaft illustrated. Take the diameter of the weld to be the same as the diameter of the shaft d.

$$\textit{Ans.}\quad s_s = \frac{5.66M}{\pi a d^2}.$$



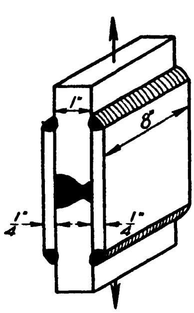
PROBLEM 54.



PROBLEM 55.

56. Analyze for the throat stresses in the welds of the reinforced butt welded joint. Base your calculations upon the fact that the three plates must stretch equally between the welds and must therefore have equal unit deformations and unit stresses. The load produces a unit stress of 20,000 lb. per sq. in. in the 1-in. plate.

Ans. f(butt weld) = 13,300; s_s(fillet) = 18,800 lb. per sq. in.



PROBLEM 56.

57. Analyze for the stresses in the welds of Problem 56 in general terms and determine the fixed ratio between these two stresses. Suggest some shape for the reinforcing plates that would produce but 8/10 as much throat shear in the fillet welds as tension in the butt weld.

Ans. Ratio = 1.414; point or slot the reinforcing plates to lengthen the fillet welds.

58. In Fig. 49(c) the data are as follows: b = 6 in., a = ½ in., and M = 30,000 in-lb. The plates are 1 in. thick. (a) Compute the controlling shear on the throat by application of the beam-flexure formula and check by use of the formula given on the figure. (b) What is the resultant shear if the plate carries a direct tension (vertical) of 3000 lb. per sq. in.? *Ans.* (a) 7070 and (b) 11,300 lb. per sq. in.

59. Same data as Problem 58 except that the moment acts as a torque as illustrated by Fig. 50(c). Do the shears produced by the torque and the direct stress combine?

Ans. (a) 7070 and (b) 11,300 lb. per sq. in.

60. In Fig. 50(b) the data are as follows: b = 4 in., a = ⅜ in., L = 10 in., T = 110,000 in-lb. which is caused by a force of 11,000 lb. parallel to the dimension L but located 10 in. out from the center of the block. Find the shear on the throat caused by this eccentric force. *Ans.* 11,600 lb. per sq. in.

61. In Fig. 53 the following data apply: a = ⅜ in., b = 20 in. Assuming that there are four identical fillet welds, compute the allowable end reaction based upon a maximum throat shear of 11,300 lb. per sq. in. Use the proper value of n from § 51.

Ans. 80,000 lb.

62. Compute the maximum allowable shear on the connection of an 18*WF*70 beam fillet welded to the face of a heavy column by an 8-in. weld across each flange and a 15-in. weld on each side of the web: these are ⅜-in. fillets. Use the beam-shear formula and compare with uniform distribution.

Detailing Structural Welds

54. Standard Welding Symbols. During the period when welding was developing as a tool for structural usage, it was natural that welds were designated in almost any conceivable manner. Wherever possible, welds were shown in side view and were indicated by a series of short

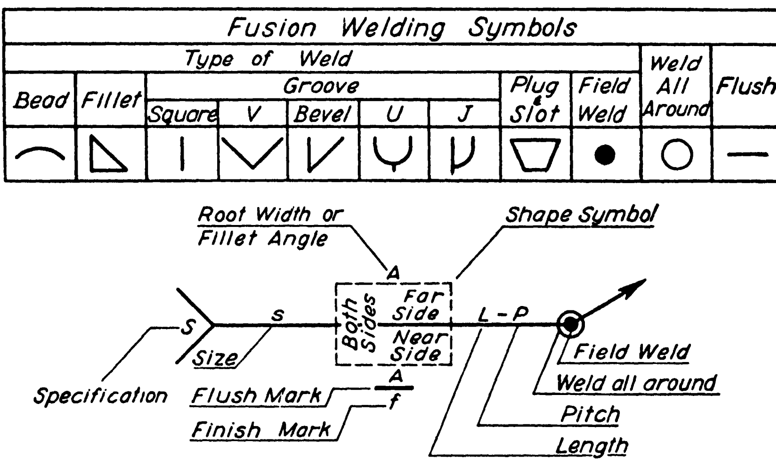


FIG. 56. WELD SYMBOLS — AMERICAN WELDING SOCIETY.

curved lines (picturing the weld bead) similar to the usual appearance of cross-hatching. Welds in cross-section were shown in solid black. This is the system that has been used here to illustrate in a realistic way the location of welds on perspective sketches. As welding developed both in usefulness and in complexity, it became necessary to adopt symbols to distinguish between *butt* welding and *fillet* welding, *shop* welding and *field* welding, welding on *far* side, *near* side or *both* sides. Symbols were also needed to designate the type of notch to be prepared for the welder and the extent to which the welder was expected to reinforce the weld by piling on extra metal beyond the amount needed to fill the notch.

The first effective attempt at standardization of symbols was the report of the American Welding Society in 1929. This report adopted a system of marking welds that was immediately accepted and widely used. It was revised in 1935, but greater simplification seemed desirable. This need was met by the second set of *AWS* Standards accepted in 1938. These

are reproduced on Fig. 56. The system eliminates entirely any actual need for picturing the welds on the drawing. Instead, all information is contained on an arrow which points to the position of the weld.

American Welding Society Symbols. In Fig. 57 the arrow for illustration (*a*) indicates that a fillet weld of ⅜-in. leg and 6-in. length is to be placed on the near side of the joint to which the arrow points. The relative position along the joint, if important, would be designated on the elevation of the joint as indicated in (*f*). The designation of "near side" is found in the fact that the right-angle triangle (45-degree fillet) is placed on

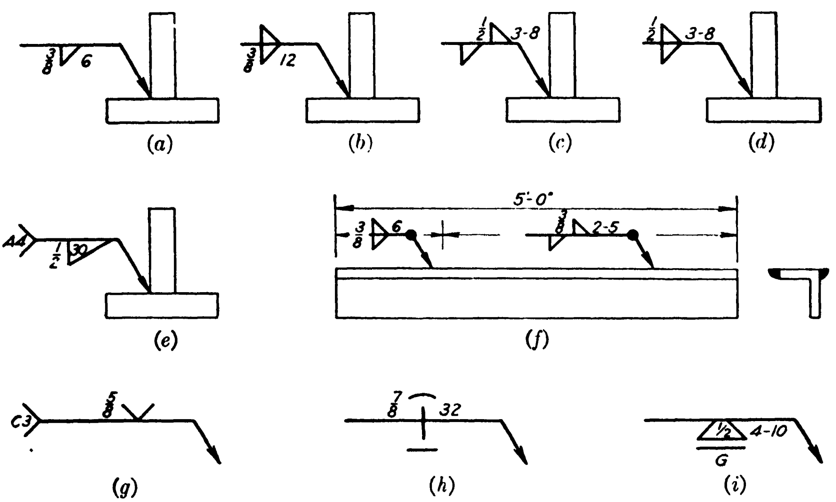


FIG. 57. SYMBOLS FOR STRUCTURAL WELDING.

the side of the arrow nearest to the reader. In (*b*) the use of two such triangles is intended to signify that fillet welds (⅜ in. and 12 in. long) are placed both "near side and far side." Other common symbols are indicated in Fig. 57, such as the dot at the break in the arrow for designating field welding and the use of the letter *V* or *U* to illustrate the kind of scarfing used for preparing the plates for welding. Finish marks such as *C* (chip), *G* (grind), *M* (machine), or *f* (finish) are placed on the mark indicating rounded bead (⌒) or on the flush mark (—), see Fig. 57(*h*) and (*i*). The specification controlling the type of weld rod and other welders' specifications may be placed in the tail of the arrow as shown on (*e*) and (*g*) of Fig. 57.

Special Symbols. If the designer wishes to place a distinctive marking on the plan or elevation showing the line of weld, he may use the marks employed in the 1929 *AWS* specifications. (See Fig. 58.) The series of small *x-marks* placed along the line on which welding is done indicates

" near side," while the other symbols are for " far side " and " both sides." These marks can be used to show where the weld starts and stops although exact dimensions should also be given. These supplementary marks should never be used alone, but they may be used along with the arrow designation.

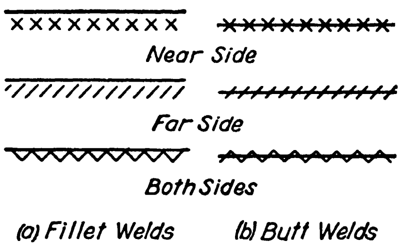


FIG. 58. SPECIAL WELD SYMBOLS.

The object in weld designation is that the beginning and end of each piece of weld and the necessary details of its cross-section and method of placement may be made clear. One hindrance to good welding has been poor detailing. One should never merely mark " weld " on a drawing or " weld all around " unless these statements are thoroughly conclusive. The detailer, draftsman, or designer is far better qualified to specify the exact *position*, *length*, and *cross-sectional dimensions* of each weld than is the welder, whose conceptions of strength are scarcely better than those of the layman. Since detailing is controlled by the methods of fabrication and erection, Specifications 132–137 should be studied.

DESIGN OF STRUCTURAL WELDS

55. Working Stresses. Standard working stresses for structural welds as specified by the American Welding Society (1938) are as follows:

Shear = 11,300 lb. per sq. in.
Tension = 13,000 lb. per sq. in.
Compression = 18,000 lb. per sq. in.

These working stresses are for welds made with lightly coated or washed electrodes. Such welds are relatively brittle and low working stresses are justified. For *high-strength ductile welds*, allowable stresses are as follows:

	1938 *AWS* Code	1942 *AISC* Code
Shear	= 13,600 lb. per sq. in.	13,600 lb. per sq. in.
Tension	= 15,600 lb. per sq. in.	16,000 lb. per sq. in.
Compression	= 18,000 lb. per sq. in.	20,000 lb. per sq. in.

The recommendations of the Lincoln Electric Company (1939) for welds made with their shielded arc electrodes (which produce a ductile weld) are for the following working stresses:

Shear = 14,100 lb. per sq. in.
Tension = 16,200 lb. per sq. in.
Compression = 18,700 lb. per sq. in.

Such working stresses might be reasonable for welds made with any heavily coated electrode that will produce tough ductile welds.

Fillet Welds. The working stress of 11,300 lb. per sq. in. on the throat section of a fillet weld may be transferred into a very convenient unit for purposes of design. For a ⅛-in. fillet (leg = 0.125 in.) this working stress gives rise to an allowable shear on each lineal inch of weld of

$$S_s = 0.125 \times 0.707 \times 11{,}300 = 1000 \text{ lb.}$$

Hence, for washed or bare electrodes we conclude that the allowable shear on a fillet weld is *1000 lb. per lineal inch per eighth inch of fillet leg,* or

2000 lb. for a ¼-in. fillet,
2500 lb. for a 5⁄16-in. fillet,
3000 lb. for a ⅜-in. fillet,
4000 lb. for a ½-in. fillet, etc.

It is evident from these results that the allowable shearing stress of 11,300 lb. per sq. in. was chosen to produce simple design calculations. Such simplification is commendable and justifies the slight variation in the desired working stress that it may necessitate.

Correspondingly, for very ductile welds made by heavily coated electrodes, an increase of 25 per cent, as recommended by the Lincoln Electric Company, permits 1250 lb. per lineal inch per eighth inch of fillet leg, or

2500 lb. for a ¼-in. fillet,
3120 lb. for a 5⁄16-in. fillet,
3750 lb. for a ⅜-in. fillet,
5000 lb. for a ½-in. fillet.

These values happen to correspond with four-place decimal fractions expressing the sizes of the fillet legs. They are therefore easily remembered.

Fatigue. Tests reported in Engineering News-Record during 1939 by W. M. Wilson and A. B. Wilder show unexpectedly low failure stresses for butt welded joints under fatigue loading. "The specimens, consisting of single-vee butt welds in ⅞-in. structural carbon steel and low alloy (manganese-vanadium) steel plates, were subjected to three relations of minimum to maximum stress: (1) from tension to an equal compression;

that is, complete reversal; (2) from zero to tension; (3) from tension to tension one half as great. The maximum stresses in the stress cycle were chosen to cause failure at 100,000 cycles and at 2,000,000 cycles. The specimens were tested (1) in the condition as welded; (2) with the welds planed flush with the base plate; (3) after stress relief at 1200° F. for one hour and cooling in the furnace. The effect of frequent periods of rest upon the fatigue strength of the butt welds in the carbon steel plates was also investigated. A summary of the results follows.

(1) In the condition as welded, the values of fatigue strengths for the complete stress reversal cycle were 21,600 lb. per sq. in. for failure at 100,000 cycles and 14,800 lb. per sq. in. for failure at 2,000,000 cycles; for the cycle from zero to tension, strengths were 32,600 lb. (100,000 cycles), and 23,100 lb. (2,000,000 cycles).

(2) For specimens with welds machined flush with the base plate, fatigue strengths for the stress reversal cycle were 29,400 lb. (100,000 cycles), and 19,800 lb. (2,000,000 cycles); for the zero to tension cycle, fatigue strengths were 47,000 lb. (100,000 cycles) and 30,100 lb. (2,000,000 cycles). That is, removing the stress raiser caused by the change in section at the edge of the weld reinforcing increased the fatigue strength 43 per cent. Moreover, the fatigue strengths of these welded specimens were equal to the fatigue strengths of the plates without welds.

(3) Stress relieving by heating had no effect upon fatigue strength, nor did frequent *rest periods* between applications of loads.

(4) For all specimens in the stress cycle from tension to tension one half as great, the yield point of the material was exceeded, so that such a cycle is not important to the structural designer.

(5) Fatigue strengths on complete reversal for welded specimens of low alloy steel plate with a static strength of 83,000 lb. per sq. in. were 24,000 lb. per sq. in. (100,000 cycles) and 16,100 lb. (2,000,000 cycles)."

56. Design for Direct Loads. This is a particularly simple problem as long as symmetry can be maintained. The total load is merely divided by the value of the weld in pounds per lineal inch to determine the required number of inches of weld. This length of weld is then placed on the member to maintain symmetry in the welded joint.

Examples *DP*11*a* and *DP*11*b*. The first problem on the design sheet 11 illustrates the design of a lap welded joint for tank plates. Such joints are used for unimportant structures or where stresses are low. A heavily stressed joint in a plate would be butt welded and reinforced, if necessary, with butt straps as illustrated by *DP*11*b*. This example shows how a joint of 100 per cent efficiency can be obtained by welding. The main features of such a design are to arrange for the base plate and the reinforcing plates to stretch equally between the welds (equal unit strains and stresses). We should make the reinforcing plate long enough so that the transfer of stress out of the base plate and into the reinforcing plates is far enough away from the butt welded joint so that the stress

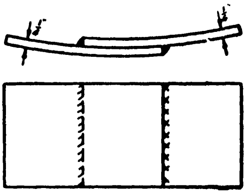
DP11a. *Design a lap welded vertical seam for a tank of 20′ diam., 55′ head of water, plates ¼″ thick, AWS spec. Weld value = 13,600#/□″ in shear.*

$$Pl.\ tension = \frac{55 \times 62.5}{144} \times \frac{20 \times 12}{2}$$

$$= 2860\#/''\ of\ height.$$

$$Weld\ leg = \frac{2860}{2} \times \frac{1.414}{13{,}600} = 0.15''.$$

Use ¼″ fillet welds which provide $\frac{1}{10}$″ for corrosion.



DP11b. *Design a butt welded seam for the full tensile value of a 12″ × ½″ plate. AWS and AISC spec. Weld value at 11,300 (shear); 13,000 (tension).*

Tensile value of pl. = ½ × 20,000 = 10,000#/″
Tensile value of ½″ butt weld = ½ × 13,000 = 6,500

Diff. = 3,500#/″

Reinforcing Plates:

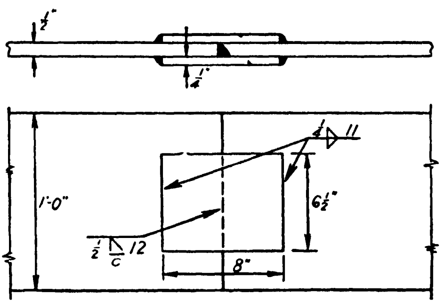
For equal unit def.

$$\frac{reinf.\ pl.\ area}{orig.\ pl.\ area} = \frac{3500}{6500} = 0.54.$$

Area of reinf. pls.
= 0.54 × 12 × 0.5
= 3.24□″ per ft.

Make reinf. pls. 6.5″ × ¼″ spaced on 12″ centers.

Area furnished
= 6.5 × 2 × 0.25
= 3.25□″.



Weld on Reinforcing Plate:

Use ¼″ fillets. Weld value at 11,300#/□″ (shear) = 2000#/″.

$$Weld\ length = \frac{3500 \times 12}{2 \times 2000} = 10.5''.\quad Use\ 11''.$$

Length of reinf. pl. Set at 8″ min. to help equalize stress in butt weld.

Remarks: *Although this joint presents a perfectly balanced design, it is evident that the discontinuity of the butt straps produces a most complex stress pattern. If welds of superior quality can be obtained, and the butt straps can be eliminated, the action of the structure will be simplified, particularly for loads above the elastic limit.*

DP12. *Design a splice between two tension members one of which is a 14WF48 split beam and the other consists of two angles 5 × 3½ × ⅜″. AISC and AWS spec.*

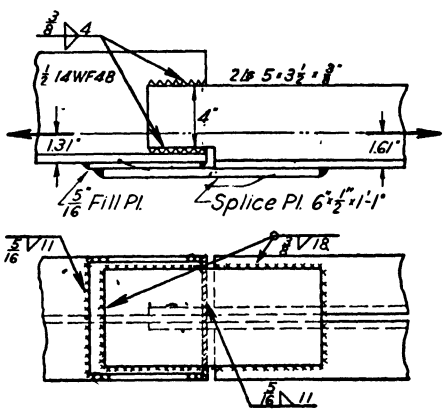
Value of Angle Legs:

Angles were designed for 80% of their gross area.

Net value = 2 × 3.05 × 0.80 × 20,000 = 97,500#.

Burn 5″ angle leg as indicated to keep c. g. of angles in line with c. g. of split beam.

Value of remaining 4″ legs $= \dfrac{4}{8.5} \times 97{,}500 = 46{,}000\#.$



Weld Lengths: *Allowable shear = 11,300#/□″.*

Length of ⅜″ fillet $= \dfrac{46{,}000}{3000} = 15.3''.$

Use 4 fillets 4″ long.

The remaining 51,500# requires $\dfrac{51{,}500}{3000} = 17.2''$ *of ⅜″ fillet.*

Place 18″ of ⅜″ fillet around each end of the 6″ × 1′–1″ splice pl.

Splice Plates:

Thickness of splice pl. $= \dfrac{51{,}500}{6 \times 20{,}000} = 0.43''.$ *Use ½″.*

Fill pl. 5/16″ thick must be connected with 22″ of 5/16″ weld.

Size of fill pl. is 7″ × 8″.

Remarks: *If no fill plate is used here, the eccentricity between the two members will be 3/10″. The corresponding moment of nearly 30,000″# would be serious.*

will be uniform in the butt weld. This will be accomplished reasonably well if the spacing and length of the butt straps are arranged so that 45-degree lines drawn from the corners of the straps *overlap* as shown by Fig. 59. The greater the overlapping the more uniform the stress on the butt weld. Naturally, the butt weld must be chipped or ground flat before the butt straps can be welded on. This marking is indicated on the design sketch.

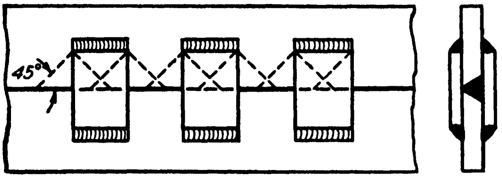


FIG. 59. SPACING OF BUTT STRAPS.

SPLICE IN A TENSION MEMBER, *DP*12. The design sheet *DS*12 illustrates a connection of a two-angle tension member to a split-beam or *T-section* tension member. The important detail to arrange is for the gravity axes of the two members to be placed in line. This is accomplished by burning away the outstanding legs of the angles and by adding a fill plate to the bottom of the split-beam section. The welds attached to the remaining 4-in. legs of the angles are then designed to develop the part of the total stress that these 4-in. legs will carry. The remainder is transferred through the lower splice plate, thence through the fill plate to the split beam. The splice is very compact and inexpensive.

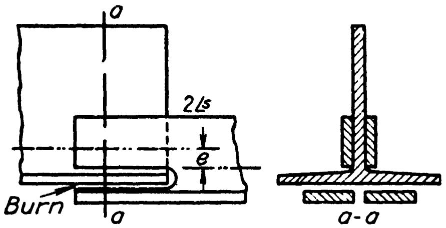


FIG. 60. SPLICE OF TENSION MEMBERS.

An even cheaper alternative would be to slot the angle as indicated in Fig. 60 and to weld the juncture so completely that the full section of the angle would be available at the end of the slot. The only difficulty involved is that it will not ordinarily be possible by this means to eliminate all eccentricity (*e* of Fig. 60) between the gravity lines of the two members.

57. End Connections for Channels and Angles. The problem of forming a welded end connection to develop a heavy channel or angle is usually complicated by the fact that the space available is quite limited. Frequently the length along the section available for welding is no greater than the width of the section.

CHANNEL CONNECTION, *DP*13*a*. In the example *DP*13*a* it is found necessary to slot the channel in order to obtain a sufficient length of standard 3/8-in. weld. Such slot welds are in reality merely additional lengths of fillet welds. Slots or round holes are sometimes made smaller and are completely filled with weld. They then become the equivalent of weld rivets, but their use is not recommended.

Balancing Welds on an Angle. The problem of connecting an angle to another member by welding is properly solved when the total length of weld required to resist the pull is so placed that the center of resistance

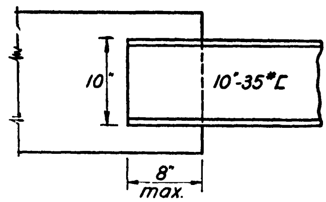
DP13a. *Design end welds to reproduce the tensile value of a 10″-35# channel. Use 3/8″ welds. Weld value = 1250#/″ per 1/8″ of fillet leg.*

Value of channel $= 10.27 \times 20{,}000 = 205{,}400\#$.

Max. length of fillet $= 8 + 8 + 10 + 10 = 36''$.

Weld value $= 205{,}400 \div 36 = 5700\#/''$.

This is too great for a 3/8″ weld which has a maximum value of 3750#/″.

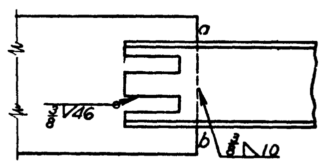


Try Use of Slot Welds: *(Spec. 131).*

Total length needed $= \dfrac{205{,}400}{3750} = 55''$.,

Slots must provide $55 - 36 = 19''$.

Use two 5″ slots 1½″ wide providing 20″ length which allows 1″ for craters at a and b.



DP13b. *Design end welds to produce a value of 20,700# for a 4 × 4 × 1/4″ angle. AWS spec. for low strength welds.*

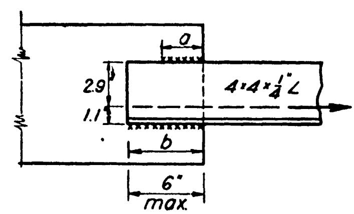
Value of 1/4″ weld $= 2000\#/''$.

Weld length $= 20{,}700 \div 2000 = 10.3''$.

Balancing Welds:

Length a $= \dfrac{1.1}{4} \times 10.3 = 2.8''$.

Length b $= \dfrac{2.9}{4} \times 10.3 = 7.5''$.

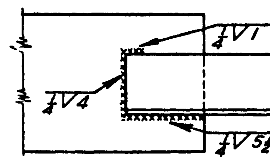


Max. $b = 6''$; *hence, weld end of angle.*

$L = 10.3''$, $c = 4''$, $x_1 = 2.9''$.

See Fig. 61, and equation (12).

$$b = \frac{Lx_1}{c} - \frac{c}{2} = \frac{10.3 \times 2.9}{4} - 2 = 5.5''.$$



$$a = \frac{Lx_2}{c} - \frac{c}{2} = \frac{10.3 \times 1.1}{4} - 2 = 0.8''. \quad \text{Use } 1''.$$

Total length $= 5.5 + 4 + 1 = 10.5''$. *Check.*

of the weld is in line with the applied load. In Fig. 61 is shown an end connection formed by three welds of lengths a, b, and c. If only the welds a and b are used to produce a total length of L, then the center of resistance will be in line with the load when the following relationship is satisfied.

$$\frac{a}{a+b} = \frac{x_2}{x_1 + x_2},$$

or

(10) $$a = \frac{Lx_2}{c} \quad \text{and} \quad b = \frac{Lx_1}{c}.$$

But, if three welds a, b, and c are used to make up the required length of weld L, the result will be more complex. We will have to make use of

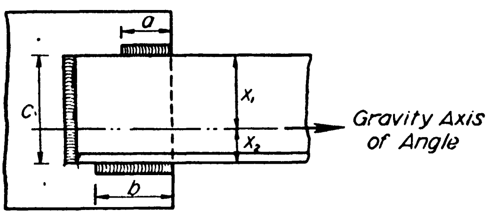


FIG. 61. BALANCING WELDS ON AN ANGLE.

an equation of moments about the lower edge of the angle or about the weld line b in Fig. 61. Thus we obtain

$$ac + \frac{c^2}{2} = Lx_2,$$

or

(11) $$a = \frac{Lx_2}{c} - \frac{c}{2},$$

and

(12) $$b = \frac{Lx_1}{c} - \frac{c}{2}.$$

For four lengths of weld, where a second length c is placed on the far side of the angle at the edge of the plate, we have

(13) $$a = \frac{Lx_2}{c} - c,$$

(14) $$b = \frac{Lx_1}{c} - c.$$

These final equations, however, may result in a *negative value for a*, obviously an impossible solution.

EXAMPLE *DP*13*b*. Here we have a case in which the application of equation (10) gave rise to an excessive length of weld *b*. A solution was then obtained by the use of three welds *a*, *b*, and *c*, with equations (11) and (12).

If an attempt is made to use four welds by a study of equations (13) and (14), a negative value of *a* is obtained resulting in an impossible solution.

58. Long Longitudinal Welds. In designing for direct loads it is often convenient to use long welds in line with the load in place of cross or transverse welds. Theory indicates that such welds have very high concentrations of stress at the ends, but tests have not borne out the seriousness of these stress concentrations for *static loads.* Nevertheless, certain rules of thumb have developed such as to discount the length of a longitudinal weld by 25 per cent, or to credit it with the first 6 in. plus one half of the length over 6 in.

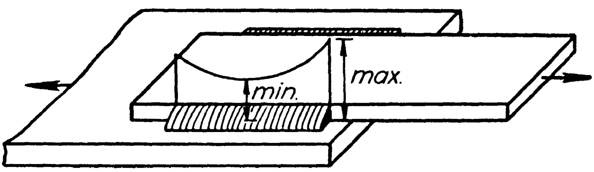


FIG. 62. DISTRIBUTION OF LONGITUDINAL SHEAR.

The theoretical analysis of the simplest possible case has been presented by H. W. Troelsch, Transactions *ASCE*, 1934, p. 409. The variation of shear is as indicated in Fig. 62 except that the ratio of the maximum shear at the end to the minimum shear at the center of the length of weld may amount to as much as 10 to 1, even for a 6-in. weld. It will be fairly evident that there would be little advantage in increasing a longitudinal weld beyond 6 in. if this condition controlled. However, test data presented by Mr. W. H. Jameson and reproduced below show rather conclusively that such high stresses do not produce failure under a single *static load.*

TABLE 5

STATIC TESTS OF FILLET WELDS

LENGTH OF LONGITUDINAL FILLETS (3/8″)	TOTAL PULL (lb.)	AVERAGE SHEAR ON THROAT (lb. per sq. in.)	THEORETICAL MAXIMUM WELD SHEAR (lb. per sq. in.)
7″	69,200	37,300	54,000
23″	241,000	39,500	97,500
31″	319,000	38,800	114,400
47″	468,000	37,500	132,000

These data can only be explained by the fact that the plastic deformation which occurs beyond the elastic limit is adequate to *distribute the shear*

more or less uniformly before failure occurs. Even the theory shows that this will be true because the end shear of the weld is expressed as

$$s_s(\text{max.}) = fc' \sqrt{\frac{D}{E}}. \tag{15}$$

Here, f is the unit plate stress, c' is a constant dependent upon the thickness of plates and the number of welds, and D is the detrusion ratio or the ratio of shearing deformation to shearing stress. Evidently, this factor D will suddenly reduce *almost without limit* as the elastic limit of the weld is passed in shear, and the end shearing stress s_s will reduce correspondingly.

Shear Reduction Formulas. For repeated stress in machines and for those bridges where fatigue is important, the high end shears mentioned will need to be considered quite seriously. In the design of buildings and other structures for fixed load, however, the matter is less serious. The empirical formula that follows has been proposed for the allowable load on long longitudinal fillets per ⅛ in. of fillet leg.

$$S_s = 1100 - 5\frac{L}{a} \left(\text{for } \frac{L}{a} > 20 \text{ and } < 150\right). \tag{16}$$

In this formula, L is the length of the fillet in inches and a is the size of the fillet leg in inches. This formula is very crude since it makes no allowance for the relative areas of the parts to be joined; however, it does allow for the size of the weld, which is important. It is of some interest that if two bars are welded together, one being of small area and the other of very large area, such that the deformation occurs mainly in the smaller bar, the unit stress at one end of the longitudinally welded joint will be zero, and, at the other end, it will be 1.414 times its amount for bars of the same size. This leads to the suggestion that the reduction term in equation (16) should be increased from $5\,L/a$ to $15\,L/a$ for such cases since the length in relation to stress variation has been in effect doubled and $1.414 \times 2 \times 5 = 15$ nearly.

$$S = 1100 - 15\frac{L}{a} \left(\text{for } \frac{L}{a} < 50\right). \tag{17}$$

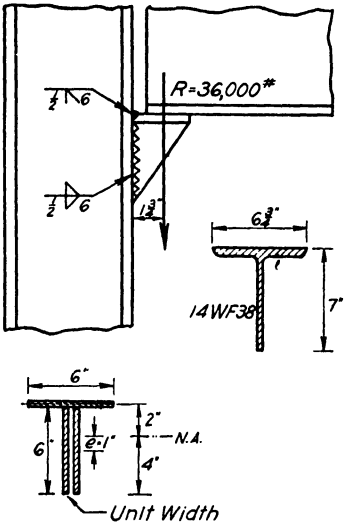
For structures where impact and fatigue are important, a faster reduction of shearing stress on longitudinal welds is required for safe design. It would seem advisable in such cases to follow the theory presented by Troelsch. As an approximation, such welds might be limited to $16a$ in length (6 in. for a standard ⅜-in. fillet) and then designed for an end shear concentration of double the average shear, which would be equivalent to a reduction of 50 per cent in working stresses.

DP14. *Design a welded bracket as a seat for an I-beam where the end reaction of 36,000# is located at 1¾″ out on the bracket.* ***Weld value*** *= 11,300#/□″.*

Data:

$M = 36{,}000 \times 1.75 = 63{,}000''\#.$
$V = 36{,}000\#.$

Length of ⅜″ welds for vertical shear $= 36{,}000 \div 3000 = 12''.$



Trial Section:

Try a split beam 14WF38 with three 6″ lengths of weld.

$$e = \frac{6 \times 3}{18} = 1''.$$

$$\begin{aligned} I = \ & 6 \times 2^2 &&= 24 \\ & 12 \times 1^2 &&= 12 \\ & 2 \times \tfrac{1}{12} \times 6^3 &&= 36 \\ \hline & I \text{ (Weld)} &&= 72 \end{aligned}$$

$$\text{Flexural stress in weld} = \frac{63{,}000 \times 4}{72} = 3500\#/''.$$

$$\text{Vertical shear} = \frac{36{,}000}{18} = 2000\#/''.$$

Resultant throat stress $= \sqrt{3500^2 + 2000^2} = 4030\#/''.$ *Use ½″ welds 6″ long.*

Remarks: *A resultant throat stress is computed here because it is in accordance with the theory explained in § 51.*

DP15. *Design a welded angle seat with stiffeners for a 24"-110# I-beam. End reaction = 30,000 lb. Hold throat stress to 11,300#/□".*

An adequate seat for this 24" beam requires a 6 × 6 × ½" angle 9" long. Three triangular stiffeners will be used as indicated by the sketch. Welds to the column are 9" long.

Case 1. *Based upon the assumption that the welds resist flexure by means of a T-C couple of arm equal to the angle leg.*

Unit vert. shear =

$$\frac{30{,}000}{18} = 1670\#/''.$$

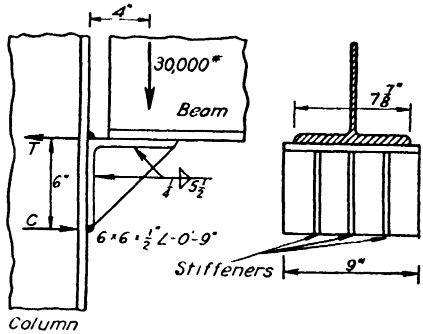
Unit horiz. shear =

$$\frac{30{,}000 \times 4}{6 \times 9} = 2220\#/''.$$

Max. unit stress (tension) =

$$\sqrt{1670^2 + 2220^2} = 2780\#/''.$$

A ⅜" weld 9" long at top and bottom of the ∠ will provide adequate strength.

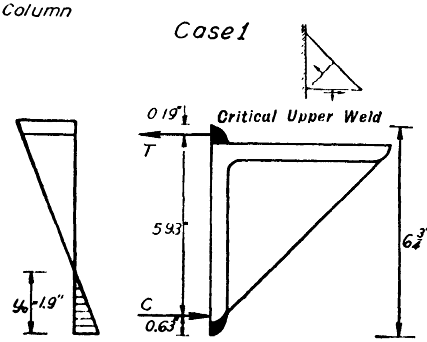


Case 1

Case 2. *Assume application of flexure formula and bearing between angle and column flange below the neutral axis.*

Cross-section consists of:

Tension area — 9 × 0.375 = 3.4□",
Comp. area — 9y_0.



Case 2

Hence, $3.4(6.56 - y_0) - 4.5y_0^2 = 0$, *or* $y_0 = 1.9''$.

Therefore, the arm from T to C is 6.75 − 0.19 − 0.63 = 5.93", which is reasonably close to the depth of the angle as assumed in Case 1.

$$\text{Unit horiz. shear} = \frac{30{,}000 \times 4}{5.93 \times 9} = 2250\#/''.$$

$$\text{Max. unit stress} = \sqrt{1670^2 + 2250^2} = 2800\#/''.$$

Remarks: *In the critical upper weld this resultant stress is actually throat tension. It is interesting to note that the maximum unit stresses computed in Case 1 and Case 2 were almost identical. Hence, the simpler calculation (Case 1) is commonly used in design.*

MOMENT RESISTANCE

59. Weld Design for Flexure. Transverse flexure should certainly never be permitted on a single line of weld and even two lines of weld separated by a plate or an angle leg of ⅜-in. or ½-in. thickness have relatively little flexural resistance. Two lines of weld separated by several inches may be designed to resist transverse flexure. Relatively long lines of weld furnish good flexural resistance when subjected to longitudinal flexure.

EXAMPLE *DP*14. We will investigate on design sheet 14 the action of longitudinal flexure on a set of welds which are shaped like the Greek letter π. Calculations are based upon the flexure formula and only the weld section is taken into consideration. Actually, the lower stem of the split-beam section will *bear* against the column and will resist some compression. If the split-beam section had been faced for bearing, we could properly take its area into consideration below the neutral axis, but, since it is commonly burned to shape, the bearing area in contact is likely to be so small that it may well be neglected as was done in this example.

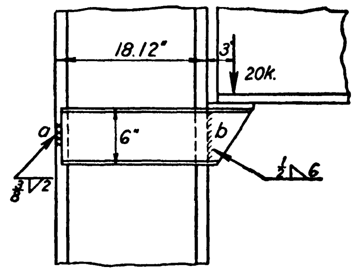
EXAMPLE *DP*15. A different case is illustrated by the example *DP*15. Here the welds are transverse and in Case 1 they are assumed to resist both the tension and compression forces caused by flexure. For Case 2 the assumption of a straight-line variation of stress is made and the angles are considered to bear properly against the flange of the column. The result in terms of required length of weld is essentially the same as for Case 1. Hence, it is suggested that Case 1 be used as a simple design procedure. The possibility exists of reducing the length of weld along the bottom of the angle to about that required for vertical shear alone, that is, 15,000 ÷ 3000 = 5 in. Such a solution would be acceptable if care had been used to clamp the lower part of the angle tightly to the face of the column before welding. A slightly longer weld would then be needed at the top of the angle to care for the larger percentage of the vertical shear resisted by the upper weld.

EXAMPLE *DP*16*a*. This example is the design of a welded seat attached to the flanges of a column. Such seats are required for *spandrel* or *wall beams* which may even have to be supported entirely outside of the width of the column. In *DP*16*a* the two vertical welds resisting the eccentric load are spaced far enough apart that the weld shears may be treated as vertical forces with slight approximation. For welds more closely spaced, the only tool of analysis readily available is the torsion formula which will be applicable only if the channel used is so thick and heavy that it cannot be distorted appreciably, the distortion being almost *entirely in the welds.*

60. Composite Connections Undergoing Flexure. A group of design problems is offered to illustrate how flexural resistance can be introduced into the end connections of beams, girders, and columns to produce continuous beams or continuous-frame action.

Wind Connection. The example *DP*16*b* is the typical welded connection used to produce wind resistance or continuity between the columns and girders of a tall building or skyscraper. Since the moment resistance to be developed is often less than the full moment resistance of the girder, the connection is not made to the entire section but is concentrated on the flanges where moment resistance can be developed with the least welding and therefore at the smallest expense. For convenience in design it is assumed that the vertical shearing force of 33,000 lb. is resisted entirely through the welds on the

DP16a. *Design a welded channel seat attached to the flanges of a 14WF38 beam as illustrated. The reaction of 20 k. is carried by a single channel.* $s_s = 11{,}300\#/\square''$.



$R_b = 20{,}000 \times 21.12 \div 18.12 =$ 23,300 lb.

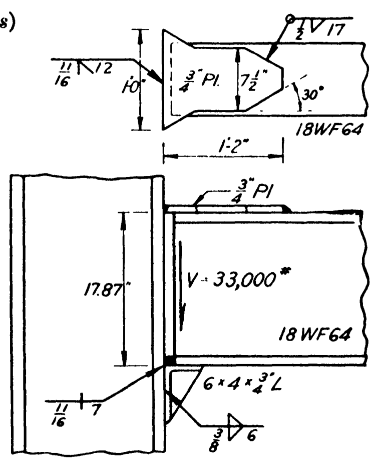
$23{,}300 \div 6 = 3890\#/''$ *of fillet.*

Use a ½" fillet 6" long at b.

$R_a = 20{,}000 \times 3 \div 18.12 = 3310$ *lb.*

Use a ⅜" fillet 2" long at a.

DP16b. *Design a welded wind connection to develop 50% of the moment resistance of an 18WF64 beam. End shear = 33,000#. AISC and AWS spec.*



Shear Connection: *(Lowest weld values)*

Welds on angle to resist vertical shear are ⅜". Value = 3000#/".

$$Length = \frac{33{,}000}{3000} = 11.0''.$$

Use a seat angle with 6" vertical leg and weld to col. with ⅜" side welds 6" long.

Moment Connection:

Sect. mod. of beam = 117.0.

$M_R = 20{,}000 \times 1.33 \times 117.0 = 3{,}120{,}000''\#$.

$$T = C = \frac{3{,}120{,}000}{2 \times 17.87} = 87{,}000\#.$$

For a 12" butt weld to col., the value needed per in. is $87{,}000 \div 12 = 7250\#/''$.

At an increase of 33⅓% for wind, an 11/16" weld has a value of $5500 \times 1.33 = 7330\#/''$.

An 11/16" or ¾" tie pl. is satisfactory.

$$\text{Length of ½'' fillet at the beam end of pl.} = \frac{87{,}000}{1.33 \times 4000} = 16.3''. \text{ Use } 17''.$$

The flg. thickness of 0.68" will permit a compression butt weld of value

$15{,}000 \times 1.33 \times 0.68 = 13{,}600\#/''$. *The length needed is*

$87{,}000 \div 13{,}600 = 6.4''$. *Use 7". The flg. width is 8¾".*

seat angle. This will not be true entirely although the upper plate is too flexible to resist much vertical shear. Actually, therefore, the shear is divided between the lower butt weld to the girder flange and the welds to the seat angle. The same dual resistance is developed to the compression force accompanying flexure if the girder is welded to its seat. Hence, it has been convenient and not unreasonable to *design each weld for a separate force* rather than to attempt to analyze an interaction that depends largely upon the procedure in placing the welds.

BEAM CONTINUITY, EXAMPLE *DP*17. Continuity is to be produced between two beams which frame into a girder. These beams might be stringers of a bridge floor or joists in a building. The device of using a channel to produce a tie spaced 2¼ in. above the top of the beam is an excellent design in that it saves the cost of *coping the beam* to raise it to the level of the girder. The channel welds must resist not only the pull across the top of the connection but also the eccentric moment caused by the 2¼-in. lift of the tie plate. Because of a modern device, the burning torch, this channel can be shaped at little cost. Other possible connections could be made by slotting the web of the girder for a tie plate or by raising the tie plate, either upon two short lengths of channel or upon two thick blocks of steel — all details of much greater cost.

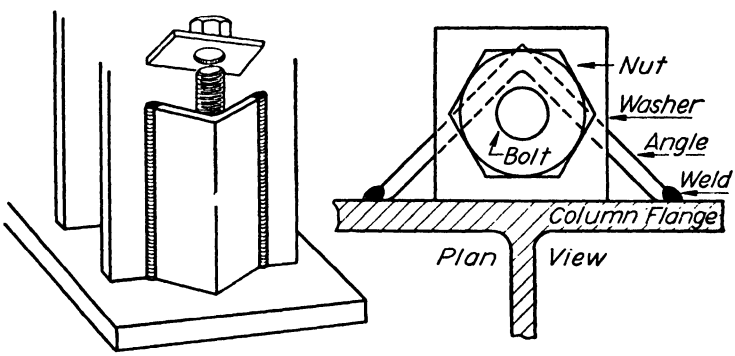


FIG. 63. COLUMN ANCHORAGE.

61. Column Fixation. The detail illustrated in Fig. 63 is one of the most successful welded connections. It makes possible rather heavy moment resistance at the end of a column. The principle of column fixation is to produce an *initial tension* in anchor bolts which will then resist all tendency toward further stretching (until the applied tension is greater than the initial tension). The result will be an entirely fixed column except for a possible rotation of the footing or deformation of the anchorage angles. The connection of Fig. 63 has the advantage that it is composed of but a single angle on each side of the column welded directly to the column flange. Deformation of the connection can therefore be reduced to a negligible factor.

EXAMPLE *DP*18. In the design problem *DP*18 the hold-down angles are 6 × 6 × ½-in. angles. They allow adequate clearance for turning the nuts on the 2-in. anchor bolts. These angles also are able to act as columns to support the load introduced by

DP17. *Design a connection for a continuous beam which is a 14WF30 section. The beams frame into a 24WF94 girder. Use AISC and lowest AWS stresses.*

Channel Tie:

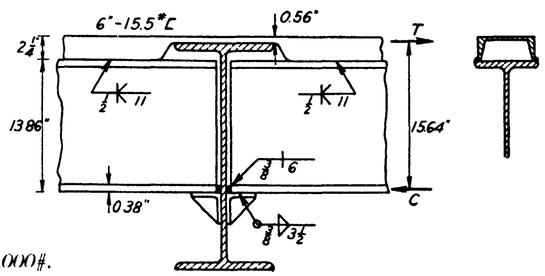
Section Modulus of 14WF30 = 41.8.

$M_R = 20{,}000 \times 41.8 = 836{,}000''\#.$

Approx. arm from T to C = $15\frac{1}{2}''$.

Approx. value of

$$T = \frac{836{,}000}{15.5} = 54{,}000\#.$$



Web area of tie channel $= \dfrac{54{,}000}{20{,}000} = 2.7\square''.$

Select a $6''$-$15.5\#$ [; $A_W = 3.3\square''$.

Welded Connection:

$$\text{True arm from } T \text{ to } C = 13.86 - \frac{0.38}{2} + 2.25 - \frac{0.56}{2} = 15.64''.$$

$$T = C = \frac{836{,}000}{15.64} = 53{,}400\#$$

Compression weld has value of $0.38 \times 15{,}000 \times 6 = 34{,}200\#$

Diff. $= 19{,}200\#$

This force is resisted by $\frac{3}{8}''$ *fillets to seat.*

$$\text{Length of fillet} = \frac{19{,}200}{3000} = 6.4''. \text{ Use } 7''.$$

The channel tie is connected to the beam by 10" fillets that undergo longitudinal shear and longitudinal flexure.

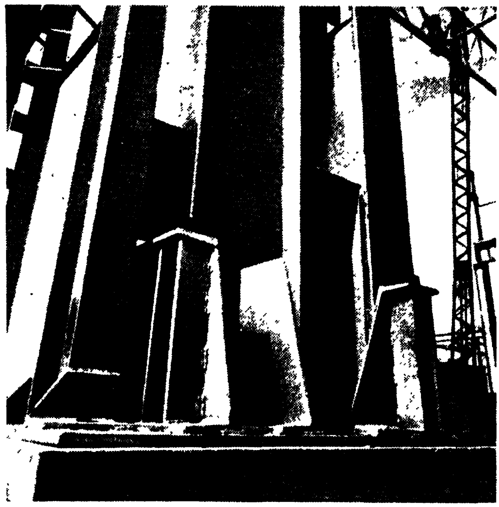
$$\text{Unit longitudinal shear} = \frac{53{,}400}{20} = 2670\#/''.$$

$$\text{Unit shear from flexure} = \frac{53{,}400\,(2.25 - 0.28) \times 5}{\frac{1}{12} \times 2 \times 10^3} = 3150\#/''.$$

$$\text{Resultant shear on throat} = \sqrt{3150^2 + 2670^2} = 4120\#/''.$$

Remarks: *A* $\frac{1}{2}''$ *fillet weld has a value of 4000#/". An 11" weld will be used to reduce the shear below 4000#/". Also the flange of the 14WF30 is only* $6\frac{3}{4}''$ *wide and the channel flange must be beveled to permit* $\frac{1}{2}''$ *welds on each side. The ratio L/a for this weld is 22 which has little effect on the working stress, equation (16).*

the anchor bolts. The side welds on the angles are designed to resist the resultant of the vertical shear produced by direct load and the horizontal shear computed from the flexure caused by the 2½-in. eccentricity of the load. It is also considered important in this instance that the allowable shear on these welds be reduced to allow for end concentration in long welds undergoing longitudinal shear.



Courtesy Eng. News-Record

FIG. 64. WELDED COLUMN ANCHORAGE.

PROBLEMS

(Lowest *AWS* working stresses except as noted.)

63. Design a lap welded circular girt seam for a water tank where the ⅜-in. plates must carry a stress of 10,000 lb. per sq. in. across the joint.

64. Design a lap welded longitudinal seam for a circular tank 10 ft. in diameter to withstand an internal water pressure of 120 lb. per sq. in. Select the plate thickness for a stress of 20,000 lb. per sq. in. and add ⅛ in. for corrosion.

65. Repeat Problem 64 for a butt welded joint with reinforcing straps to obtain 100 per cent efficiency for the plate. Use a pressure of 240 lb. per sq. in.

66. Design a splice joining two tension members one of which is one half of a 12*WF*36 beam and the other is composed of two angles 3 × 3 × ⅜ in. with the legs turned out. The connection is to develop the full value of the angles.

67. Repeat Problem 66 for a split-beam section obtained from a 21*WF*112 beam and two 6 × 6 × ¾-in. angles. Design the splice for 80 per cent of the value of the angles.

68. Design an end connection to develop the full tensile value of an 8-in., 16.25-lb. channel in a length of 5 in. The channel is connected to a ⅜-in. plate and welds are limited to ⅜ in. Use *AISC* allowable stresses from § 55.

69. Repeat Problem 68 for an 18-in., 58-lb. channel with ½-in. welds limited to a 10-in. length along the member.

70. Design an end connection to develop the full tensile value of a 6 × 4 × ½-in. angle connected by its 6-in. leg to a ½-in. plate. The length of the connection is not limited. Use two ½-in. fillets with no reduction in working stress.

DP18. *Design a column anchorage for 25% of the moment resistance of a 12WF106 column section. AISC spec.; weld stresses from equation (16).*

Anchor Bolts:

Sect. mod. = 144.5.

$M_R = 20{,}000 \times 144.5 = 2{,}890{,}000''\#.$

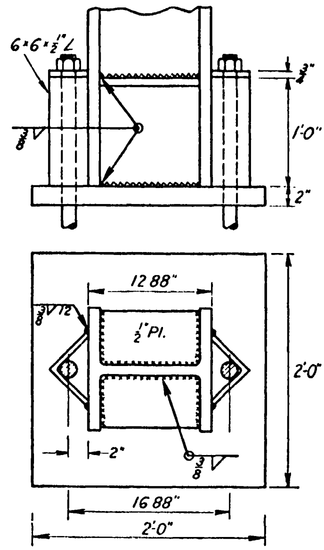
$$\text{Bolt stress} = \frac{2{,}890{,}000}{4 \times 16.88} = 42{,}800 \text{ lb.}$$

$$\text{Net area needed} = \frac{42{,}800}{20{,}000} = 2.14\square''.$$

Root diam. of a 2″ bolt = 1.71″.

Diam. reduced $\frac{1}{16}$″ for stress concentration at root of notch = 1.65″.

$$\text{Net area furnished} = \frac{\pi \times 1.65^2}{4} = 2.14\square''.$$



Welded Connection:

Weld length needed for shear for a ⅜″ fillet $= \dfrac{42{,}800}{2 \times 3000} = 7.1''.$

$$\text{Moment acting on one weld} = \frac{42{,}800 \times 2.0}{2} = 42{,}800''\#.$$

Guess weld length at 12″ and reduce working stress by formula (16);

$$S = 3\left(1100 - 5\frac{12}{0.375}\right) = 2820\#/''.$$

$$\text{Shear from direct pull} = \frac{21{,}400}{12} = 1790\#/''.$$

$$\text{Shear from flexure} = \frac{42{,}800 \times 6}{\frac{1}{12} \times 12^3} = 1790\#/''.$$

Resultant shear on throat of weld =

$$\sqrt{1790^2 + 1790^2} = 2530\#/''.$$

Make the welds 12″ long. A 6 × 6 × ½″ angle will provide clearance for the bolt. Add ½″ diaphragm plates as illustrated.

71. Repeat Problem 70 but reduce the working stress on welds over $20a$ in length according to equation (16).

72. Repeat Problem 70 but limit any weld to a length of 10 in.

73. Design a connection for the angle of Problem 70 with no weld more than 6 in. long.

74. Design a welded bracket seat using one half of a 16*WF*58 beam to resist an end reaction of 46,000 lb. located 1⅞ in. from the face of a column. The welds are designed to provide all resistance. Use *AISC* allowable stresses from § 55.

75. In Problem 74 assume that the face of the *tee* is finished for bearing against the column. Allow for bearing of metal against metal below the neutral axis as well as for the resistance of the welds and determine the sizes of welds required.

76. Design a split-beam seat for the 26,000-lb. end reaction of a 27*WF*91 beam. The reaction acts 2¼ in. from the face of the column.

77. Repeat the calculations of the example *DP*15 for an end reaction of 36,000 lb. acting 3¾ in. from the face of the column. Allow the highest *AWS* working stresses.

78. Design an angle seat with stiffeners as in the example *DP*15 for an end reaction of 58,000 lb. acting 4 in. from the face of the column. Do not increase the welds beyond ½ in. but add welds on the edges of the angle if necessary. The column flange is 10 in. wide. Allow the highest *AWS* working stresses.

79. Redesign the connection of *DP*16*a* for a reaction of 32 k. acting 4½ in. from the face of the column. Allow the highest *AWS* working stresses.

80. Design a welded seat angle to fit between the flanges of a 14*WF*314 column as in the upper illustration of Fig. 40*a*. This seat carries a 12-in. beam having an end reaction of 15,000 lb. eccentric 2 in. from the center line of the column (nearer to one flange than the other). The angle should be stiffened since it is connected only by its vertical leg to the inside of the column flanges. The reaction line is 1½ in. outside of the center of the vertical leg. Use working stresses as recommended by the Lincoln Electric Company.

81. Design a welded wind connection as in the example *DP*16*b* for 50 per cent of the moment resistance of a 21*WF*73 beam. The flange width of the column is 16 in.

82. Repeat Problem 81 for the full moment value of the beam and for an end shear of 30,000 lb. Allow the highest *AWS* working stresses.

83. Design a welded end connection to the web of a 12*WF*92 column for a 12-in., 31.8-lb. I-beam. Develop a moment resistance of 500,000 in-lb. and an end shear of 22,000 lb. Use standard working stresses as given in a City Building Code.

84. Design a connection similar to the one illustrated by the example *DP*17 to replace the moment resistance of a 16*WF*45 beam and thus provide full continuity across the girder. The top of the girder is 2 in. above the beam.

85. Repeat Problem 84 but slot the girder to permit a splice plate to pass through. Allow the highest *AWS* working stresses.

86. Redesign the base connection of the example *DP*18 for 40 per cent of the moment resistance of the 12*WF*106 column. Use stiffener plates perpendicular to the face of the column in place of the angle in order to obtain double the number of lines of weld.

87. Design a welded base connection to develop 30 per cent of the moment resistance of a 14*WF*150 column. Use 2 anchor bolts on each face of the column. For such heavy work, stiffener plates provide a better detail than angles, but such plates should be joined together by a welded plate outside of the anchor bolts. Use the highest *AWS* allowable stresses.

62. The Past and Future of Structural Welding. Structural welding passed through the preliminary or trial stage from 1920 to 1930 and was then accepted as a practical method of connecting structural members.

The decade from 1930 to 1940 seems to have been a period of adjustment for the assimilation of the new tool. Naturally, there have been some fabricators who welcomed the opportunity to enter the welding field while others preferred to use the time proved method of riveting. Almost universally, however, steel fabricators have accepted welding as a useful device for attaching column base plates, clip angles, beam seats, etc.

Impact Resistance. A question that has been raised repeatedly concerns the impact resistance of weld metal. Charpy tests on weld metal placed by the bare electrode, made under the author's direction in 1930–31, showed consistently low values of impact resistance.* In contrast, weld metal placed by heavily coated electrodes proved to be very *ductile* and showed a *high Charpy value,* in fact even greater than that of structural steel. This is convincing proof that there need be little question as to the impact resistance of weld metal placed by use of proper electrodes heavily coated to shield the weld from the atmosphere.

Shrinkage and Plastic Deformation. Cooling stresses are serious in welded structures since unequal shrinkage is inevitable. It is known, however, that equally high stresses occur from unequal cooling of structural sections after they have passed through the rolls in the mill. Furthermore, these sections must be straightened in the fabrication shop and this procedure leaves residual stresses. An occasional failure of a brittle piece of steel from a light blow indicates that such stresses may approach the elastic limit of the material. The property that protects structural steel from injury by these internal stresses is its ductility. Welds of ductile metal will be equally safe.

To the designer, ductility has another significance. Unequal stress distribution from welding or from other fabrication or erection methods will be ironed out by plastic deformation. Those fibers stressed to the yield point deform and do not resist additional load that may come on the structure. Less heavily stressed parts or particles must resist such load increments until they in turn reach the elastic limit. Near the ultimate load the result is a structure with stresses that approach those which would have existed if there had been no initial shrinkage stresses.

Fatigue Failure. It is interesting to speculate upon the importance of these shrinkage stresses when failure occurs by impact or fatigue. Fatigue failures occur without evident stretch or plastic deformation. It does not seem reasonable, therefore, to assume that the plastic deformation above the yield point is available for redistributing relatively low stresses that may eventually result in fatigue failure. On the other hand, localized stresses will not ordinarily exceed the yield point. *Impact failure* does involve considerable plastic deformation and there is probably a redis-

* H. C. Givens and B. W. Farquhar, Thesis Studies for the M.S. Degree, Texas A. and M. College.

tribution of stress through such plastic deformation, but it may not be as effective as it is known to be under static loading.

Tests reported at the University of Illinois in 1939 by W. M. Wilson seem to show conclusively that butt welded joints will fail at about 23,000 lb. per sq. in. plate stress under 2,000,000 repetitions of a given kind of stress irrespective of the *static strength* of the plate. These tests were conducted upon joints of full size. It is probable that some reduction factor should be considered in the design of bridge welds since two million repetitions are within the number to be expected in the life of a major bridge structure.

Welding Versus Riveting. There has been much discussion of the possibility that welding will eventually replace riveting entirely. Such a result is not to be expected. Rivets have not eliminated the use of pins and concrete did not replace steel as a universal structural material. Timber is still in wide use. The criterion is primarily economic usefulness and welding will therefore replace riveting only where it is a cheaper method of construction. The factor of appearance enters and is important, but the major influence will be relative costs.

CHAPTER 4

PINS AND BOLTS FOR CONNECTIONS

63. Bolts, Rivets and Pins. Bolts may be used in place of rivets under most specifications but at reduced working stresses. Rough unturned bolts in punched holes are usually allowed the same values as old fashioned hand driven rivets. Turned bolts in drilled or reamed holes are allowed the same values as power driven rivets under the *AISC* specifications, but the *AREA* specifications allow them only the values of hand driven rivets. It is observed that structural fabricators often take advantage of the *economy of bolted connections* in constructing their own buildings. Since bolts properly tightened produce plate friction that resists shearing loads, there would be every reason to allow greater shear and bearing values for high strength alloy steel bolts than for those of low carbon steel.

Tension in Bolts. A distinction between bolts and pins is that the former may resist tension. A pin should not be required to resist longitudinal tension since such stress produces friction that prevents free turning of the pin — one of its principal functions. Bolt tension is limited by the *net area* at the root of the thread. The threaded part of a turned bolt is smaller than the shank as governed by the following common specification.

Turned bolts used to transmit shear shall be 1/32 in. less in diameter than the hole and the threaded portion shall be 1/16 in. smaller than the shank. The length of the shank shall be 1/16 in. greater than the grip. There shall be a washer not less than 1/4 in. thick under the nut.

The concentration of tensile stress that occurs at the root of the thread of a bolt can be allowed for either by a reduction of working stress, which is required by some specifications, or by a reduction of effective area. A specification sometimes recommended is to use as effective area the area of a circle of diameter 1/16 in. smaller than the diameter at the root of the thread.

64. Structural Uses for Pins. A pin functions essentially as a single rivet or bolt. Its size may range from the common cotter pin of 3/8-in. or 1/2-in. diameter, used for connecting strap-iron bars, to the railway bridge pin 12 in. or more in diameter. Fixed shafts or trunnions for bascule bridges are also pins and they may be of much larger size. Pins of more than the 9-in. diameter have a 2-in. hole drilled longitudinally on

the center line to aid in the relief of locked-up stresses. This hole may be used to carry a 2-in. bolt for clamping circular caps on the ends of the pin to take the place of large lock nuts.

Building Structures. Light diagonals can often be connected to column or beam flanges by pins, thus permitting the use of tie rods at less expense than diagonal angle bracing. An end clevis, as shown in Fig. 65, may be used for such connections. Pin connected tension rods, with adjustable turnbuckles to introduce initial tension, form excellent diagonals for water towers, elevated bins, etc. Hinged arches are used for coliseums and for other large open buildings. There may be as many as three pins introduced into each arch to control its structural action. Since a pin is considered to be a *point of zero moment*, the introduction of each pin simplifies the analysis of an indeterminate structure. Pin connected trusses, similar

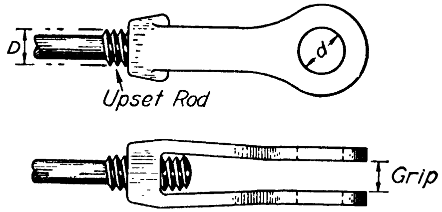
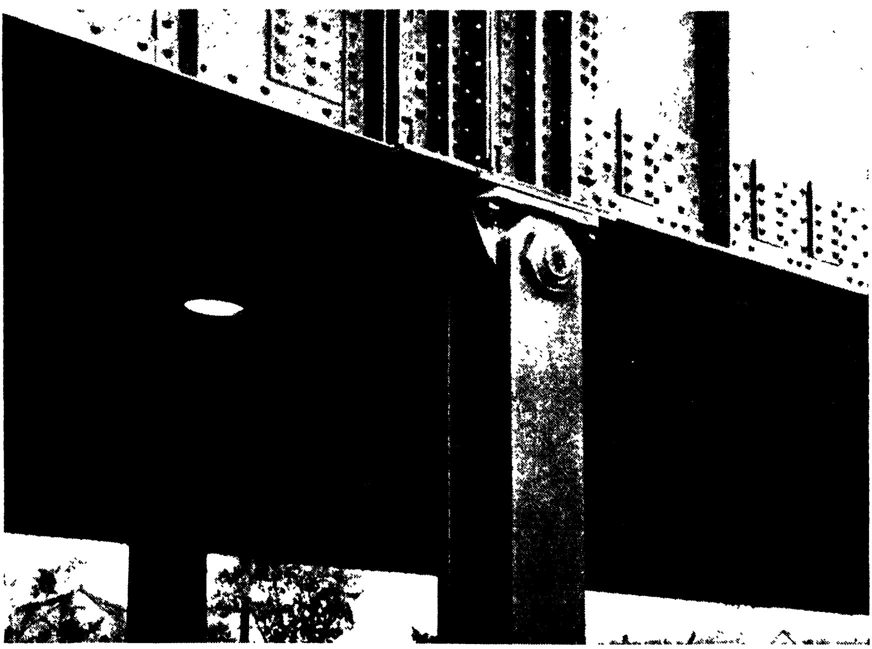


FIG. 65. CLEVIS.



Courtesy C. M. St. P. & P. R.R. Co.

FIG. 66. PIN CONNECTED COLUMN SUPPORTING GIRDER.

Observe in the photograph the pin casting, the pin nut, girder stiffeners, floor stringers, and open drain hole.

to those used for bridges, are occasionally used in buildings to carry extremely heavy column loads across ballrooms or auditoriums.

Bridge Structures. In light bridge trusses it is reasonably common to use pin connected tie rods for diagonal bracing, although angle bracing is often specified for stiffness. End pins and end rollers are provided in all except the smallest bridge trusses to allow for expansion and to permit the end rotation that accompanies deflection. Viaduct columns, as shown by Fig. 66, may be pin connected at both top and bottom to provide for the *expansion of great lengths of roadway.* Suspension bridge towers have even been hinged at the base (requiring very large pins) to provide for change in length of the straight back-stay cables under stress. Perhaps, however, the most extensive use for pins in bridge structures has been for large pin connected railway truss bridges where a pin occurs at each panel point of each chord. The advantages of using pins are the reduction of secondary stresses that should accompany freedom of pin rotation and the common use of heat treated eye bars of high strength as tension members. Disadvantages are the lack of rigidity which produces a loose noisy bridge under traffic vibration (particularly for light structures) and the necessity of expensive machine work on pins and pin holes. The cost of machine work along with the cost of pin plates, that must be attached to the compression members to provide adequate bearing area against the pins, just about balances the saving of weight and cost achieved by the use of eye-bar tension members. The result is that riveted and pin connected bridges compete on about equal terms in regard to cost.

Pin Design

65. Factors in Pin Design. The design of a pin follows essentially the same procedure as the design of a rivet. The process will be simplified if we think of a pin as a single large rivet. *Shear, bearing,* and *flexure* must be investigated.

Shear may determine the diameter of the pin and since the pin is a very important part of the structure, we should investigate the shear properly by use of the beam-shear formula. The pin is, in fact, a deep beam. For a circular section this formula reduces to

$$s_s = \frac{VQ}{Ib} = \frac{4}{3}\frac{V}{A} = \frac{16V}{3\pi d^2}. \tag{1}$$

The unit shear allowance for pins is usually found to be the same as the maximum value permitted for power driven rivets.

Bearing need not be treated any differently for pins than for rivets, although the allowable unit bearing value may be changed. It is usually possible to increase the bearing area by riveting or welding extra pin plates

to the end of the member without increasing the diameter of the pin. *Bearing on eye bars determines the minimum pin diameter* as the width of the widest bar (b) times the ratio of the allowable stress in tension to the allowable stress in bearing, that is, $d = b(f_s/f_b)$. *AREA* specifications further limit the minimum pin diameter to $\frac{8}{10}$ of the width of the widest bar attached to it. (Spec. 185.) Allowable bearing values between rocker pins and cast steel rockers should be reduced to resist wear. This reduction is 50 per cent in the *AREA* specifications. (Spec. 164.)

Flexure is more serious in pins than in rivets or bolts since the bearing plates on a pin are separated to permit clearance for free rotation. If two built-up riveted members bear on a pin, the adjacent faces of these two members may have to be separated as much as $1\frac{5}{8}$ in. This will be the case if the 1-in. rivet heads are not flattened or countersunk and are separated the required distance of $\frac{1}{4}$ in. Evidently, the bending moment in the pin will be increased greatly by such separation of members and the pin diameter may therefore be controlled by flexure. The allowable flexural stress in a pin is usually 50 per cent greater than the allowable tension in a truss member. The facts that (1) secondary stresses do not occur in pins, (2) large pins (over 7 in.) are forged and annealed, and (3) there are fewer possibilities of introducing fabrication and erection stresses in pins than in truss members are justifications for a higher working stress. For the purpose of computing bending moments, it is usual to assume that each plate or united group of plates which bears on the pin will produce a concentrated bearing load. This is a conservative procedure which simplifies calculations and does not add appreciably to the design moment.

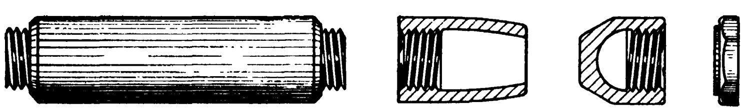


FIG. 67. BRIDGE PIN AND NUTS.

The moment of inertia of a circular area is $\pi r^4/4$ or $0.049d^4$. The section modulus for a cylindrical pin becomes $I/c = 0.049d^4 \div d/2 = 0.098d^3$. Hence, for a circular pin, we may write

$$d = \sqrt[3]{\frac{M}{0.1f}} = \sqrt[3]{\frac{10M}{f}}. \tag{2}$$

66. Chain-Link Pin. Suspension bridges may be supported either by parallel wire cables, by wire rope strands, or by chain-link cables. The

latter are used for spans of medium length. Such chain links are connected by pins that are subject to bearing, shear, and flexural stresses.

EXAMPLE *DP*19. The example *DP*19 illustrates the design of a pin to develop the strength of a pair of 8-in. by 2-in. eye bars. Two procedures are always possible in such design problems. (1) We can calculate the actual stresses for an assumed size of pin and compare them with the allowable stresses, thus determining whether a revision of size is necessary. This is the procedure used in the example *DP*19 for checking the minimum pin diameter for shearing and bearing stresses. (2) We can use the allowable unit stresses in bearing, shear, and flexure to determine the pin size to resist each stress and then we must choose the largest pin required by any type of stress. This method is used to select the pin diameter for flexure in the example *DP*19. Some designers prefer the first procedure and some prefer the second. The author usually attempts to select any structural member or part according to his best judgment as to the probable controlling stress. Then this tentative design is checked to determine its resistance to the other stresses involved.

Capacity to Resist Load. A third procedure in design is to determine the capacity of a part tentatively selected and to compare it with the applied load. *This seems the least convenient procedure of all.* It would be exemplified by the following calculation relative to the problem *DP*19. These calculations are for a 6-in. pin.

$$\begin{aligned}
\text{Bearing resistance} &= 27{,}000 \times 6 \times 2 &&= 324{,}000 \text{ lb.}\\
\text{Shearing resistance} &= 13{,}500 \times 0.75 \times \pi \times 6^2/4 &&= 287{,}000 \text{ lb.}\\
\text{Flexural resistance} &= 0.1 \times 6^3 \times 27{,}000/2.125 &&= 274{,}000 \text{ lb.}
\end{aligned}$$

Since the load is 288,000 lb. per bar, it is evident that the 6-in. pin is adequate for bearing and shear resistance but not for flexure.

BRIDGE PINS

67. Pin Packing. When diagonals, posts, and chords are brought together on a pin, there are several possibilities of arrangement or of "packing" the members. For instance, we might arrange for the diagonal composed of two eye bars to be on the outside of the top chord and for the beam-section post to be placed on the inside of the chord member. (See Fig. 68.) But we must realize that the post forms the reaction to the stress in the diagonal, for vertical flexure of the pin.

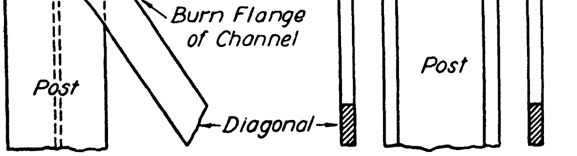


FIG. 68. INCORRECT PIN PACKING.

Hence, it is evident that this *improper arrangement* of separating the post and diagonal by the chord web produces a large vertical flexural moment in the pin. An equally heavy horizontal flexure may be produced by

DP19. *Design a pin to connect two pairs of parallel eye bars of size 8″ × 2″. The eye bars of a single member are spaced at least 4″ apart and those of adjacent members at least ⅛″ apart. AASHO spec.*

Minimum Pin Diameter:

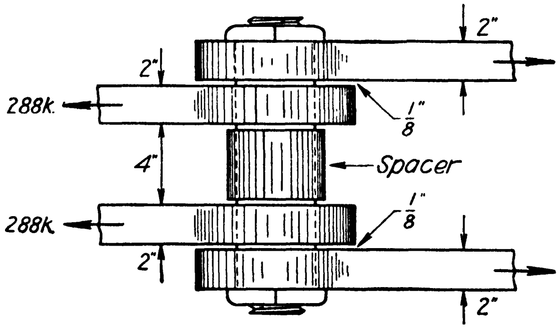
Minimum diameter is ¾ of the width of the widest bar or 6″.

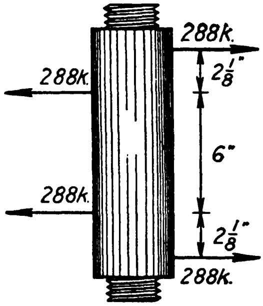
Bar tension. $8 \times 2.0 \times 18{,}000 = 288{,}000\#$.

Bearing unit stress. $288{,}000 \div 6 \times 2.0 = 24{,}000\#/\square''$ $(< 27{,}000)$.

Average shear stress. $288{,}000 \div (\pi \times 6^2/4) = 10{,}200\#/\square''$ $(< 13{,}500)$.

Arrangement of bars. Bars must be alternated to maintain symmetry.





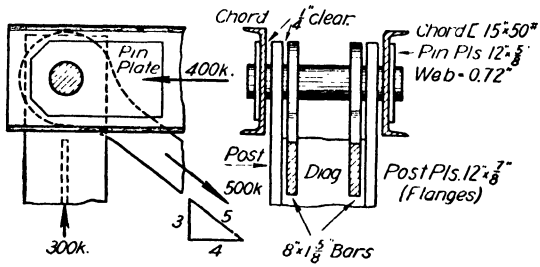
Bending moment. $M = 288{,}000 \times 2.125 = 613{,}000''\#$.

Diameter for flexure. $d = \sqrt[3]{\dfrac{613{,}000}{0.1 \times 27{,}000}} = 6.1''$. [*Equation 2.*]

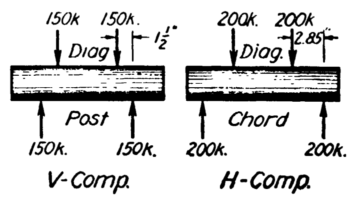
Remarks: *The minimum pin diameter of 6″ would be adequate for shear and bearing but it must be increased to 6¼″ for flexure.*

DP20. *Arrange a horizontal chord member, a vertical post, and an eye-bar diagonal of a building truss to meet on a pin so that the pin diameter is a minimum. The dimensions of the post may be varied. Allow ¼″ between adjacent plates and bars. AISC spec.*

Case 1: *Chord outside; diagonal inside; post between.*



Bending moment.



H-comp. $= 200{,}000 \times 2.85 = 570{,}000''\#.$

V-comp. $= 150{,}000 \times 1.50 = 225{,}000''\#.$

$M_{resultant} = \sqrt{570{,}000^2 + 225{,}000^2} = 612{,}000''\#.$

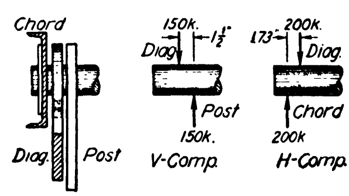
$$\textit{Diameter} = \sqrt[3]{\frac{612{,}000}{0.1 \times 30{,}000}} = 5.9''.$$

Bearing stress. (Post controls) $D = \dfrac{150{,}000}{32{,}000 \times 0.87} = 5.4''.$

Shearing stress. (Max. $V = 250$*k.)* $D = \sqrt{\dfrac{16 \times 250{,}000}{3 \times \pi \times 15{,}000}} = 5.3''.$

Case 2: *Chord outside; post inside; diagonal between.*

Bending moment.



H-comp. $= 200{,}000 \times 1.73 = 346{,}000''\#.$

V-comp. $= 150{,}000 \times 1.50 = 225{,}000''\#.$

$M_{resultant} = \sqrt{346{,}000^2 + 225{,}000^2} = 413{,}000''\#.$

$$\textit{Diameter} = \sqrt[3]{\frac{413{,}000}{0.1 \times 30{,}000}} = 5.2''.$$

Remarks: *Since the arrangement of Case 2 gives a smaller diameter for flexure than for bearing, we will use this arrangement and make the pin 5½″ to care for bearing stress. The AISC spec. do not require the pin to be 75 or 80 per cent of the width of the widest eye bar.*

placing the post inside of the chord and by arranging the diagonal eye bars inside of the post so that the chord and the diagonal are then separated by the post. Thus, a large lever arm is produced between the horizontal component of the diagonal stress and its reaction which is the stress in the horizontal chord.

EXAMPLE OF PACKING, *DP*20. The arrangement just mentioned is illustrated by the sketch for Case 1 of the example *DP*20. The lever arm is 2.85 in. which gives rise to a resultant moment of 612,000 in-lb. The moment desired, of course, is the resultant of the moments computed separately from the vertical and horizontal components of the forces on the pin.

Minimum Size of Pin. The most desirable arrangement from the point of view of pin selection is shown as Case 2 of the example *DP*20. The diagonal is placed between the chord and the post so that its two reactions (chord and post) are both *adjacent to it.* The greatest lever arm then reduces to 1.73 in. with a reduction of pin moment to 413,000 in-lb. A pin of 5¼-in. diameter would then be adequate for flexure, but a 5½-in. pin was found in Case 1 to be necessary for bearing or shear. In some instances bearing stresses will become so serious that an oversize pin may be desirable to reduce the number of pin plates required on the compression members. Under such circumstances an arrangement such as Case 1 of *DP*20 may not be objectionable because a reduction of pin moment obtained by placing the diagonal between the chord member or members and the post would not permit a reduction in pin diameter. Other specifications regarding the minimum pin diameter may also influence pin packing.

68. Bridge Pin Packing and Design. The members of railway bridge trusses of long span are often pin connected. Since there may be a dozen or more panel points on one side of the center line of the truss, we are faced with many possibilities in arranging the members on the pins. Of course, the packing at one joint influences the packing at an adjacent joint because built-up members have a constant width from end to end and eye bars are permitted to *slope* no more than 1/16 in. per foot of length.

Upper chord members butt end to end at the pin as shown in Fig. 69(*a*). Diagonals are packed between the upper chord members and the vertical posts or hangers. At the lower chord it is desirable to alternate pairs of eye bars from opposite directions to reduce the lever arm for bending moment on the pin (see joint *d* in Fig. 69(*c*)). Counters may be fitted in as found convenient. A single counter such as *cD* is satisfactory at the center line of the pin if it is maintained in position by the use of ring fills. Spacers are needed between eye bars wherever the spacing is more than ¼ in. Parallel bars of the same member are placed at least 1 in. apart for painting.

Controlling Pins. At upper chord panel points of the common Pratt truss, the diagonal provides the load on the pin, the vertical post furnishes vertical reactions and the horizontal chord provides horizontal reactions. At the hip joint, both the hanger and the diagonal produce the loads (the end post and chord providing reactions); thus, the largest pin is usually needed at the hip joint. Since the pin forces must be in equilibrium, we should use stresses in the connecting members produced by a single position

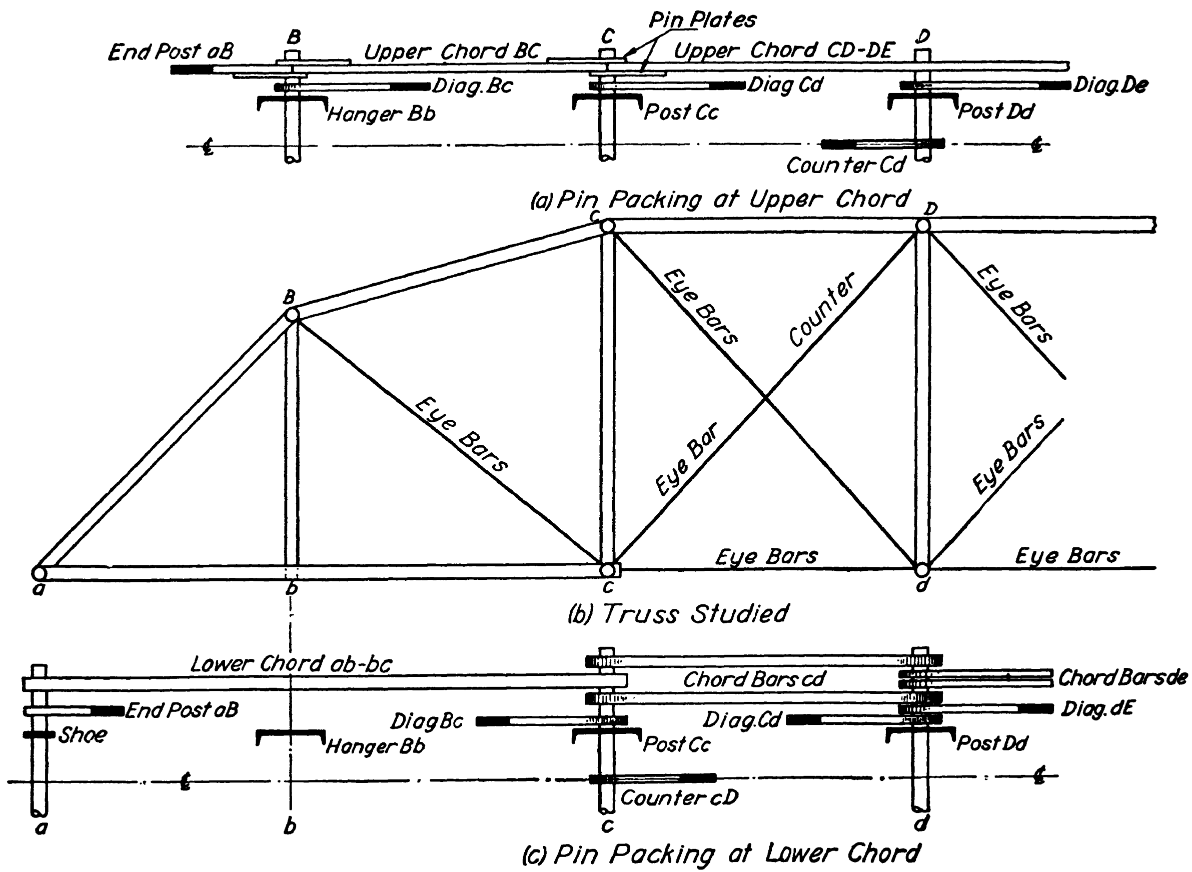


Fig. 69. Pin Packing for a Bridge Truss of Pratt Type.

of the live loading. Hence, we choose the maximum hanger stress and compute the greatest diagonal stress consistent with the proper loading for maximum hanger stress. End post and chord stresses can then be obtained for static equilibrium. These forces are increased for impact, based upon the length of bridge covered by the live load, the pin stresses being computed for *dead load, live load,* and *impact.*

It is customary to make the pin at the reaction as large as the one at the hip joint (to avoid numerous pin sizes) even though a smaller reaction pin might be used in most cases. The lower chord pin of maximum size occurs at the panel point *c* since the diagonal of greatest stress joins the lower chord at the joint *c*. The heaviest pin in the upper chord between the hip joints occurs at *C* but its size is usually controlled by the specification that the minimum pin diameter is 80 per cent of the width of the widest eye bar attached thereto. Therefore, the pin sizes can normally be reduced to two or three: (1) hip and reaction, controlled by the hip joint (2) other lower chord pins, controlled by the joint *c* (3) other upper chord pins, controlled by specification for minimum diameter at the joint *C*. The designation *C* or *c* indicates the third panel point as shown in Fig. 69 irrespective of the number of panels in the truss.

Pin Plates

69. Design of Reinforcing Plates. The minimum size of an eye-bar pin usually is specified so that the eye bar will not be overstressed in bearing. The thin web or flange plates of a built-up member require reinforcement for bearing unless a pin of extremely large diameter is used. Reinforcing plates or pin plates may be either riveted or welded to the member. As illustrated by Fig. 70, the pin plates must build up the net section *a-b* on the center line of the pin to 40 per cent more than the section through the member itself. The section required beyond the pin along *c-d* is specified for the case of tension members as equal to the cross-sectional area of the member.

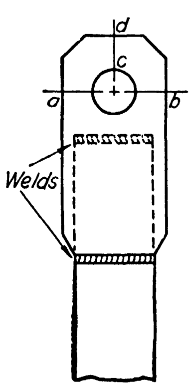


Fig. 70. Net Section at Pin Hole.

The main problem in the design of pin plates is their proper attachment to the member so that the stress received by each pin plate through bearing on the pin will be *transferred back into the main parts of the member*. This is accomplished by proper design of rivets or welds. The connection must be designed to produce a uniform distribution of stress over the entire cross-section of the member beyond the end of the longest pin plate. A typical riveted pin plate connection arranged to accomplish this result is shown in Fig. 72.

Courtesy C. M. St. P. & P. R.R. Co.

FIG. 71. RIVETED AND PIN CONNECTED JOINT WITH PIN PLATES.

The photograph shows pin plates which thicken the gusset plate so that the eye-bar diagonal will not produce an excessive bearing stress through its pin connection.

70. Pin Plates on a Compression Chord Member. EXAMPLE *DP*21. This example shows the calculations for riveting a group of pin plates to the outside of the web plates of a standard top chord section for a railway bridge truss. As shown in Fig. 72, the usual plates are a *fill plate* of the same thickness as the thickest angle (supplemented by a thin fill on the face of the thinner angle), a *pin plate* of the maximum possible width to overlap

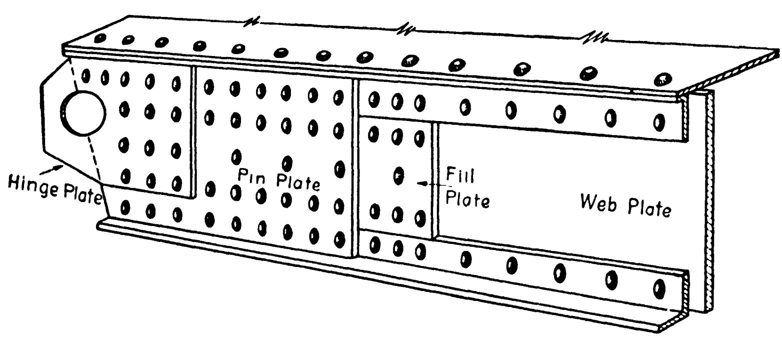


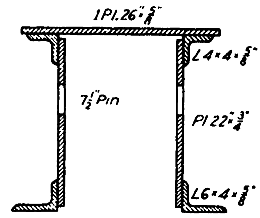
FIG. 72. MULTIPLE PIN PLATES ON A COMPRESSION CHORD MEMBER.

the angles, and a *hinge plate* riveted over the pin plate and top angle only. The hinge plate makes a proper insertion of the pin possible even though the pin plate and the fill plate are merely finished to bear against the pin.

The total width of bearing needed against the pin is calculated as 2.78 in. in the example *DP*21, then the load to be transferred through each plate is determined according to the relative thickness that it furnishes in bearing on the pin. The hinge plate

DP21. *Design riveted pin plates for the compression chord member of a railway bridge. The pin is 7½" diam. The load is 1,000,000#. Use 1" rivets. AREA spec.*

Pin Plates:



Area $= 16.3 + 33.0 + 9.2 + 11.7 = 70.2\square''$.
cov. webs top ∠s *bot.* ∠s

Unit stress $= \dfrac{1{,}000{,}000}{70.2} = 14{,}250\#/\square''$.

Thickness for bearing $= \dfrac{1{,}000{,}000}{2 \times 7.5 \times 24{,}000} = 2.78''$.

Pin pl. thickness $= 2.78 - 0.75 = 2.03''$. *See Fig. 72 for arrangement.*

Reinforcing pls. used will be a ⅝" fill pl., a ⅞" pin pl., and a ⅝" hinge pl.

Rivets through Plates:

Bearing unit stress $= 1{,}000{,}000 \div 2(7.5 \times 2.87) = 23{,}200\#/\square''$.

Rivet value, single shear controlling $= 13{,}500 \times 0.785 = 10{,}600\#$.

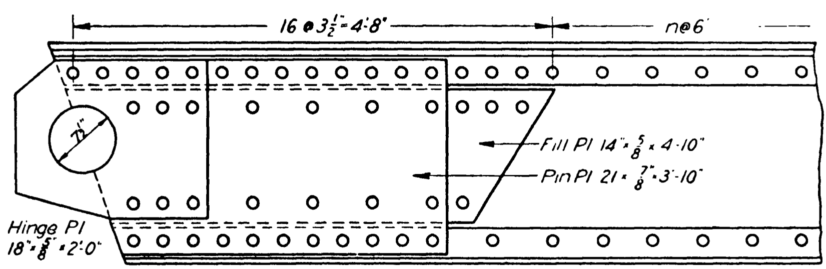
Rivets through hinge pl. $= \dfrac{7.5 \times 0.625 \times 23{,}200}{10{,}600} = 10.3$ *(11 used).*

" " *pin pl.* $= \dfrac{7.5 \times 1.50 \times 23{,}200}{10{,}600} = 24.6$ *(38 used).*

" " *all pls.* $= \dfrac{7.5 \times 2.12 \times 23{,}200}{10{,}600} = 35.0$ *(42 used).*

" " *top* ∠ $= \dfrac{14{,}250 \times (16.3 + 9.2)}{2 \times 10{,}600} = 17.1$ *(17 used).*

" " *bot.* ∠ $= \dfrac{14{,}250 \times 11.7}{2 \times 10{,}600} = 7.9$ *(11 used).*



Remarks: *In total 11 rivets pass through the hinge plate. There are 11 rivets through the bottom angle and the pin plate which make a total of 38 rivets through the pin plate instead of the required 25. However, we have only 13 rivets through the top angles in the length of the pin plate. Hence, 4 extra rivets are placed through the fill plate beyond the pin plate. Thus we use 4 extra rivets through the top angle for stress transfer from the web, a total of 17 as required.*

being on the outside must have enough rivets to develop its bearing stress by rivets in single shear; 11 rivets are required and 11 furnished. Next, a sufficient number of rivets in single shear must be placed through the hinge plate and pin plate, taken as a group, to develop their combined loads. We calculate that 25 rivets are necessary, but as will be seen, a much larger number is furnished. Finally, there must be enough rivets through all 3 reinforcing plates, considered as a group, to transfer their combined stress to the web and flange of the member through the use of rivets in single shear. We find that 35 rivets are required and 42 are furnished.

Stress Equalization. The minimum number of rivets mentioned above for each plate will transmit the bearing stress through the reinforcing plates and into the main member. The excess rivets furnished are necessary to allow each part of the section to receive its proper share of the total stress. For instance, there must be enough rivets through the lower angle within the length of the pin plate to transfer stress equal to the full value of the angle by rivets in single shear. For this purpose 8 rivets are needed. The corresponding group of rivets through the top angle must have a resistance in single shear equal to the value of the top angle and one half of the value of the cover plate. Since this requirement may necessitate excess length of pin plates, as is the case for the example *DP*21, it is often better to place one plate *inside* of the web plate to make the rivets act in double shear. In lieu of this, we may extend the fill plate beyond the pin plate, as in the example *DP*21, in this way forcing the web to transfer stress from the fill plate to the upper angle. Thus 4 rivets in the angle beyond the pin plate become a part of the 17 rivets needed through the angle for complete stress transfer. The result of these requirements is that a considerable number of excess rivets is used. In fact, 42 rivets are shown where 35 would have been adequate if they could have been distributed for full effectiveness. The pin plate shown might be shortened to include only 8 rivets through the bottom angles, but this arrangement would necessitate too great a transfer of stress from the fill plate to the upper angle *through the web*.

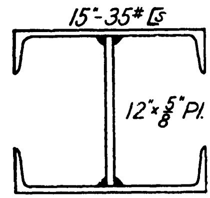
71. Pin Plates on a Welded Tension Member. EXAMPLE *DP*22. A hanger or other built-up tension member is usually of I shape. In welded construction the flanges of the I can be formed rather effectively from channels. These channels will then be bored for an end pin connection after pin plates are added both inside and outside of the channel webs to provide the proper bearing area. The total thickness of metal needed in the example *DP*22 is 1.75 in., of which 0.42 in. is furnished by the web. The inside pin plate is shown slotted to straddle the web plate of the hanger. Net area through the pin hole, 40 per cent in excess of the cross-sectional area of the member, is provided to meet the *AISC* requirements for tension members. The area of section beyond the pin and on the center line of the member to be provided is equal to the cross-sectional area of the member. This requires an 8-in. extension of the pin plates beyond the pin.

The inside pin plate must transfer load to the web plate and, therefore, its welds to the channel must overlap the welds between the channel and the web plate of the hanger for a length sufficient to effect such transfer of stress. This length is calculated to be 12.5 in. Each pin plate must also be welded to the channel with a sufficient length of weld to develop its bearing pressure against the pin. The outside pin plate is welded to the channel with about 40 in. of ⅜-in. continuous fillet weld. The slotted inner pin plate is welded continuously with a ⅜-in. fillet weld along the sides of the slot (26 in.) while intermittent fillet welds 2 in. long at 5-in. centers along the sides and upper edge of the plate provide more than enough additional length of weld, the total requirement being only 39 in.

Riveted Pin Plates for a Tension Member. The important variation from the procedure of the example *DP*22 necessitated by the use of rivets would be to deduct from the net section all rivet holes on any transverse line within the length covered by the pin.

DP22. *Design welded pin plates for a hanger of a building truss of the cross-section shown. The pin is of 5″ diameter which is adequate for shear and also for flexure, with special pin packing. The load is 550,000#. Use ⅜″ welds. AISC and AWS spec.*

Pin-Plate Design:



Cross-sectional area $= 2 \times 10.23 + 7.50 = 28.0\square''$.

Value of member $= 20{,}000 \times 28.0 = 560{,}000\#$.

Thickness for bearing $= \dfrac{560{,}000}{2 \times 5 \times 32{,}000} = 1.75''$.

Pin-pl. thickness $= 1.75 - 0.42 = 1.33''$.

An outside pin plate ⅝″ thick and an inside pin plate ¾″ thick (slotted) will be tried. Make pin plates about 14″ wide.

Net sect. at pin
$= 2 \times 10.23 + 2 \times 14 \times 1.37 - 2 \times 5 \times 1.79$
$= 40.9\square''$.

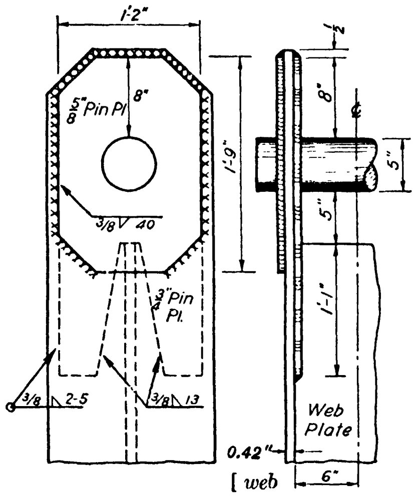
Reqd. sect. at pin
$= 1.4 \times 28.0 = 39.2\square''$.

Extension beyond pin
$= 28.0 \div 2 \times 1.79 = 7.8''$.

Weld Design:

Length of four ⅜″ web welds to develop web $= 20{,}000 \times 12 \times 0.625 \div 12{,}000 = 12.5''$.

Extend inside pin plate 13″ along web. Outside pin plate may be cut off when developed for bearing on pin.



Outside pin plate — value in bear. $= \dfrac{560{,}000}{2} \times \dfrac{0.625}{1.79} = 98{,}000\#$.

Length of ⅜″ weld $= 98{,}000 \div 3000 = 33''$. *About 40″ is shown.*

Inside pin plate — value in bear. $= \dfrac{560{,}000}{2} \times \dfrac{0.75}{1.79} = 117{,}000\#$.

Length of ⅜″ weld $= 117{,}000 \div 3000 = 39''$. *Use an intermittent ⅜″ weld 2 in. long at 5-in. centers along sides and end but use continuous ⅜″ welds along sloping cut.*

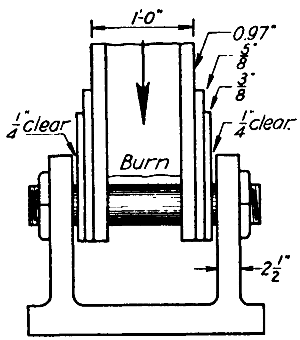
Remarks: *The compactness of the welded connection is one of its desirable features. Even greater compactness could be obtained if necessary by use of slot or plug welds.*

Again, a similar deduction would be advisable when consideration is given to the required length beyond the pin. Some of the rivets must either be flattened or countersunk on one side since the pin will have other members connected to it. Countersunk rivets are often discounted 50 per cent in design.

PROBLEMS

88. Design a chain-link pin to connect 2 pairs of parallel eye bars of size 5 in. $\times$ 1¾ in. Other details are similar to *DP*19. Use *AASHO* spec. *Ans.* Use a 4⅞-in. pin.

89. Revise *DP*20 for a load of 450,000 lb. in the eye bars which have a slope of 45 degrees with the horizontal or vertical. Study both cases as in *DP*20.



PROBLEM 90.

90. Design a pin to bear on the steel base for the case illustrated. Use *AISC* spec. The total force on the pin is 420,000 lb. *Ans.* d = 5.6 in.

91. Repeat Problem 90 for *AREA* spec. after selecting the width of the supporting standards from working stresses for cast steel. Note that the allowable bearing between a pin and an end rocker by *AREA* spec. is only 50 per cent of the allowable bearing between the pin and the member.

92. Select a bolt to act as a pin for a standard No. 5 clevis. The tie rod is of 1½ in. diameter and is upset. Other details can be obtained from the *AISC* handbook. The pin should preferably be able to develop the tie rod according to the *AISC* spec. The clevis has a grip of ⅞ in. for connection to a ⅝-in. plate. *Ans.* Use 1¾-in. bolt.

93. Revise the example *DP*21 for a section composed of a cover plate 28 $\times$ ⅝ in., web plates 24 $\times$ ¾ in., upper angles 4 $\times$ 4 $\times$ ½ in. and lower angles 6 $\times$ 4 $\times$ ⅝ in. Use an 8-in. pin.

94. Revise the example *DP*22 for a section composed of two 12-in., 30-lb. channels and a 12 $\times$ ½-in. web plate. Use a pin of 4½-in. diameter.

95. Select pin plates and arrange welds to connect a 12*WF*65 section to a 4¼-in. pin which will be adequate for shear and flexure with special pin packing. The flanges are drilled for the pin and the connection must develop the tension value of the section at 20,000 lb. per sq. in. Use *AISC* spec.

96 & 97. Obtain plans of a pin connected truss and check the pin packing, pin size, and use of reinforcing plates at one upper and one lower chord joint.

72. Special Functions of a Pin Connection. Evidently, the usual purpose in placing a pin in a structure is to obtain a point of free rotation or of zero moment. This will not be accomplished fully unless friction is reduced to a minimum. In a few instances a crude pin arrangement will suffice since the purpose may be merely to reduce to a small value the moment at a particular section. This is the usual case where pins are assumed in the analysis but not actually introduced into concrete structures. Evidently, this result can be accomplished by the *sudden restriction of a cross-section* to perhaps one half or one third of its full breadth for a relatively short length. With welding as a structural tool, this idea of sharply reducing the overall

size of a member to reduce its moment resistance without permitting a reduction of cross-sectional area becomes a practical device for simulating pins in a steel structure. The result might be a means of controlling the moments in a continuous steel frame.

CHAPTER 5

TIMBER CONSTRUCTION

73. Wood Structures. Timber is used for temporary structures because of its low cost. When maintained either continuously dry or continuously wet, timber structures have a life that may exceed twenty or even thirty years. Submerged piles of Roman origin have been found in good condition. Timber will have a short life if it is alternately wet and dry. It is also attacked by certain insects and worms such as the termite and the teredo, or marine borer.

Defects in Timber. Natural defects in wood as shown in Fig. 73 influence its strength. Knots on the top or bottom of a structural piece reduce the flexural strength in proportion to the relative width covered by the knot. Shakes are separations between fibers of the wood along the grain. Checks and splits are similar defects that cross the annual rings. Such defects reduce the shearing strength parallel to the grain. A wane, which is merely bark along one corner of the timber, is not very serious in its effect on strength since only a small wane is permitted in structural timber. Slope of the grain is objectionable in that it reduces the strength of both beams and columns. The strength is reduced for slopes greater than 1 in 20; the necessary reduction is about 50 per cent for a slope of 1 in 6 either in beams or columns.

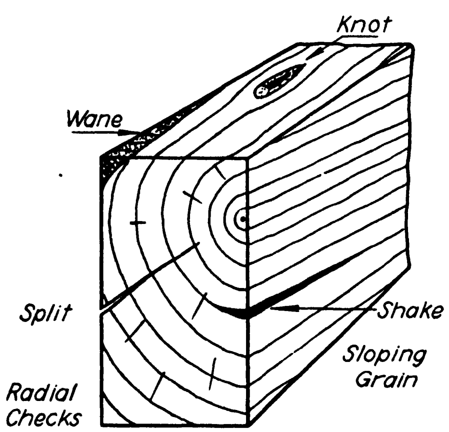


FIG. 73. FAULTS IN TIMBER.

74. Structural Timber Classifications. Structural timbers are classified under three groups: (1) *joists* and *planks* of nominal thicknesses 2 in., 3 in., and 4 in. and nominal widths up to 16 in. by even inches, (2) *beams* and *stringers* of nominal widths and depths of 5 in., 6 in. and up to 20 in. by even inches, and (3) *posts*, or timbers carrying longitudinal loads, in nominal sizes of 5 in., 6 in., and of larger dimensions in even inches. Dressed

joists and planks are reduced ⅜ in. in thickness and also in width up to 6 in. Commercial beams, stringers, posts and also joists or planks of widths greater than 6 in. are reduced ½ in. by planing.

Strength Determined by Stress Grading. Timber classification is based upon strength either in flexure for joists and beams or in compression for posts. Such classifications are given in Tables 8 to 10 for commercial timbers. The strength of clear timber not of commercial grade is given in Table 6.

TABLE 6

SAFE UNIT STRESSES FOR CLEAR TIMBER

(See Tables 8 to 10 for commercial timbers)

Species	Flexure Extreme Fiber	Compression		Horizontal Shear		Modulus of Elasticity
		Perpendicular to Grain	Parallel to Grain	Beams	Details	
Cedar, northern and southern white	1000	175	730	90	130	800,000
Cedar, western red	1200	200	930	100	150	1,000,000
Cypress, southern	1730	300	1470	130	190	1,200,000
Douglas fir, west coast	2000	325	1470	120	180	1,600,000
Douglas fir, inland	1470	275	1070	110	160	1,200,000
Hemlock, eastern	1470	300	930	90	130	1,100,000
Larch, western	1600	325	1470	130	190	1,300,000
Oak, red and white	1870	500	1330	170	250	1,500,000
Pine, southern yellow	2000	325	1470	140	210	1,600,000
Pine, southern yellow, dense	2330	375	1700	170	250	1,600,000
Redwood	1600	250	1330	90	130	1,200,000

In order to maintain the strength requirements of commercial timber, rigorous specifications have been accepted by the lumber industry. Such specifications control the allowable sizes and locations of knots, shakes, checks, splits and wanes. Also, the maximum slope of the grain is specified. The qualifying adjectives "dense" and "close grained" are defined with respect to the *number of annual rings per inch* for each species of wood. The designer can now feel that the specification of "1800*f* dense shortleaf southern pine" or "1200*c* close-grained Douglas fir (coast)" will produce timber adequate to resist the unit stress designated by the classification.

75. Allowable Unit Stresses. Comparisons may be made from Table 6 of the relative unit stresses permitted in timber for different load conditions. The allowable compression parallel to the grain is roughly 75 per cent of the value in flexure while allowable compression perpendicular to the grain is only about 20 per cent of the flexure value. Horizontal

TABLE 7

Strength Factors for Commercial Timber [a]

Designation	Beams and Stringers		Joist and Plank		Posts and Timbers
	Fiber Stress	Beam Shear	Fiber Stress	Beam Shear	Compression
Clear	100%	100	100	100	100
Select	75%	75	67	75	75
Common	60%	60	57	60	60

[a] These factors can be applied to the data of Table 6, but the stress factors from Tables 8, 9, and 10 are preferred.

TABLE 8

Allowable Unit Stresses for Commercial Joists and Planks

(Load applied to either face)

Grades and Species	Fiber Stress in Bending or Tension	Maximum Horizontal Shear	Compression Perpendicular to Grain	Modulus of Elasticity
1800#f Dense Douglas fir (coast and inland)	1800	120	380	1,600,000
1800#f Dense larch	1800	120	380	1,300,000
1800#f Dense longleaf or dense shortleaf southern pine	1800	120	380	1,600,000
1600#f Close-grained Douglas fir (coast)	1600	100	345	1,600,000
1600#f Close-grained Douglas fir (inland)	1600	80	335	1,500,000
1600#f Close-grained larch	1600	100	345	1,300,000
1600#f Dense longleaf or dense shortleaf southern pine	1600	120	380	1,600,000
1600#f Close-grained redwood	1600	80	267	1,200,000
1400#f Tidewater red cypress	1400	120	300	1,200,000
1400#f Oak	1400	120	500	1,500,000
1400#f Dense longleaf southern pine	1400	100	380	1,600,000
1400#f Close-grained redwood	1400	80	267	1,200,000
1200#f Port Orford cedar	1200	100	250	1,200,000
1200#f Douglas fir (coast)	1200	100	325	1,600,000
1200#f Douglas fir (inland)	1200	80	315	1,500,000
1200#f Larch	1200	100	325	1,300,000
1200#f Dense shortleaf southern pine	1200	100	380	1,600,000
1200#f Close-grained redwood	1200	70	267	1,200,000
1100#f Port Orford cedar	1100	80	250	1,200,000
1100#f Tidewater red cypress	1100	100	300	1,200,000
1100#f Oak	1100	100	500	1,500,000
1000#f Western red cedar	1000	100	200	1,000,000

TABLE 9

ALLOWABLE UNIT STRESSES FOR COMMERCIAL BEAMS AND STRINGERS

(Load applied to narrow face)

GRADES AND SPECIES	FIBER STRESS IN BENDING OR TENSION	MAXIMUM HORIZONTAL SHEAR	COMPRESSION PERPENDICULAR TO GRAIN	MODULUS OF ELASTICITY
1800#f Dense Douglas fir (coast and inland)	1800	120	380	1,600,000
1800#f Dense larch	1800	120	380	1,300,000
1800#f Dense longleaf or dense shortleaf southern pine	1800	120	380	1,600,000
1600#f Close-grained Douglas fir (coast)	1600	100	345	1,600,000
1600#f Close-grained Douglas fir (inland)	1600	80	335	1,500,000
1600#f Close-grained larch	1600	100	345	1,300,000
1600#f Dense longleaf or dense shortleaf southern pine	1600	120	380	1,600,000
1600#f Close-grained redwood	1600	80	267	1,200,000
1400#f Tidewater red cypress	1400	120	300	1,200,000
1400#f Oak	1400	120	500	1,500,000
1400#f Dense longleaf southern pine	1400	100	380	1,600,000
1400#f Close-grained redwood	1400	80	267	1,200,000
1200#f Douglas fir (inland)	1200	80	315	1,500,000
1200#f Larch	1200	100	325	1,300,000
1200#f Dense shortleaf southern pine	1200	100	380	1,600,000
1200#f Close-grained redwood	1200	70	267	1,200,000
1000#f Western red cedar	1000	100	200	1,000,000

beam shear is limited to less than 10 per cent of the allowable stress in flexure although some specifications would allow a 50 per cent increase for unit horizontal shear in timber details. These percentages vary greatly for different species of timber and should not be used in computations.

*Bearing Oblique to the Grain.** An expression has become commonly accepted for the calculation of the allowable unit compression or bearing at any angle to the grain. It is

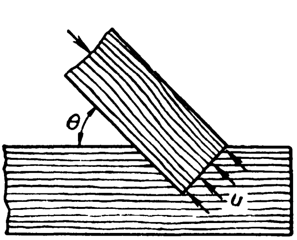


FIG. 74. BEARING AT AN ANGLE TO THE GRAIN.

$$u = \frac{pq}{p \sin^2\theta + q \cos^2\theta}. \tag{1}$$

* The formula given is from the *Wood Handbook* of the Forest Products Laboratory. Much of the data to be given on resistance of nails, screws, bolts and connectors is taken from the publications of the Forest Products Laboratory, to which the reader is referred for important information on special cases not covered here.

In this relation u is the unit allowable compression or bearing at the slope θ with the grain (see Fig. 74), p the allowable unit compression parallel to the grain, and q the allowable unit compression perpendicular to the grain.

TABLE 10

Allowable Unit Stresses for Commercial Posts

(Timbers carrying longitudinal loads)

Grades and Species	Compression Parallel to Grain, Short Columns	Modulus of Elasticity [a]
1200#c Tidewater red cypress	1200	1,200,000
1200#c Close-grained Douglas fir (coast)	1200	1,600,000
1200#c Close-grained Douglas fir (inland)	1200	1,500,000
1200#c Close-grained larch	1200	1,300,000
1200#c Dense longleaf or dense shortleaf southern pine	1200	1,600,000
1200#c Close-grained redwood	1200	1,200,000
1100#c Douglas fir (coast)	1100	1,600,000
1100#c Douglas fir (inland)	1100	1,500,000
1100#c Larch	1100	1,300,000
1100#c Oak	1100	1,500,000
1100#c Close-grained redwood	1100	1,200,000
1000#c Port Orford cedar	1000	1,200,000
1000#c Tidewater red cypress	1000	1,200,000
1000#c Oak	1000	1,500,000
1000#c Dense longleaf southern pine	1000	1,600,000
1000#c Close-grained redwood	1000	1,200,000
800#c Western red cedar	800	1,000,000

[a] Modulus of elasticity for loads perpendicular to the grain varies from 1% to 10% of values given.

Moisture, Decay, and Timber Treatment. The allowable stress in compression parallel to the grain or in flexure might be increased slightly for timber continuously dry. Timber continuously wet is weakened about 30 per cent in compression perpendicular to the grain. Frequent variation of moisture results in decay and should be considered by a reduction of working stress up to 25 per cent. If the decay hazard is sufficiently serious to require a greater stress reduction, it is desirable to use *treated timber.* Creosote and other timber preservatives requiring the use of heat, vacuum, and pressure weaken the timber up to as much as 25 per cent, but the gain in decay resistance is usually considered to justify the use of unreduced working stresses.

Impact and Fatigue. Timber is able to withstand greatly increased loads momentarily. Hence, impact is seldom considered in the design of a wood structure. The fatigue limit for rectangular beams is about one third of the modulus of rupture. Since the allowable working stress is only about one sixth of the modulus of rupture, the danger of fatigue failure is negligible. Wood stressed slightly below the fatigue limit may

be expected to withstand at least *15,000,000* repetitions of stress. For a stress slightly above the fatigue limit, this factor will drop to *2,000,000* repetitions of stress.

Holding Power of Nails and Screws

76. Holding Power of Wire Nails and Spikes. There is considerable variation of test results on the holding power of nails because of the number of variables involved. These may be listed as (1) quality and species of wood (2) edge distance (3) nearness of a knot or fault (4) method of driving (5) moisture content when driven and when pulled. Since these factors are not all under control, particularly the human element in driving, it has seemed best to reduce ultimate holding power by a factor of 6 for working loads. Thus we obtain the following approximate relationship,

$$P = 1150G^{2\frac{1}{2}}d. \qquad (2)$$

In this formula, P is the safe load in tension per lineal inch of penetration, G is the specific gravity of the wood (oven dry) and d is the diameter of the nail or spike in inches.

The only important case in which this formula needs correction is for application to southern yellow pine. Then, a factor of 80 per cent should be introduced up to the twelvepenny size and a factor of 70 per cent is used above the twentypenny size. Thus, we get the values offered in Table 11 which is obtained from the *Wood Handbook* of the Forest Products Laboratory.

TABLE 11

Safe Holding Power of Wire Nails and Spikes [a]

(Values are in pounds per lineal inch of penetration)

Timber Species	Specific Gravity (oven dry)	Size of Nail (penny designation)							
		8	12	16	20	30	40	50	60
Douglas fir	0.51	28	32	35	41	44	48	52	56
Oak, red or white	0.69	60	67	74	87	94	102	111	120
Longleaf pine	0.64	39	45	47	50	55	59	64	69
Shortleaf pine	0.59	32	36	38	41	44	48	52	57
Ponderosa pine and redwood	0.42	17	19	21	25	27	30	32	35

[a] Reduce 40 per cent if driven parallel to the grain.

Sizes of Wire Nails and Lag Screws. The common designation of wire nails and small spikes is by *weight per thousand.* Thus a thousand sixpenny

nails weigh six pounds, etc. The diameter of the nail or screw is needed in the determination of holding power and is given in Table 12.

TABLE 12

SIZES OF ORDINARY WIRE NAILS AND LAG SCREWS

Size of Nail	Diameter, d	$d^{3/2}$	Length	Diameter Lag Screw	Length
Eightpenny	0.13	0.047	2½	5/16	1½–6
Twelvepenny	0.15	0.058	3¼	⅜	1½–6
Sixteenpenny	0.16	0.065	3½	7/16	1½–8
Twentypenny	0.19	0.083	4	½	1½–10
Thirtypenny	0.21	0.096	4½	9/16	2–12
Fortypenny	0.22	0.105	5	⅝	2–12
Fiftypenny	0.24	0.120	5½	¾	2½–12
Sixtypenny	0.26	0.135	6	⅞	3–12

Spikes are larger in diameter than nails for equal lengths. They are available in lengths from 3 in. to 12 in. Boat spikes are square and have a chisel point. They are obtainable in sizes varying by sixteenths of an inch from ¼ in. to ⅝ in. and in lengths up to 14 in.

Variations of Holding Power. Nails are available in special shapes. A long sharp point increases the holding power but also increases the splitting tendency. The reverse is true of blunt points. The Forest Products Laboratory has developed a chemically etched nail with holding power increased at least 90 per cent under all conditions. This nail may become commercially available. Green wood shrinks, causing nails to lose a considerable part of their holding power. In order to avoid much of this loss of strength, the designer may specify either barbed or spirally grooved nails for use in green wood. Prebored holes of smaller size than the diameter of the nail, and slant driving, increase the holding power. However, it is not usual to increase the values given by equation (2). Nails driven parallel to the grain should be reduced 40 per cent in holding power.

77. Holding Power of Drift Bolts. A drift bolt is a pin driven into a hole about ⅛ in. smaller in diameter than the bolt. Although test results vary considerably, the following formula seems to be reasonably satisfactory for the holding power of drift bolts in dry wood, since it is based upon a factor of safety of from 5 to 6 for different species.

$$P = 1000G^2d. \tag{3}$$

In this equation, P represents the permissible tension in pounds per lineal inch of penetration, G is the specific gravity of the wood (oven dry) and d is the bolt diameter in inches.

78. Holding Power of Screws. The equation recommended by the Forest Products Laboratory is

$$P = 1700G^2d. \tag{4}$$

Again, P is the tension resistance in pounds per lineal inch of penetration (penetration is ⅔ of length), G is the specific gravity of the wood (oven dry) and d is the diameter of the shank in inches. Screws in ordinary timber are inserted into holes about 70 per cent of the root diameter of the screw. The holding power should be reduced 40 per cent for screws inserted parallel to the grain.

TABLE 13

FORMULAS FOR SAFE LATERAL RESISTANCE OF NAILS AND SCREWS[a]

TIMBER SPECIES	NAILS	SCREWS
Eastern hemlock	$F = 900d^{3/2}$	$F = 2100d^2$
Southern cypress	$F = 1125d^{3/2}$	$F = 2700d^2$
Redwood	$F = 1125d^{3/2}$	$F = 2700d^2$
Douglas fir (coast)	$F = 1375d^{3/2}$	$F = 3300d^2$
Southern yellow pine	$F = 1375d^{3/2}$	$F = 3300d^2$
Oak, red and white	$F = 1700d^{3/2}$	$F = 4000d^2$

[a] F is the total safe lateral load per nail or per screw; d is the shank diameter in inches.

79. Lateral Shear Resistance of Nails and Screws. The values to be specified for shear resistance of bolts and screws are intended to give a factor of safety of about 6.0 against ultimate failure or 1.6 against elastic failure. There is usually an initial inelastic slip of about 0.01 in. Penetration is assumed to be ⅔ of the length and at least 7 diameters for screws. Values of shear resistance as computed from Table 13 should be reduced 25 per cent where unseasoned wood is used. They should be reduced 40 per cent if the screw is inserted parallel to the grain of the wood. The lateral shear resistance of the nail or screw may be increased 25 per cent for a metal-to-timber connection.

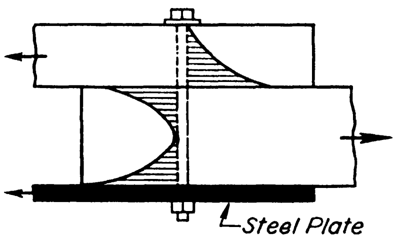


FIG. 75. VARIATION OF BEARING PRESSURE UNDER A BOLT.

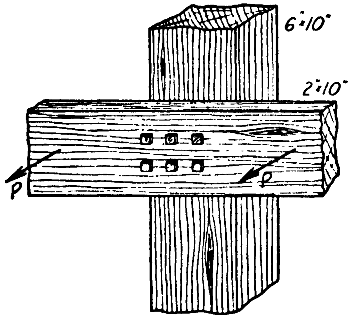
BOLTED JOINTS IN TIMBER

80. Bearing Pressure under Bolts. The actual bearing pressure under a bolt is never uniform and becomes extremely variable as indicated by Fig. 75 for long slender bolts. The action of a bolted timber joint under load is not elastic in the usual sense. There is, instead, a *slip* which is proportional to the load. *The limit beyond which the slip increases more*

DP23. *Find the number and size of nails, drift bolts, and wood screws to resist a pull of 5000# between a 2" and a 6" thickness of shortleaf pine. The nails passing through the 2" plank penetrate the 6" timber. The force is in the direction of the nails.*

Nails: *In order for the penetration to be ⅔ of the length of the nail, the nail must be 6" long. This is a 60d nail. The holding power from Table 11 is 57#/" or 250# per nail for a penetration of 4⅜".*

Use $\dfrac{5000}{250}$ *= 20 nails.*



Drift Bolts:

Try ¼" bolts 7" long, penetration of 5⅜".

$P = 1000G^2d = 1000\,(0.59^2)\,0.25 = 87\#/''$. *Equation (3)*

Pull per bolt $= 87 \times 5.37 = 467\#$.

Use $\dfrac{5000}{467}$ *= 11 drift bolts (¼" × 7").*

Lag Screws:

Try 5/16" lag screws 6" long, penetration = 4⅜".

$P = 1700G^2d = 1700\,(0.59^2)\,0.31 = 183\#/''$. *Equation (4)*

Pull per screw $= 183 \times 4.37 = 800\#$.

Use $\dfrac{5000}{800}$ *= 6.2 or 6 lag screws (5/16" × 6").*

Remarks: *The illustration shows the design for lag screws. This problem indicates how the use of lag screws may be made to simplify timber joints. It would be difficult to find space either for 20 nails or 11 drift bolts.*

rapidly than the load is treated as a proportional limit. Working stresses for average bearing pressure under bolts as given in Table 14 are based upon this limit.

81. Corrections for Determining Bolt Resistance. The values in Table 14 represent safe bearing pressures in pounds per square inch for loads parallel or perpendicular to the grain. They apply only to the woods specifically mentioned at the bottom of the table. These values are multiplied by the projected area of the bolt (diameter times length through controlling thickness of timber). There are increased values given for high strength bolts (elastic limit 125,000 lb. per sq. in.) since the strength

TABLE 14

SAFE UNIT BEARING PRESSURE UNDER BOLT [a]

L/d Length[b] ÷ Diam.	Parallel to Grain		Perpendicular to Grain		Correction Factor[c] Perpendicular to Grain	
	Common Bolt	High Strength Bolt	Common Bolt	High Strength Bolt	Bolt Size	Factor
1	1040	1040	275	275	½	1.68
2	1040	1040	275	275	⅝	1.52
3	1020	1040	275	275	¾	1.41
4	960	1020	275	275	⅞	1.33
5	830	990	275	275	1	1.27
6	690	930	275	275	1¼	1.19
8	520	750	240	275	1½	1.14
10	410	620	185	250	2	1.07
12	350	520	140	200	3	1.00

[a] These values are intended to apply only to the following species of wood: southern cypress, Douglas fir (coast), western larch, southern yellow pine, and redwood. For other species consult the *Wood Handbook* of the Forest Products Laboratory. *The values given are to be halved* if the load is applied to one end only of the bolt, i.e., single shear plane.
[b] Length is taken as the length through the thickest timber.
[c] The correction factor is applied only for loads perpendicular to the grain.

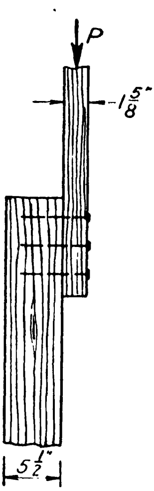
of a joint is influenced both by the resistance of the timber and by the tensile resistance of the bolt. The main variable, however, is the ratio of the bolt diameter to the length through the thickest piece of timber, *L/d*. A correction factor is listed in Table 14 that varies with the bolt diameter. This factor is to be applied only for loads perpendicular to the grain. It accounts for an observed increase in the relative strengths of small bolts. All sizes over 3 in. are to be treated as the 3-in. diameter, that is, the correction factor in this case is unity.

Unseasoned or Damp Wood. Timber that changes occasionally in moisture content should be discounted 25 per cent with reference to bolt resistance. Continuously wet timber should be discounted 33 per cent.

Metal Side Plates. The safe bearing pressures given in Table 14 can be increased 25 per cent for load parallel to the grain when metal side

DP24. *Find the number and size of nails, lag screws, and bolts to resist a lateral force of 5000# between a 2" and a 6" thickness of shortleaf southern pine (1400f).*

Nails:



For a penetration of ⅔ of the length of the nail into the 6" timber, the nail should be 6" long. This requires a 60d nail. The lateral resistance of a nail in southern yellow pine (Table 13) is given as

$$P = 1375d^{3/2} = 1375(0.26)^{3/2} = 183\#.$$

Use $\dfrac{5000}{183} = 28$ *nails (60d).*

This number of nails seems impractical.

Lag Screws:

From Table 13 we may compute the resistance to lateral shear of a $\frac{5}{16}" \times 6"$ *lag screw as*

$$P = 3300d^2 = 3300 \times 0.31^2 = 320\#.$$

Use $\dfrac{5000}{320} = 16$ *lag screws* $(\frac{5}{16}" \times 6")$.

Even 16 screws would be too many for convenient construction.

Bolts:

The allowable bearing under bolts parallel and perpendicular to the grain for shortleaf southern pine are as given in Table 14. Try ½" bolts.

$$L/d = \frac{5\frac{1}{2}}{\frac{1}{2}} = 11.0.$$

By interpolation in Table 14 we obtain bearing values of 380 and 162#/□". These values are halved for load applied to one end only of the bolt. Values in Table 14 are for symmetrical loading.

$$P = 0.5 \times 0.5 \times 5.5 \times 380 = 520\#.$$
$$P' = 0.5 \times 0.5 \times 5.5 \times 162 \times 1.68 = 375\#.$$

The correction factor (1.68) is taken from the final column of Table 14.

Use $\dfrac{5000}{520} = 10$ *bolts for load parallel to the grain, or,*

$\dfrac{5000}{375} = 14$ *bolts for load perpendicular to the grain.*

Remarks: *Nails would be used only for a temporary structure. Bolts are more satisfactory than lag screws if the structure is an important one, although lag screws are in common use.*

plates are used. It is not recommended that the values for bearing perpendicular to the grain be increased.

Loads Applied Obliquely to the Grain. The common formula for determining allowable bearing pressure at an angle to the grain is

$$u = \frac{pq}{p \sin^2\theta + q \cos^2\theta}. \tag{5}$$

Here, u is the allowable unit bearing stress for load at the angle θ to the grain, p is the allowable unit bearing stress *parallel* to the grain, and q is the allowable unit bearing stress *perpendicular* to the grain. The allowable values of p and q for bolts can be obtained from Table 14 and the value of u follows from equation (5). The example *DP*25 illustrates the use of this formula in a practical design. ✓

82. Bolt Spacing and Edge Requirements. Bolts should be spaced at least 4 diameters apart center to center. Net section requirements in tension members may require even a greater lateral spacing. The end margins for a compression member may be 4 diameters from the center of the bolt, but a margin of 7 diameters is recommended for structural tension members to avoid splitting at the end. The edge distance where the load is parallel to the edge need be no more than 1½ diameters, but an allowance of 4 diameters is required if the bolts bear against the wood forming the edge margin. These factors were considered in arranging the detail for the example *DP*25.

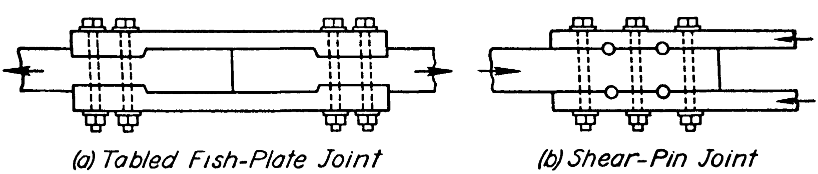


FIG. 76. SPLICES IN TIMBER MEMBERS.

TIMBER CONNECTORS

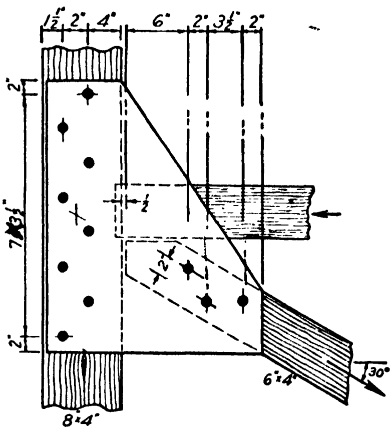
83. Purpose and Usefulness. The idea of using either metal or wood shear developers between the interior surfaces of timber joints is nearly as old as the use of bolts. Two such devices are shown in Fig. 76. The tabled fish-plate joint (*a*) is a semi-obsolete form that can be replaced to advantage by the use of shear pins (steel pipes) as illustrated in (*b*). Wood blocks have also been introduced in place of the shear pins. Such hardwood blocks are tapered for a driving fit.

The modern device to take the place of fish plates and shear pins is the *ring* or *toothed connector*. Since 1933 their use has developed exten-

DP25. *A tension diagonal is joined to a vertical compression member at an angle of 60° as shown. Design a proper bolted connection with the use of ¼″ gusset plates to develop the net tension value of the diagonal member at 900#/□″ stress.*

Materials:

Steel plate, AISC spec. Compression member: 1200c close-grained Douglas fir (coast). Tension member: 1600f dense Douglas fir (coast). Actual thickness = 3.62″.



Net Section:

Allow for two 1″ holes. (Required because spacing $<4d$)

$A_{net} = (5\frac{1}{2} - 2) \cdot 3\frac{5}{8} = 12.7$□″. ✓

Tension value = 900 × 12.7 = 11,400#.

Bolt Value:

L/d ratio = 3.62 ÷ 1.0 = 3.62. Nominal L/d ratio = 4.0.

Bearing parallel to grain = 960 (Table 14).

Bearing perpendicular to grain = 275 × 1.27 = 350.

Bearing at 60° to grain $= \dfrac{960 \times 350}{960 \times 0.866^2 + 350 \times 0.5^2} = 416$#/□″.

Bearing resistance per bolt with steel plates = 416 × 3.62 × 1.0 = 1500#.

Number of bolts required = 11,400 ÷ 1500 = 7.6. Use 8 bolts through post.

Number of bolts in tension member where bearing is parallel to grain = 11,400 ÷ (1.25 × 960 × 3.62) = 2.6 (use 3 bolts).

Details: *The details of bolt spacing and margins are arranged to agree with the specifications from § 82 for bolts of 1″ diameter. The factor 1.25 is for metal gussets.*

sively in the United States although they have long been used in Europe and have been available in a few forms for more than half a century. In all of its forms, the modern connector is simply a *shear developer* which takes the place of nails or bolts for shear resistance. Bolts must still be used to hold the joint together, but the bolts are often placed in oversized holes and consequently they can resist little shear.

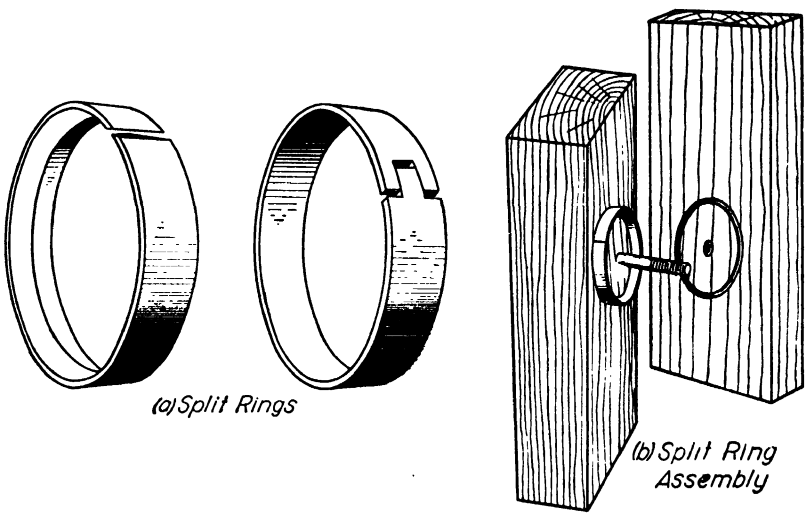


FIG. 77. SPLIT RING CONNECTORS.

84. Split Rings. A split ring of either type shown in Fig. 77(*a*) is introduced between two pieces of timber as shown in (*b*). The grooves for the ring are cut by a special tool. They are slightly larger than the ring so that the ring must be spread before it is inserted. Thus the ring is tight in its groove and initial slip is reduced to a small factor. These rings are readily available in diameters of 2½, 4, 6, and 8 inches.

Load capacities of split ring connectors may be corrected for special conditions from the values given in Table 15. An illustration of the use of these values is given in the design example *DP*26*a*.

Wood Classification. Values in Table 15 are for timber of the No. 4 classification. They should be corrected for other classifications as indicated below.

No. 1, 65%; No. 2, 75%; No. 3, 85%; No. 4, 100%; No. 5, 115%.

The standard classifications of the common species of timber grouped for connector design are as follows:

No. 1. Black ash, basswood, paper birch. eastern hemlock, ponderosa pine. red and white spruce.

No. 2. Rocky Mountain Douglas fir, gum, western hemlock, soft maple, Norway pine, sycamore.

No. 3. Southern cypress, redwood.

No. 4. White and Oregon ash, beech, sweet and yellow birch, west coast Douglas fir, hickory, western larch, hard maple, commercial oak, southern yellow pine.

No. 5. Dense Douglas fir (coast), dense southern pine.

TABLE 15

SAFE LOADS[a] FOR SPLIT RING CONNECTORS IN PAIRS[b]

DIAMETER OF CONNECTOR	DEPTH OF CONNECTOR	BOLT SIZE	SAFE LOAD PARALLEL TO GRAIN	SAFE LOAD PERPENDICULAR TO GRAIN
2½ in.	0.75 in.	½ in.	5700 lb.	4000 lb.
4 in.	1.00 in.	¾ in.	12,000 lb.	8400 lb.
6 in.	1.25 in.	¾ in.	18,000 lb.	10,800 lb.
8 in.	1.50 in.	¾ in.	23,000 lb.	11,500 lb.

[a] Safe loads on all types of connectors are set to maintain a factor of safety of 4.0 with respect to ultimate or failure loads. The factor of safety is about 1.6 with respect to the elastic limit. Data for safe loads for all connectors have been drawn from tests made at the United States Forest Products Laboratory and at George Washington University.

[b] Correct for (1) *wood classification* (2) *moisture content* (3) *direction of load* (4) *edge distance;* values of safe load are for a *pair* of connectors.

Moisture Content. Test results are obtained on seasoned wood specimens of 15 per cent moisture content. Green wood has about 28 per cent moisture and is often discounted 40 per cent for connector design. Between the limits of 15 and 28 per cent moisture content, the discount factor may be approximated by direct proportion based upon a linear variation. This correction is not as essential in the design of split ring connectors as it is for connectors of fixed diameter. In fact, it is not common to introduce a moisture correction for the split ring type of connector unless the moisture change is very severe.

Direction of Load. The resistance of a split ring connector for load perpendicular to the grain is less than the resistance for load parallel to the grain as is shown by Table 15. If no data are available for load resistance perpendicular to the grain for special connectors, it is usual to discount values of load resistance parallel to the grain by at least 25 per cent. The load resistance oblique to the grain of the wood may be obtained by proportion between the two known values based upon a straight line variation for angles between 0 and 90 degrees.

Edge Distance. The minimum width of lumber for the 2½-in. and 4-in. connectors is respectively the nominal 4-in. and the nominal 6-in. size. The 6-in. and 8-in. rings may be used in nominal 8-in. and 10-in. timber respectively. The corresponding edge distances from the outside surface to the cut groove are about ½ in. *If the load acts at 30 degrees or more to the direction of the grain,* the specified load from Table 15

should be reduced 15 per cent for these minimum margins. No reduction is necessary for a margin of 1½ in., while between ½ in. and 1½ in. the reduction of load may be approximated by proportion.

Ring Spacing. Split rings and other ring connectors are properly spaced at least 1½ diameters apart in a single line for load parallel to the line of connectors. If the load is normal to the line of connectors, they may be spaced closer together. The minimum spacing is the diameter of the ring plus ½ in., but a greater spacing is preferred in order that the wood between rings may not be damaged.

TABLE 16

LUMBER SIZES FOR CONNECTOR DESIGN [a]

Diameter of Groove			Width of Timber		Edge Margin	Groove Depth	Ring Depth	Thickness of Timber[b]	
Nominal	Inside	Outside	Nominal	Actual				Single Groove	Double Groove
2½ in.	2.56	2.92	(4) in.	3⅝	0.35	0.37	0.75	1 5/16 (1½)	2⅝ (3)
4	4.08	4.50	(6)	5½	0.50	0.50	1.00	1⅝ (2)	2⅝ (3)
6	6.12	6.66	(8)	7½	0.42	0.62	1.25	2⅛ (2½)	3⅝ (4)
8	8.14	8.82	(10)	9½	0.31	0.75	1.50	2⅝ (3)	4½ (5)

[a] Nominal or commercial sizes of timber are placed in parentheses.
[b] For thinner material, reduce value of connector in proportion to reduction of thickness.

End Margin. The proper minimum end margin measured along the grain is 1½ in. The safe load should be reduced one third if the end margin is only 1 in. It is doubtful that any value should be permitted where the end margin is less than 1 in.

Minimum Lumber Sizes. The information needed for providing precut grooves for the installation of split ring connectors is collected in Table 16. The minimum sizes given should be increased wherever possible in order to increase connector resistance and to reduce the possibility of splitting the wood.

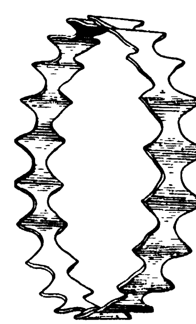


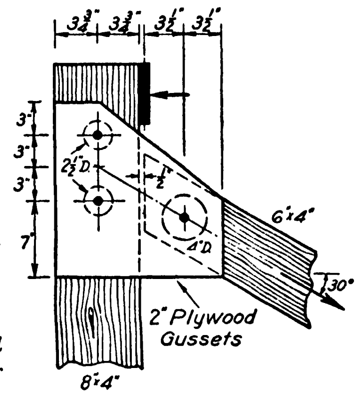
FIG. 78. ALLIGATOR CONNECTOR.

85. Alligator Rings. The alligator ring functions in much the same manner as the split ring, but it is designed for installation without the need for a cut groove. As shown in Fig. 78, the sharp teeth of the alligator connector will cut into the timber if sufficient pressure is applied. The necessary pressure is produced by tightening down on an *alloy steel bolt* of high tensile strength that passes through the center of each ring. The hole for the bolt is drilled 1/16 in. oversize so that the bolt of alloy steel can easily be removed and replaced by an ordinary steel bolt after the

DP26a. *A tension diagonal joins a vertical compression member at an angle of 60° as shown. Design a proper split ring connection with plywood gussets to develop the net section of the diagonal at 600#/□″ stress. Seasoned timber and gusset material are of No. 5 classification.*

Net Section:

The cross-sectional area removed by the ring groove amounts to only ¼□″ which is neglected. Deduct for the ¾″ bolt hole. Net width = 5½ − ¾ = 4¾″.



Allowable Load:

(5½ − ¾) × 3⅝ × 600 = 10,300#.

Ring Values:

Value of 4″ split rings used in pairs. Load parallel to grain for diagonal member; averages 45° to grain for plywood gusset; gusset controls.

1.15 × 0.5(12,000 + 8400) = 11,700#. (Values from Table 15)

(The factor 1.15 is for wood of No. 5 classification; the edge margin exceeds 1½″ for the gusset.) Use a pair of 4″ split ring connectors.

Value of 2½″ split rings. 5700# and 4000# respectively (parallel and perpendicular to grain). Load at 60° to grain controls their design.

At 60° to grain, usual value = 4000 + ⅓ × 1700 = 4570#.
Increase 15% to 5250# for No. 5 timber. (Margins exceed 1½″.)
Use 10,300/5250 = 2 pairs of connectors as shown.

DP26b. *Repeat problem DP26a for alligator connectors with bolts placed through loose holes in No. 5 timber. Develop the allowable load of 10,300#.*

Value of connected member = 10,300#.
Parallel to grain. Try 2 pairs of 3⅜″ connectors.
Value = 1.10 × 2 × 5200 = 11,400#. (1″ margins OK; load ∥ to grain.)
The factor 1.10 is the correction factor for No. 5 timber.
Oblique to grain 45–60°. Discount 25%. Try 2 pairs of 4″ connectors.
Value = 1.10 × 2 × 0.75 × 6300 = 10,400#. (Margins exceed 1½″.)

Remarks: *The use of one pair of 4″ ring connectors as shown in the illustration would produce a pin connected joint that should permit a slight rotation under load to relieve flexural stresses. Clearly, the split ring connector seems the better solution for this case.*

connector has been embedded in the wood. Safe loads for alligator connectors may be obtained by correcting the values of Table 17. An illustration is offered as the design problem *DP*26*b*.

TABLE 17

SAFE LOADS FOR ALLIGATOR CONNECTORS IN PAIRS [a]

DIAMETER OF CONNECTOR	BOLT SIZE	SAFE LOAD PARALLEL TO GRAIN	
		Bolt tight in hole	Oversize hole
2 in.	½ in.	2400 lb.	2200 lb.
2⅝	⅝	4200	3600
3⅜	¾	5800	5200
4	¾	6900	6300

[a] Correct for (1) *wood classification* (2) *moisture content* (3) *direction of load* (4) *edge distance;* values of safe load are for pairs of connectors.

Wood Classification. Adjust values in Table 17 by using the following factors for correction where the timber classification is other than No. 4. (See § 84 for the classification of various species of timber.) No. 1, 80%; No. 2, 85%; No. 3, 90%; No. 4, 100%; No. 5, 110%.

Moisture Content. Use the same specification as for split ring connectors, § 84. This specification should be applied rigorously to alligator connectors and to other rings of fixed diameter.

Direction of Load. The safe load values of Table 17 should be discounted 25 per cent for loads applied perpendicularly to the grain or at any angle of more than 45 degrees to the direction of the grain. At less than 45 degrees, the safe load value may be obtained by proportion upon the assumption of a straight line variation from 0 to 45 degrees.

Edge Distance. Minimum requirements are the same as for split ring connectors, although greater edge distances are desirable because of the splitting action when the connector is installed. Connectors in line with the load should be spaced 1½ diameters apart. Connectors in a line perpendicular to the load should be spaced at least 1 in. clear.

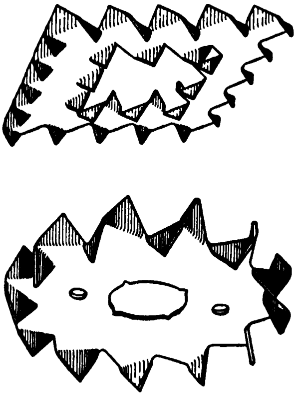


FIG. 79. BULLDOG CONNECTORS.

86. Bulldog Plates. The bulldog plate is different from the ring shear developer in that its teeth may be distributed over its entire surface. It must be embedded by bolt pressure in the same manner as the alligator connector. The bolt passes through the center of either the circular or square connector illustrated by Fig. 79. The metal used is slightly less

than $\frac{1}{16}$ in. thick. Safe load values are given in Table 18. These values apply with bolts in oversize holes. An illustrative example *DP27a* is given to show the use of these values.

TABLE 18

SAFE LOADS FOR BULLDOG PLATE CONNECTORS IN PAIRS [a]

Diameter of Connector	Round Connectors		Square Connectors		
	Bolt Size	Safe Load	Size of Connector	Bolt Size	Safe Load
3 in.	$\frac{3}{8}$ in.	2200 lb.	4 in.	$\frac{1}{2}$ in.	4300 lb.
"	$\frac{1}{2}$	2800	"	$\frac{5}{8}$	4900
"	$\frac{5}{8}$	3400	"	$\frac{3}{4}$	5500
$3\frac{3}{4}$ in.	$\frac{3}{8}$	3000	"	$\frac{7}{8}$	6100
"	$\frac{1}{2}$	3500	5 in.	$\frac{3}{4}$	7300
"	$\frac{5}{8}$	4200	"	$\frac{7}{8}$	7800
"	$\frac{3}{4}$	4900	"	1	8200
			"	$1\frac{1}{4}$	10,500

[a] Correct for (1) *wood classification* (2) *moisture content* (3) *direction of load* (4) *edge distance;* values of safe load are for pairs of connectors.

Wood Classification. Adjust values in Table 18 by using the following factors for correction where the wood is classified as other than No. 4. (See § 84 for the classification of various species of timber.) No. 1, 80%; No. 2, 85%; No. 3, 90%; No. 4, 100%; No. 5, 110%.

Moisture Content. Apply the correction given in § 84 rigorously for bulldog plates and other connectors of fixed diameter.

Direction of Load. Whenever loads are applied perpendicularly to the direction of the grain, the load values for circular bulldog connectors are to be reduced 10 per cent and those for square bulldog connectors are to be reduced 25 per cent. Whenever loads are applied obliquely to the direction of the grain, an adjustment should be made for a straight line variation of safe load with angle variation from 0 to 90 degrees.

Edge Distance. The same margins and spacing should be specified as for alligator connectors, § 85.

87. Flanged Plate Connectors. In many cases timbers are connected between steel plates or straps which serve either as gusset plates at the joints, as splice plates, or which may be brought together on a pin to form a pin connected joint. Evidently, we need a ring connector of only one half the usual depth for this purpose, but there must be a hub provided for bearing against the bolt. The pressed steel or cast iron connectors shown in Fig. 80 fit into precut grooves to lie flush with the timber. They are used singly in a timber-to-metal joint or back to back for a timber-to-timber joint. In either case there is a tight fitting bolt which passes through the central hole and bears on the metal plate or hub.

They offer ease of assembly and disassembly. They may be installed before shipment since they are flush with the face of the timber and can be screwed tightly in place if so desired.

The safe load characteristics of flanged plate connectors may be obtained by correcting the values given in Table 19. An illustration of the design of a tension member splice is given as *DP27c*.

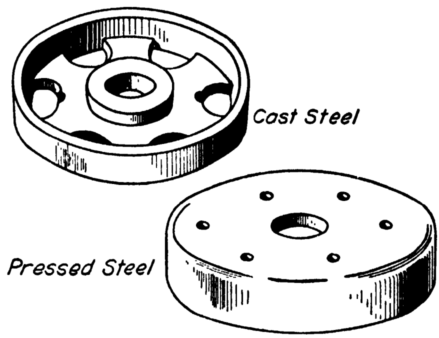


Fig. 80. Flanged Plate Connectors.

Wood Classification. Adjust the values in Table 19 by using the following factors for correction where the timber is classified as different from No. 4. (See § 84 for the classification of various species of timber.) No. 1, 65%; No. 2, 75%; No. 3, 85%; No. 4, 100%; No. 5, 115%.

Moisture Content. This requirement was discussed in § 84 for split ring connectors. The specification for load reduction with increase of moisture content must be applied rigorously for these connectors of fixed diameter.

TABLE 19

Safe Loads for Flanged Plate Connectors in Pairs [a]

Diameter of Connector	Material	Bolt Size	Safe Load Parallel to Grain
2⅝ in.	pressed steel	¾ in.	5500 lb.
4	pressed steel	¾	7500
4	malleable casting with hub	⅞	8300

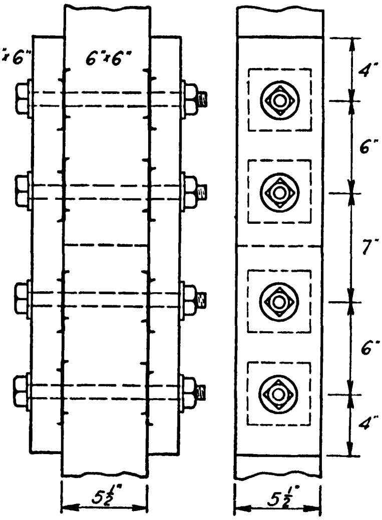
[a] Correct for (1) *wood classification* (2) *moisture content* (3) *direction of load* (4) *edge distance;* values of safe load are for pairs of single or double connectors.

Direction of Load. The values given in Table 19 should be reduced 25 per cent for loads applied perpendicularly to the grain. The allowable loads applied obliquely should be reduced by a straight line relationship between the values for 0 and 90 degrees.

Edge Distance. The specifications that were given for split ring connectors in § 84 may be applied with flanged plate connectors.

88. Claw-Plate Connectors. These connectors serve the same purpose as flanged plate connectors but they are installed differently. Their installation requires a precut circular dap into which the face plate is recessed, but the teeth must be forced into the uncut timber by bolt

DP27a. *Design a column splice to transfer 33 per cent of the allowable load through side timbers by the use of bulldog connectors. Make allowance for margins under 1½" even though load is parallel to grain because wood used splits easily.*



Timber is of 1000c classification.

Value of a 6" × 6" post at 1000#/□" = 5½ × 5½ × 1000 = 30,200#.

Splice value = 0.33 × 30,200 = 10,000#.

Value of a pair of 4" square bulldog connectors (No. 4 timber, ⅞" bolt, ¾" side margins) = 6100 × 0.88 = 5300#.

The factor 0.88 follows from § 84 (edge margin).

Use two pairs of connectors on each side of the splice.

DP27b. *Redesign the splice of DP27a for use with claw-plate connectors.*

Diameter. With 3⅛" connectors, edge margins are 1¼"; allow 4% reduction for edge margin.

Connector value in No. 4 timber = 5200 × 0.96 = 5000#.

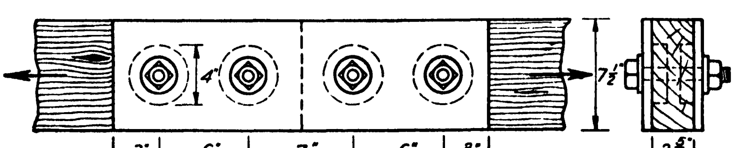
Use 2 connectors on each side of the splice spaced 1½ diameters apart and with 1½" end margins.

DP27c. *Design a tension splice for a 3" × 8" redwood timber to develop its net value in tension at 800#/□". Use flanged plate connectors with ¼" splice plates.*

Value of timber (⅞" bolts) = 2.62 (7.5 − 0.87) 800 = 13,900 lb.

Value of 4" connector (No. 3 timber) = 8300 × 0.85 = 7050# (pair).

Use two pairs of 4" malleable connectors on each side of the splice.



pressure. These claw plates are castings and they are furnished in male and female types as shown in Fig. 81. The male or hubbed connector is used with the plain or female connector in timber-to-timber connections. Shear is then transferred through the hub and not through the bolt. The arrangement in a timber-to-metal joint is for the hub to fit into a hole in the metal strap so that shear is again transferred through the hub rather than through the bolt. This arrangement is intended to permit the teeth to remain fixed in the wood after shrinkage since the hubbed connector can separate slightly from the plain connector or metal strap without developing structural weakness.

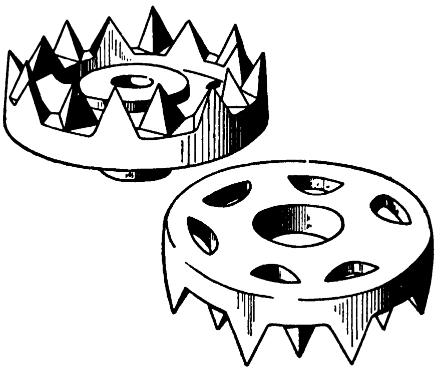


FIG. 81. CLAW-PLATE CONNECTORS.

The safe load values for claw-plate connectors may be obtained by correcting the values as given in Table 20. See the example *DP27b* for an illustration of the use of this table.

TABLE 20

SAFE LOADS FOR CLAW-PLATE CONNECTORS IN PAIRS [a]

Diameter Connector	Bolt Size	Safe Load Parallel to Grain	
		Timber-to-timber	Timber-to-metal
2⅝ in.	½ in.	4200 lb.	5000 lb.
3⅛	⅝	5200	6400
4	¾	7000	8600

[a] Correct for (1) *wood classification* (2) *moisture content* (3) *direction of load* (4) *edge distance;* values of safe load are for a pair of single or double connectors.

Wood Classification. The values in Table 20 should be corrected by use of the following factors where the timber is classified as different from No. 4. (See § 84 for the classification of various species of timber.) No. 1, 65%; No. 2, 75%; No. 3, 85%; No. 4, 100%; No. 5, 115%.

Moisture Content. Since claw-plate connectors can follow the shrinkage of the wood without developing looseness, it is not common to allow for shrinkage unless the change in moisture content is quite severe. The resistance of any connector used in entirely green timber should be discounted, however. An allowance has been suggested in § 84.

Direction of Load. The safe load resistances for loads perpendicular to the direction of the grain should be obtained by reducing the values in Table 20 by 25 per cent. Safe loads applied obliquely to the grain are obtained by direct proportion on the assumption of a straight line variation between these limits.

Edge Distance. The same minimum requirements as given for split ring connectors in § 84 must be provided, but greater margins are recommended for toothed connectors, where splitting is always a possibility.

Timber Members and Connections

89. Usage. Timber is widely used for all temporary construction such as concrete form supports, arch centering, trestles, and scaffolding. In the Pacific coast region, timber bridge trusses are rather widely used. The heavy timber warehouse designated as "slow burning" or "mill" construction is the most common example of a large timber building. Such structures as elevated bins, mine head frames, wharves and docks are as commonly constructed of timber as of other materials.

The main structural elements in timber construction are the beam and the post or column. Timber is an excellent material for compression members since tight knots do not seriously weaken the piece and because *its normal dimensions provide it with proper stiffness* to resist buckling. Timber beams are also satisfactory when they are selected so that only small tight knots are permitted near the extreme fibers. Wood is not ordinarily used for tension members because steel rods are usually more economical. However, the use of modern connectors has made timber tension members practical. Working stresses for timber in highway bridge design are given by Spec. 78.

90. Beams and Joists. The important stresses to be considered in the design of timber beams are (1) *fiber stress* in flexure (2) *beam shear* at the neutral axis (3) *bearing perpendicular to the grain* at the reactions and under concentrated loads.

Flexural Resistance. The safe moment resistance of a rectangular wood beam may be computed from the ordinary relation

$$M = \frac{fI}{c} = \frac{fbd^2}{6}, \text{ or } bd^2 = \frac{6M}{f}. \tag{6}$$

Of course, f is the allowable stress in flexure of the specified grade of structural timber, and I/c is the section modulus, $bd^2/6$. For design, the value of M is known; the grade of timber is selected tentatively to fix f so that an economical commercial size of timber may be selected. For reasonable economy, the depth of the beam should be at least 50 per cent larger than the breadth, and a greater relative depth is preferred.

Form Relationships. Economy will be obtained by the use of deep narrow timbers. A depth-thickness ratio of from 3 to 6 is common for joists, and a ratio of from 2 to 4 is used for stringers.

Shearing Resistance. The safe shear resistance of a rectangular wood beam may be computed from the *beam-shear formula*

$$s_s = \frac{VQ}{Ib} = \frac{3}{2}\frac{V}{bd}, \tag{7}$$

or

$$V = \frac{2}{3} s_s bd. \tag{8}$$

V is the maximum shear or maximum end reaction and s_s is the allowable unit shear *parallel to the grain.* Such shear often governs the sizes of wood beams.

Notched Beams. Beams are often notched near the end as shown in Fig. 82. The shearing resistance is reduced thereby not only because the effective depth in equation (8) must be reduced to d', the remaining depth of the beam, but also to allow for a reduction of the allowable horizontal shear s_s to s'_s where

$$s'_s = s_s d'/d. \tag{9}$$

This relationship has been established by tests at the Forest Products Laboratory.

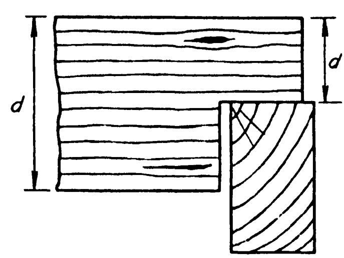


Fig. 82. End Notch.

Bearing Resistance. The normal values of compression perpendicular to the grain apply for end bearing areas and for interior bearing areas more than 6 in. in length. *Shorter interior areas* may be loaded more heavily. The increase in allowable compression perpendicular to the grain

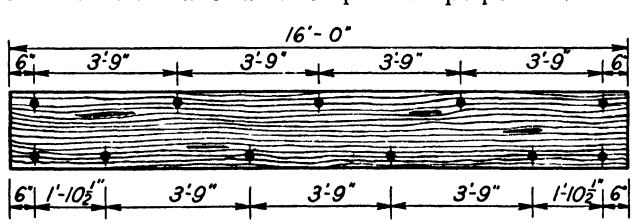


Fig. 83. Bolt Spacing of Laminated Beams Used in Tests at the U. S. Forest Products Laboratory.

may be 10 per cent for a length of 4 in., 30 per cent for a length of 2 in., and 60 per cent for a length of 1 in. No increase is permitted for short end bearing areas.

91. Built-up Beams. Timber beams may be built up of planks placed side by side and bolted, nailed or glued together. Figure 83 shows how a

beam constructed with vertical laminations (five planks 2 in. by 12 in.) was bolted together for testing in the Forest Products Laboratory. This beam was found to be as strong in flexure as a structural timber of comparable size.

Beams may also be built up with two or more *horizontal laminations.* The use of such beams cannot be recommended unless they are constructed with proper shear developers, such as modern connectors, between successive laminations. The design of proper shear developers is not complicated since the horizontal shear between laminations can be found by use of the beam-shear formula ($s_s = VQ/Ib$) and the safe load on modern connectors has been given in Tables 15 to 20. This design process is illustrated by the example *DP*28.

92. Buckling and Deflection. Beams that are narrow and deep show a tendency toward lateral buckling and failure by *torsion.* If the compression surface is restrained against lateral deflection, such failure cannot occur. The total *buckling load* for a simply supported beam where the ends are supported laterally, but are not fixed by being "built in" to a wall, is

$$W = \frac{28.3\sqrt{EI'GK'}}{L^2}. \tag{10}$$

In this expression, I' is the moment of inertia of the beam about its vertical axis or $db^3/12$, G is the torsional modulus of rigidity which is approximately $E/16$, and K' is a factor similar to I' which varies from $db^3/4$ to $db^3/3$ as the depth-breadth ratio increases from 2.0 to ∞. Hence, we may write an expression for W' the *safe load with a factor of safety against buckling of about 6 or 7,*

$$W' = \frac{2EI'}{L^2} = \frac{db^3E}{6L^2}. \tag{11}$$

Similar results for different end conditions may be derived from formulas given in the *Wood Handbook* of the Forest Products Laboratory.

For the example *DP*28, this formula would give a safe total load of

$$W' = \frac{16.5 \times 7.5^3 \times 1{,}600{,}000}{6 \times 240^2} = 32{,}000 \text{ lb.}$$

The actual load is only $785 \times 20 = 15{,}700$ lb. Therefore, the beam will not be endangered by buckling.

Deflections of Wood Beams. The usual formulas for deflection may be applied with the use of the values of E as given in Table 6. The computed deflection, however, does not represent the result to be expected after *time yield* has taken place. Such yielding will be especially marked for timber which is placed green and which seasons gradually under load. It

DP28. *Design a built-up beam from 6″ × 8″ Douglas fir timbers (1600f coast) to carry a uniform load of 750#/′ on a 20′ span. Timber is No. 4 classification.*

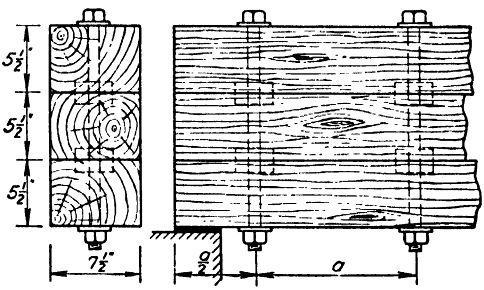
Beam Section:

Weight of beam.	*Estimate weight at 50#/′.*
Bending moment.	$M_{max.} = \frac{1}{8} \times 800 \times 20^2 \times 12 = 480{,}000''\#$.
Section modulus.	$S = 480{,}000/1600 = 300$.
Dimensions.	*For* $b = 7\frac{1}{2}''$, $d = \sqrt{\frac{6 \times 300}{7.5}} = 15.5''$.
	Use three 6″ timbers; $d = 3 \times 5\frac{1}{2} = 16\frac{1}{2}''$.
Weight of beam.	*Nominal size 8″ × 18″, weight = 35#/′.*
Reaction or end shear.	$V_{max.} = (750 + 35)10 = 7850\#$.
Horizontal shear.	$s_s(max.) = 3/2\frac{V}{bd} = 3/2 \times \frac{7850}{7.5 \times 16.5} = 95\#/\square''$.
	(allowable shear = 100#/□″)

s_s on planes of lamination at reaction obtained from beam-shear formula,

$$\frac{VQ}{Ib} = \frac{7850 \times 7.5 \times 5.5 \times 5.5}{\frac{1}{12} \times 7.5 \times 16.5^3 \times 7.5} = 85\#/\square''.$$

Shear per lineal foot = $85 \times 12 \times 7.5 = 7650\#$.



Shear Connectors: *(Used singly; reduce table values 50%.)*

Connectors may be chosen and spaced as follows for use with ¾″ bolts:

(a) *Split ring type* 4″ d. at 0.5 × 12 × 12,000/7650 = 9½″.
(b) *Alligator type* 4″ d. at 0.5 × 12 × 6300/7650 = 5″.
(c) *Bulldog type* 4″ sq. at 0.5 × 12 × 5500/7650 = 4¼″.
(d) *Claw-plate type* 4″ d. at 0.5 × 12 × 7000/7650 = 5½″.
(e) *Flanged type* 4″ d. at 0.5 × 12 × 7500/7650 = 6″.

Spacing of connectors may be increased toward the center of the span and may be doubled at the quarter point if the uniform load is fixed in position.

End Bearing:

For 1600f Douglas fir, compression perpendicular to the grain is limited to 345#/□″.

Bearing length = 7850 ÷ (7.5 × 345) = 3″. Use a 3″ × 7½″ bearing plate attached to beam with countersunk screws.

is common to double the value of the dead loading for use in deflection computations to allow in some measure for time yield. Such computed deflections should then be less than the deflections permitted by the specifications.

EXAMPLE. For the beam of the example *DP*28, the deflection $\left(\frac{5}{384}\frac{wL^4}{EI}\right)$ will be computed and doubled to represent the approximate ultimate deflection including an allowance for time yielding.

$$D = 2 \times \frac{5}{384} \times \frac{785}{12} \times \frac{240^4}{1{,}600{,}000 \times \frac{1}{12} \times 7.5 \times 16.5^3} = 1.25 \text{ in.}$$

Since this deflection is $\frac{1}{192}$ times the span while $\frac{1}{360}$ is the limitation set for beams to which plaster is attached, this deflection might need to be decreased by deepening or widening the beam.

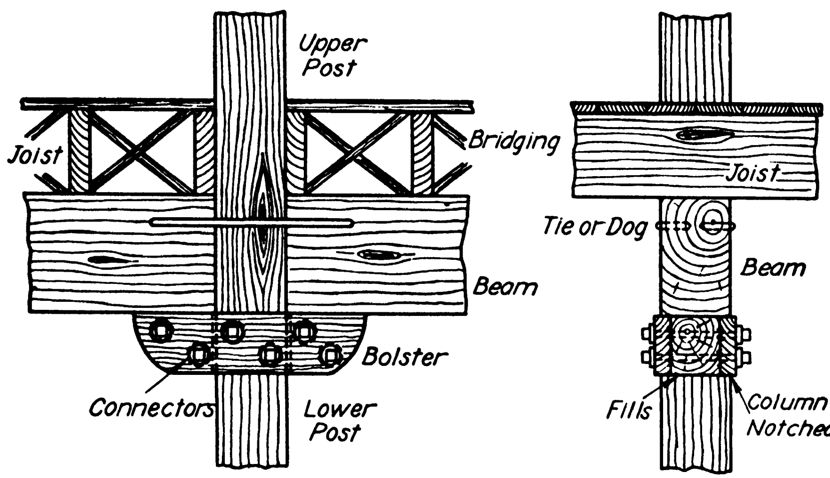


FIG. 84. SATISFACTORY STRUCTURAL DETAIL THAT IS WASTEFUL OF HEADROOM.

93. Beam and Column Details. Shrinkage occurs in all timber. This influence may be particularly serious where the load is applied perpendicularly to the grain. For instance, Fig. 84 illustrates a beam-to-column connection arranged to let the upper and lower posts bear directly on each other. If, instead, the *bolster* is placed in between the upper and lower post, the bolster is in compression across the grain. Its normal shrinkage and crushing will permit considerable settlement of the upper floor of the building. This objectionable settlement will be exaggerated by bringing the beams together between the upper post and the top of the bolster since the thickness of wood acting in compression across the grain is thereby increased.

Shrinkage is even a problem in the use of joist hangers as shown in Fig. 85. Such hangers can be used to save the headroom that is clearly wasted in Fig. 84. However, the joist becomes subject to settlement from vertical shrinkage of the beam and also from any crushing of the beam fibers where the hanger rests on the top of the beam. The type of hanger shown in Fig. 85(*b*) was designed to help reduce this settlement.

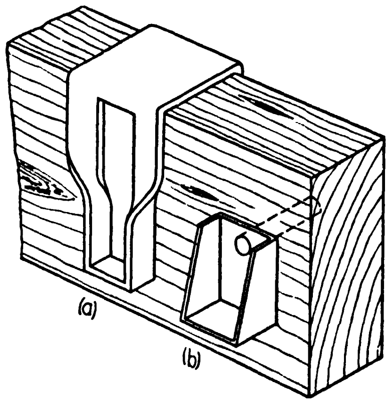


FIG. 85. JOIST HANGERS.

Welded Beam Seat. Simple column caps and beam seats of welded steel can be made as shown in Fig. 86. Patented caps of many types are also available. The steel column cap and beam seat permits direct bearing of the upper and lower posts through the metal cap and provides stability by allowing an adequate seat for the beams. It is advisable to tie the two beams together across the post with a steel *strap or dog* so that the building cannot separate under severe vibration, which might drop a beam from its seat. In the other direction, a tie detail should be arranged between the ends of the joists and the beams, or else tie rods must be run through the building.

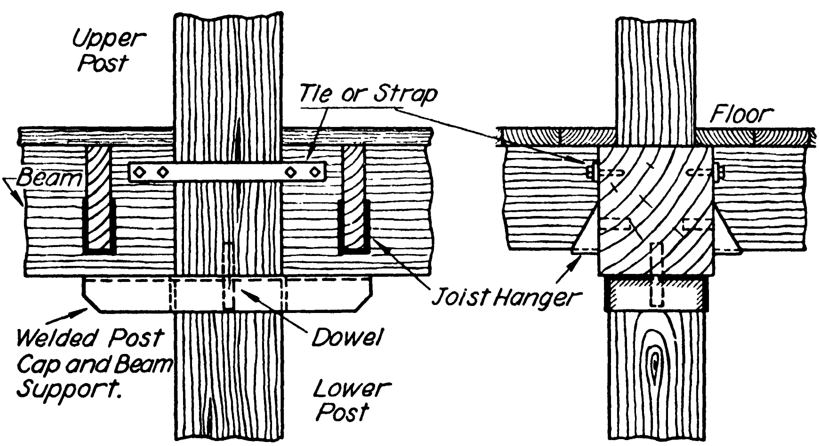


FIG. 86. USE OF WELDED STEEL COLUMN CAP AND JOIST HANGERS TO SAVE HEADROOM.

Pintels in Mill Construction. An objection to the use of steel beam seats and exposed ties and joist hangers is that such thin metal is quickly weakened by fire. *Heavy timber construction* with all of the metal con-

nectors enclosed within the timber itself burns very slowly and has an excellent fire rating.* The use of a pintel as illustrated by Fig. 87 makes the abutting of the beams possible to obtain excellent bearing and still provides end-to-end bearing of the upper and lower columns through the pintel. The pintel itself and the beam ties are actually buried in the timber. The lower column cap is partially exposed but it is cast of thick metal. The detail would not be weakened appreciably even if these exposed edges should be overheated. Mill construction usually does not make use of closely spaced joists. The joists are widely spaced to justify a *thick subfloor* which will also be slow burning. The pintel is designed as a short cast iron column usually at a working stress of 12,000 lb. per sq. in. The design of a building joint where a pintel forms an important part of the connection is given in the example *DP*29.

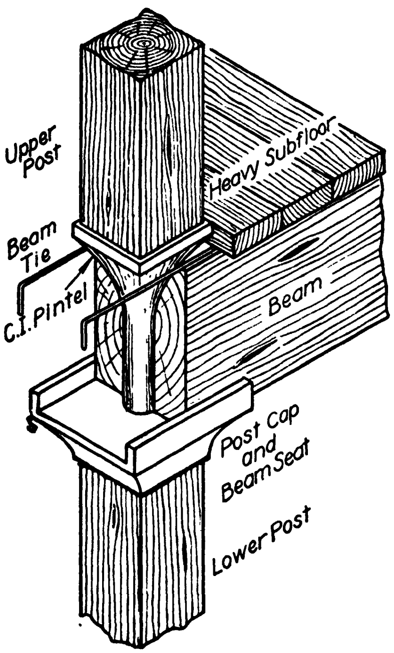


FIG. 87. USE OF PINTEL TO JOIN POSTS IN MILL CONSTRUCTION.

94. Columns and Posts. As has been shown by Table 10, structural timber is classified into stress grades for use as compression members and posts. The stress grade such as 1200*c*, 1000*c*, etc. refers to the permissible unit compressive stress parallel to the grain for short compression members, that is, where the length-depth ratio, L/d, is less than 12. For members of greater slenderness ratio, a column formula should be applied. Some early column formulas for wood members that are now seldom used were as follows:

$$\frac{P}{A} = \frac{f_c}{1 + \dfrac{L^2}{1000d^2}}, \tag{12}$$

$$\frac{P}{A} = f_c - 10\frac{L}{d}, \tag{13}$$

$$\frac{P}{A} = f_c\left(1 - \frac{L}{60d}\right). \tag{14}$$

* See *Building Code*, National Board of Fire Underwriters.

The column formula now in use both by the *AASHO* and *AREA* specifications was developed by the Forest Products Laboratory and is known as a *fourth power parabolic formula.* It is used for columns of intermediate stiffness or slenderness ratio for which L/d is less than the critical value K defined below. Hence, this column formula is used for ordinary design, the Euler formula being applied to slender struts.

$$\frac{P}{A} = f_c\left[1 - \frac{1}{3}\left(\frac{L/d}{K}\right)^4\right], \tag{15}$$

where P = total load in pounds,
A = area of cross-section in square inches,
f_c = allowable unit stress in compression parallel to the grain,
L = unsupported length in inches,
d = least dimension of post in inches,

$$K = \frac{\pi}{2}\sqrt{\frac{E}{6f_c}} \quad \text{for any grade or species, and} \tag{16}$$

E = modulus of elasticity.

For slender columns, where L/d is greater than the critical value K, the Euler formula is applied.

$$\frac{P}{A} = \frac{\pi^2 E}{36\left(\frac{L}{d}\right)^2}. \tag{17}$$

The upper limit to be set on slenderness is that L/d shall not exceed 50. The use of these column formulas is illustrated by the example *DP*29.

Column Design. The procedure in designing a timber column is first to choose the stress grade needed, which may be controlled by general requirements; for example, the recommended practice of the *AREA* is as given in Tables 21 and 22. Then, the value of K is obtained from equation (16) and is substituted into equation (15). Here, the factor L/d enters and it must be approximated by use of the designer's best judgment (enhanced by scratch calculations) as to the approximate area of cross-section that will be required for an estimated value of P/A. The resulting computed value of P/A is the working stress from which an actual cross-sectional area can be selected. This gives rise to an actual value of L/d which makes it possible to revise the value of P/A from equation (15). Thus the design of a timber column becomes a " guess and check " process similar to the procedure followed for any long compression member. If the timber is rather short and stiff, a direct design is possible by use of the unreduced working stress for compression parallel to the grain. A

TABLE 21

BRIDGE AND CONSTRUCTION TIMBER

(*AREA* Recommendations)

COMBINATION AND HOWE TRUSS SPANS	
(*a*) Compression Members	1200*c* Structural Posts and Timbers
(*b*) Tension Members	1600*f* or 1400*f* Structural Joist and Plank
(*c*) Diagonals Subject to Reversal of Stress	1200*c* Structural Posts and Timbers
(*d*) Floor Beams, (*e*) Stringers	1800*f* or 1600*f* Structural Beams and Stringers
(*f*) Ties [a]	1200*c* Structural Posts and Timbers
(*g*) Guard Timbers	1100*c* Structural Posts and Timbers
(*h*) Railing	No. 1 Dimension or Timbers
(*i*) Stiffeners	No. 1 Dimension
(*j*) Splices	No. 1 Dimension
(*k*) Nailing Strips	No. 1 Dimension
(*l*) Grillage	1100*c* Structural Posts and Timbers
(*m*) Deck Plank	No. 1 Dimension
(*n*) Bridging	No. 2 or No. 3 Boards
PILE AND FRAMED TRESTLES	
(*a*) Sills and Mud Sills	1200*c* Structural Posts and Timbers
(*b*) Posts	1200*c* Structural Posts and Timbers
(*c*) Caps [a]	1200*c* Structural Posts and Timbers
(*d*) Sash Bracing	1200*f* or 1100*f* Structural Joist and Plank
(*e*) Cross Bracing	1200*f* or 1100*f* Structural Joist and Plank
(*f*) Longitudinal Bracing	1200*f* or 1100*f* Structural Joist and Plank
(*g*) Girts	1100*c* Structural Posts and Timbers
(*h*) End Planks	1200*f* or 1100*f* Structural Joist and Plank
(*i*) Stringers	1800*f* or 1600*f* Structural Beams and Stringers
(*j*) Ties [a]	1200*c* Structural Posts and Timbers
(*k*) Guard Timbers	1100*c* Structural Posts and Timbers
(*l*) Planking for Ballasted Decking	1600*f* or 1400*f* Structural Joist and Plank
(*m*) Railing	No. 1 Dimension or Timbers
FALSEWORK	
(*a*) Sills and Mud Sills	1100*c* Structural Posts and Timbers, or No. 1 Timbers
(*b*) Posts	1100*c* Structural Posts and Timbers, or No. 1 Timbers
(*c*) Caps [a]	1100*c* Structural Posts and Timbers, or No. 1 Timbers
(*d*) Stringers	1600*f* or 1400*f* Structural Beams and Stringers
(*e*) Centering	No. 1 Dimension
(*f*) Lagging	No. 1 Dimension
(*g*) Bracing	1200*f* or 1100*f* Structural Joist and Plank
(*h*) Wedges	No. 1 Dimension
(*i*) Scaffolding	No. 1 Dimension

[a] Where strength in bending is a factor, Beam and Stringer grades should be specified.

TABLE 22

BUILDINGS — HEAVY FRAME CONSTRUCTION

(*AREA* Recommendations)

TYPES OF STRUCTURES

A. Engine Houses
B. Large Machine Shops
C. Warehouses
D. Large Ice Houses
E. Large Station Buildings

ROUGH CARPENTRY

(*a*) Foundation Timbers (Sleepers)	1400*f* or 1200*c* Structural Posts and Timbers
(*b*) Posts and Columns	1400*f* or 1200*c* Structural Posts and Timbers
(*c*) Sills	1400*f* or 1200*f* Structural Joist and Plank
(*d*) Beams, Stringers, Girders, Purlins	
4 Inches and Thinner	1600*f* or 1400*f* Structural Joist and Plank
5 Inches and Thicker	1600*f* or 1400*f* Structural Beams and Stringers
(*e*) Joists and Headers	1600*f* or 1400*f* Structural Joist and Plank
(*f*) Bridging	No. 1 or No. 2 Boards or Dimension
(*g*) Subflooring	No. 2 or No. 3 Boards, or No. 2 Dimension
(*h*) Sleepers or Screeds	No. 1 or No. 2 Dimension
(*i*) Flooring, Plank or Laminated	No. 1 or No. 2 Dimension
(*j*) Studs, Plates, Caps, Bucks	No. 1 or No. 2 Dimension
(*k*) Wall Sheathing	No. 2 or No. 3 Shiplap or D&M
(*l*) Partitions, Plank or Laminated	No. 1 or No. 2 Dimension
(*m*) Rafters	No. 1 Dimension
(*n*) Ribbon Boards, Collar Beams,	No. 1 or No. 2 Dimension
Ridge Boards, Bracing	No. 1 or No. 2 Boards or Dimension
(*o*) Roofing, Pitched	No. 2 or No. 3 Shiplap or D&M
(*p*) Roofing, Flat	No. 1 or No. 2 Shiplap or D&M or Dimension
(*q*) Shingles, Roof	No. 1 or No. 2 Grade
(*r*) Furring and Grounds	No. 2 or No. 3 Boards
(*s*) Lath, Wall and Ceiling	No. 1 or No. 2 Lath
(*t*) Stair Stringers	1400*f* or 1200*f* Structural Joist and Plank, or No. 1 Dimension
(*u*) Cellar and Attic Stair Treads and Risers	No. 1 or No. 2 Boards, No. 1 Dimension
(*v*) Shelving, Heavy	No. 1 or No. 2 Dimension

relatively great reduction of the allowable stress will be necessary for long slender columns controlled by the Euler formula (17). These points are illustrated by the example *DP*29.

95. Tension Resistance of Timber. Since structural joist or stringer timbers are selected for flexural resistance, these classifications can also be used for built-up or even single piece tension members. The availability of modern connectors makes the use of wood tension members practical. Structural joist or stringer timber is selected so that knots near the edges are limited to small sizes. However, large knots are permitted near the neutral axis of the timber covering as much as 40 per cent of the face width for some species. (See Table 23.)

DP29. *Design a column for a ceiling height of 18′ to carry a load of 120,000#. Design a cast-iron pintel to pass through a 14″ × 16″ beam and support the 10″ × 10″ column above. AREA spec.*

Size of Section:

Materials. *Use 1200c close-grained redwood (Table 22).* $E = 1{,}200{,}000$.
Use gray cast-iron for pintel at 12,000#/□″C.

Column formula. Apply equations (15), (16) and (17).

Approx. column size. At 1000#/□″ the area required would be 120□″. Try nominal dimensions of 12″ × 12″.

Ratio $L/d = 18 \times 12 \div 11.5 = 18.8$.

$$\text{Constant } K = \frac{\pi}{2}\sqrt{\frac{E}{6f_c}} = 1.57\sqrt{\frac{1{,}200{,}000}{6 \times 1200}} = 20.2.$$

Since L/d is less than K, the column design is controlled by equation (15).

$$P/A = f_c\left[1 - \frac{1}{3}\left(\frac{\frac{L}{d}}{K}\right)^4\right] = 1200\left[1 - \frac{1}{3}\left(\frac{18.8}{20.2}\right)^4\right] = 900\#/□''.$$

Allowable Load:

$900 \times 11.5 \times 11.5 = 119{,}000\#$.

Load on pintel = value of 10″ × 10″ column.

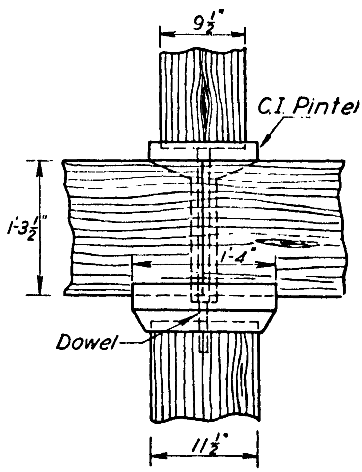
Ratio $L/d = 18 \times 12 \div 9.5 = 22.8$.

Since L/d is larger than K, equation (17) applies.

$$P/A = \frac{\pi^2 E}{36\left(\frac{L}{d}\right)^2} = 0.275\,\frac{1{,}200{,}000}{22.8^2} =$$

630#/□″.

Allowable load $= 630 \times 9.5 \times 9.5 =$ 57,000#. *This is the pintel load.*



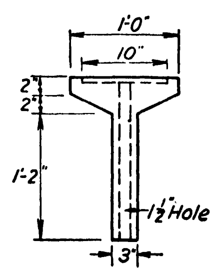
Pintel Design:

$$\text{Area of pintel} = \frac{57{,}000}{12{,}000} = 4.75□''.$$

Try a pipe stem 3″ O.D. and 1½″ I.D.

$$\text{Area} = \frac{\pi}{4}(3^2 - 1.5^2) = 5.3□''.$$

The column cap should be flared from 14″ × 14″ to 16″ × 16″ to receive the 14″ beam and bear on the 12″ column to which it should be doweled.



Remark: *The ties joining the beams are not shown.*

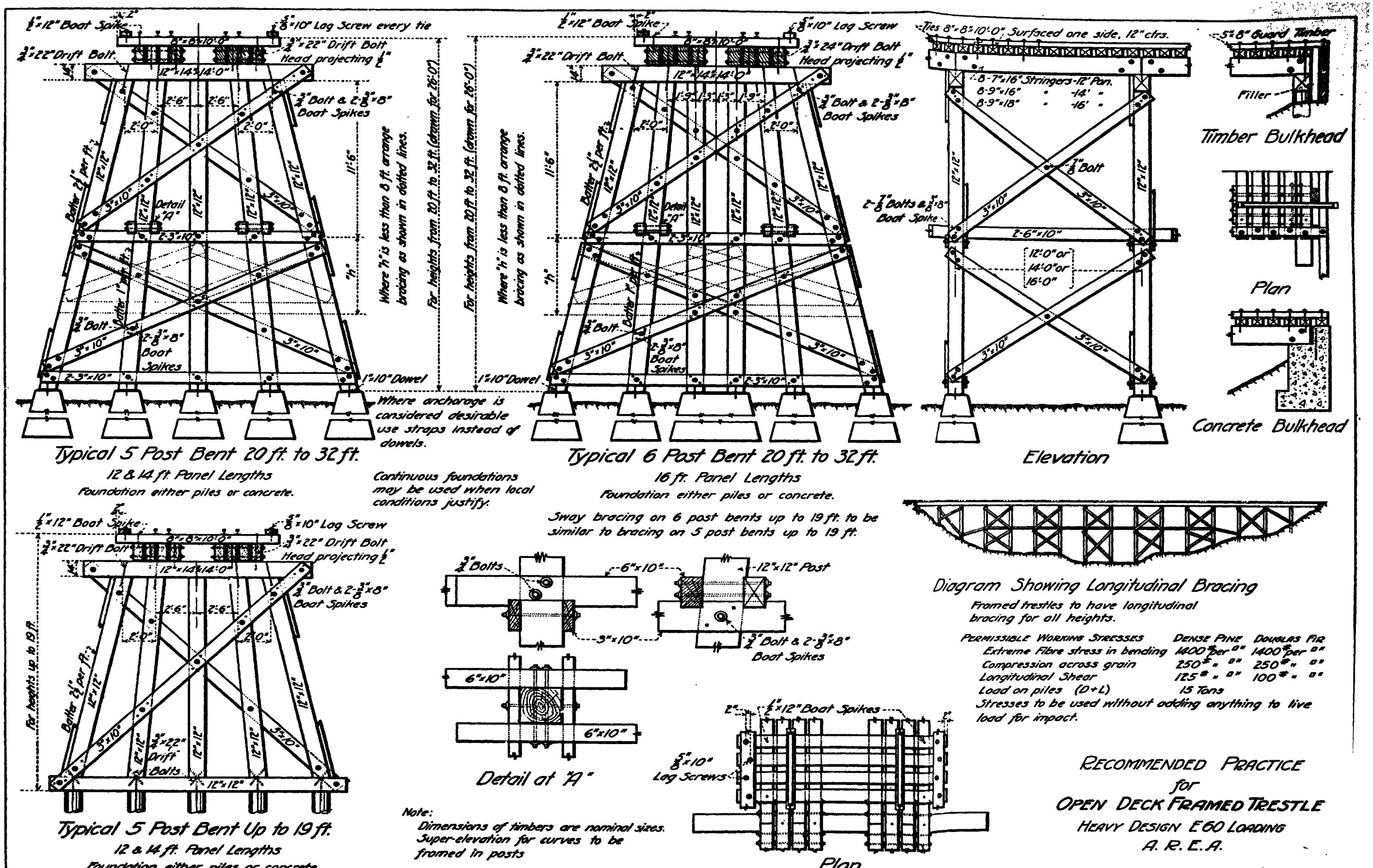

Typical 5 Post Bent 20 ft. to 32 ft.
12 & 14 ft. Panel Lengths
Foundation either piles or concrete.
Where "h" is less than 8 ft. arrange bracing as shown in dotted lines.
For heights from 20 ft. to 32 ft. (drawn for 26'-0")
Where anchorage is considered desirable use straps instead of dowels.
Continuous foundations may be used when local conditions justify.
Typical 6 Post Bent 20 ft. to 32 ft.
16 ft. Panel Lengths
Foundation either piles or concrete.
Sway bracing on 6 post bents up to 19 ft. to be similar to bracing on 5 post bents up to 19 ft.
Elevation
Ties 8"×8"×10'-0" Surfaced one side, 12" ctrs.
Timber Bulkhead
Plan
Concrete Bulkhead
Diagram Showing Longitudinal Bracing
Framed trestles to have longitudinal bracing for all heights.
Permissible Working Stresses
Dense Pine
Douglas Fir
Extreme Fibre stress in bending
Compression across grain
Longitudinal Shear
Load on piles (D+L)
15 Tons
Stresses to be used without adding anything to live load for impact.
Typical 5 Post Bent Up to 19 ft.
12 & 14 ft. Panel Lengths
Foundation either piles or concrete.
For heights up to 19 ft.
Detail at "A"
Note:
Dimensions of timbers are nominal sizes.
Super-elevation for curves to be framed in posts
Plan
Recommended Practice
for
Open Deck Framed Trestle
Heavy Design E60 Loading
A. R. E. A.

Fig. 88

Timbers for use as tension members should preferably be restricted as to defects *in any position* in the same way that defects in joists and stringers are limited near the heavily stressed fibers within the middle half of the length.* If such selection is not possible, it seems desirable to reduce the working stresses in flexure by about the percentages given in Table 23 to obtain reasonable working stresses in tension.

TABLE 23

ALLOWABLE PERCENTAGE OF FACE WIDTH COVERED BY KNOT [a]

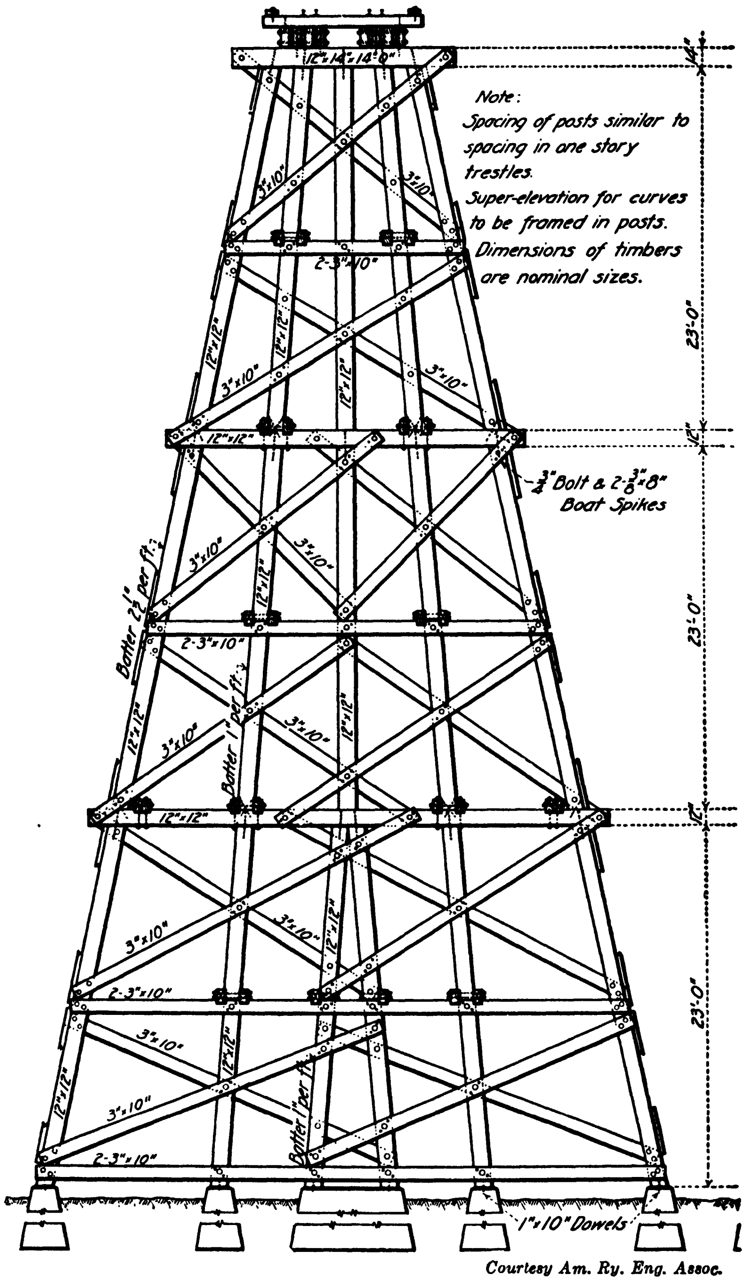
Classification		Joist and Plank	Beam and Stringer
1800*f*	Dense Douglas fir (coast or inland)	28%	18%
	Dense southern pine (long or shortleaf)	28	18
1600*f*	Close-grained Douglas fir (coast)	28	18
	" " " " (inland)	25	13
	Dense southern pine (long or shortleaf)	32	25
	Close-grained redwood	19	5
1400*f*	Tidewater red cypress	26	15
	Dense longleaf southern pine	38	32
	Close-grained redwood	25	14
1200*f*	Douglas fir (inland)	35	26
	Dense shortleaf southern pine	43	38
	Close-grained redwood	32	24
1000*f*	Western red cedar	25	13

[a] Reduce allowable stresses in flexure by the percentage given to obtain working stresses in tension where timbers cannot be selected to eliminate large knots.

Net Section. It is necessary to design tension members upon the basis of the net section after bolt holes have been deducted. The author recommends that *all holes on a zigzag line* be deducted from a right cross-section provided that the slope of the zigzag line is at no point greater than 45 degrees to the right section. The fibrous nature and weak shear resistance of timber *along the grain* dictates a more severe deduction than has been common for steel members.

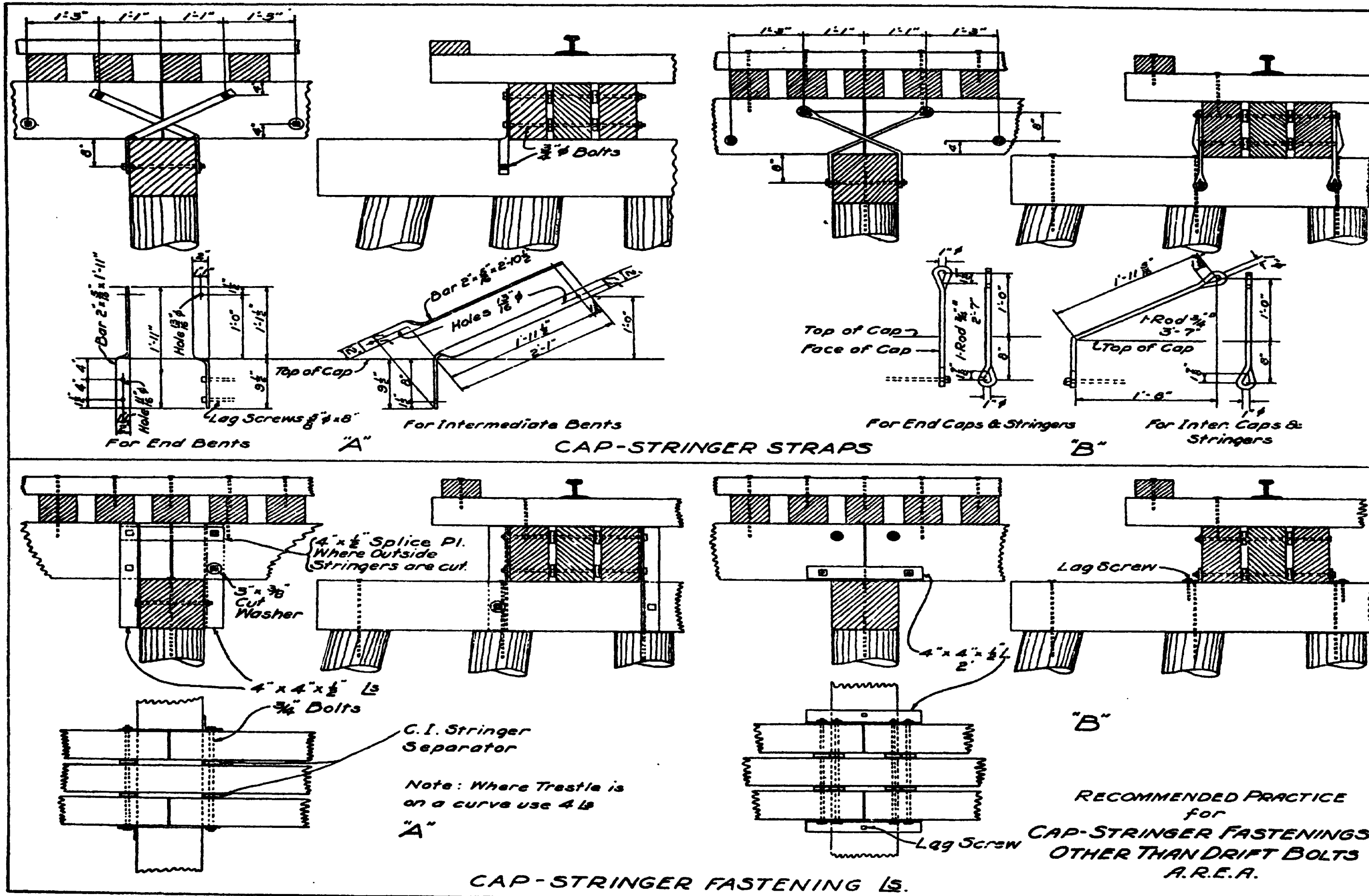
96. Timber Trestles. Perhaps the most common use that has been found for timber as an important modern structural material is in pile trestles for railroad work. The details have been standardized rather carefully by the American Railway Engineering Association. The illustrations of Fig. 88 and Fig. 89 show these details for three trestles that are quite typical. The two-panel bent is limited to a height of 30 ft. The six-panel bent of Fig. 89 is designed for a height of 72 ft. The details of

* These restrictions are given in full in *Standard Specifications for Highway Bridges*, *AASHO*, 1935, and in "Specifications for Structural Timbers," *AREA Manual*, 1936.



Courtesy Am. Ry. Eng. Assoc.

FIG. 89. TYPICAL 5 POST BENT.



Bar 2"×5/16"×1'-11"
Hole 11/16" φ
Hole 13/16" φ
Lag Screws 5/8" φ × 8"
For End Bents
Top of Cap
Bar 2"×5/16"×2'-10½"
Holes 13/16" φ
For Intermediate Bents
"A"
3/4" φ Bolts
CAP-STRINGER STRAPS
Top of Cap
Face of Cap
1-Rod 3/4" φ 2'-7"
For End Caps & Stringers
1-Rod 3/4" φ 3'-7"
Top of Cap
For Inter. Caps & Stringers
"B"
4"×½" Splice Pl. Where Outside Stringers are cut.
3"×3/8" Cut Washer
4"×4"×½" Ls
3/4" Bolts
C. I. Stringer Separator
Note: Where Trestle is on a curve use 4 Ls
"A"
Lag Screw
4"×4"×½" L 2'
"B"
CAP-STRINGER FASTENING Ls.
RECOMMENDED PRACTICE for CAP-STRINGER FASTENINGS OTHER THAN DRIFT BOLTS A.R.E.A.

FIG. 90

attaching stringers properly to the cap of the bent are illustrated by Fig. 90 where both straps and angle fastenings are shown. Caps are drift bolted to the piles or posts as shown by the illustration. A typical section through an overhead wood highway bridge is shown by Fig. 91. For

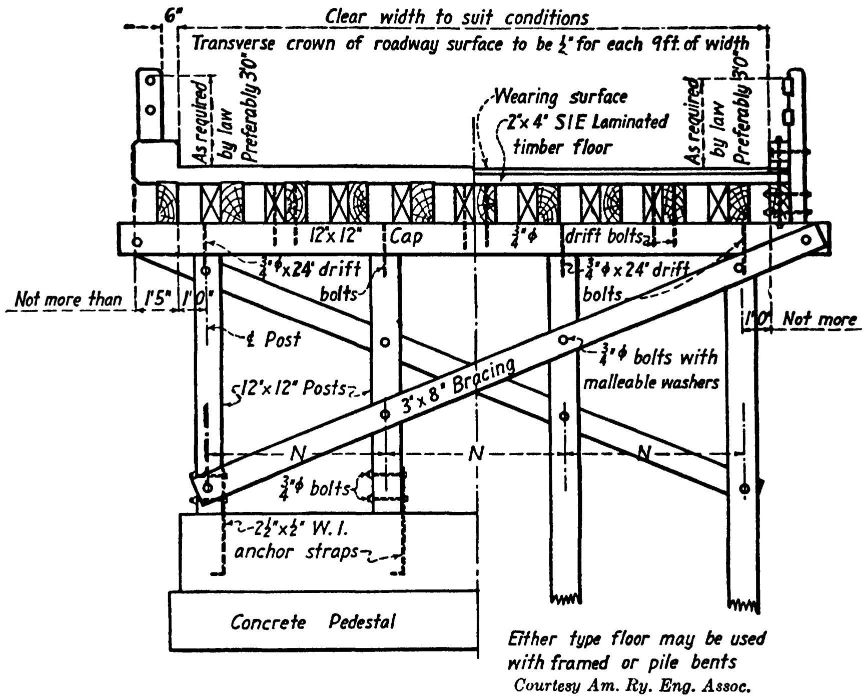


FIG. 91. OVERHEAD WOOD HIGHWAY BRIDGE, H-15 LOADING.

higher structures the posts will have to be battered, the design being similar to the railway bent of Fig. 89. All of these structures are shown merely bolted together. Added strength and stiffness can be provided cheaply by placing alligator or bulldog connectors around each bolt used.

PROBLEMS

98. Find the allowable bearing stress for load at 30° to the grain for 1600*f* or 1200*c* close-grained Douglas fir (inland). *Ans.* 730 lb. per sq. in.

99. Find the allowable bearing stress for load at 45° to the grain for 1000*f* or 800*c* western red cedar. *Ans.* 320 lb. per sq. in.

100. Revise *DP*23 for (*a*) oak, (*b*) redwood timber.

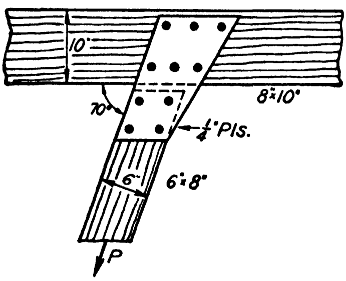
101. Determine the required number and size of nails and of lag screws to connect a steel plate ½ in. thick to a 4-in. timber (white oak) to develop a holding power of 2000 lb.
Ans. Use 9 nails 16d or 3 lag screws ⅜" × 3½".

102. Revise *DP*24 for (*a*) oak, (*b*) redwood timber.

103. Determine the required number of $\frac{3}{8}$-in. × $3\frac{1}{2}$-in. lag screws or $\frac{1}{2}$-in. bolts to develop a lateral shear resistance of 2000 lb. between a $\frac{1}{2}$-in. steel plate and a 4-in. southern yellow pine timber. Permit a 25 per cent increase of safe load where the load is applied through a steel plate. The load is at 30° to the grain. Read footnotes to Table 14. *Ans.* 4 lag screws or 4 bolts.

104. Determine the required number of $\frac{1}{2}$-in. × 6-in. lag screws and also of $\frac{1}{2}$-in. bolts to develop a lateral shear of 3000 lb. between a 3-in. redwood plank and a 6-in. redwood timber. The load is at 45° to the direction of the grain.

105. Revise *DP*25 for 1100*c* and 1400*f* close-grained redwood timber.



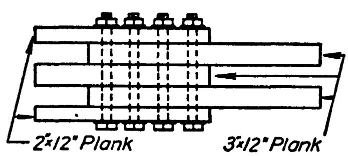
PROBLEM 106.

106. Design and detail a bolted gusset-plate connection of the type shown to develop a load *P* of 6000 lb. The timber is southern yellow pine. Fulfill all specifications as to edge and end margins and proper spacing. Note eccentricity of load on upper group of rivets.

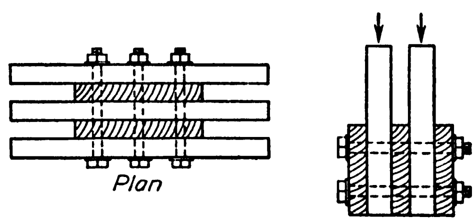
107. Design and detail the bolted splice shown to develop the value of southern cypress timber in tension at 800 lb. per sq. in. on the net section.

108. Design and detail the bolted joint shown to develop a total load of 6000 lb. The planks are 3-in. × 12-in. redwood.

109. Redesign the joint of *DP*26*a* and *b* for southern cypress timber and allow for $21\frac{1}{2}$ per cent moisture content.



PROBLEM 107.



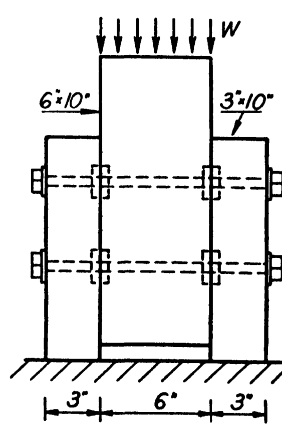
PROBLEM 108.

110. Select size and number of pairs of connectors to resist the load *W* of 49,000 lb. acting parallel to the grain. The timber is seasoned shortleaf southern pine. Carry out the design for (*a*) split ring, (*b*) alligator, (*c*) bulldog, (*d*) flanged plates, (*e*) claw plates.

Ans. (*a*) Three 6-in. diam.; $\frac{3}{4}$-in. bolts.
(*b*) Eight 4-in. diam.; $\frac{3}{4}$-in. bolts.
(*c*) Six 5-in. square; 1-in. bolts.
(*d*) Six 4-in. diam.; $\frac{7}{8}$-in. bolts.
(*e*) Seven 4-in. diam.; $\frac{3}{4}$-in. bolts.

111. Redesign the column splice of *DP*27*a* and *b* for Rocky Mountain Douglas fir (1100*c*).

112. Revise Problem 110 to account for the following special conditions: load applied at right angles to the grain of seasoned Rocky Mountain Douglas fir with 1-in. end margins.



PROBLEM 110.

113. Revise Problem 110(*d*) to account for the following special conditions: load applied parallel to the grain through steel side plates in place of wood side timbers. Use values for dense Douglas fir (coast).

114. Solve Problem 106 by use of (*a*) flanged plates and (*b*) claw plates.

115. Solve Problem 106 by use of split ring connectors for No. 2 wood classification. Do not fail to check on required timber thickness for double ring grooves from Table 16.

116. Solve Problem 108 by use of split ring connectors. Check required thickness of timber in Table 16. Use a load of 20,000 lb.

117. Same as Problem 115 but use bulldog plates.

118. Same as Problem 116 but use bulldog plates.

119. Design a beam to span 22 ft. and carry a uniformly distributed load of 550 lb. per lineal ft. including its own weight. Choose a commercial timber (1600*f*) and check shear and end bearing.

120. Design a beam of 10-ft. span to carry a concentrated center load of 30,000 lb. Check shear, and design bearing plates under the load and at the reactions. Grade, 1200*f*. Allow for the weight of the timber.

121. Revise the design of *DP*28 for 1400*f* dense longleaf southern pine.

122. Assume that timbers of all species are available up to size 12 in. × 12 in. Design a built-up beam held together with bolts and timber connectors to carry a uniform load of 1000 lb. per ft. on a span of 21 ft.-6 in. Allow for the weight of the beam.

123. Revise the design of *DP*29 for 1100*c* redwood. Reduce ceiling heights to 15 ft.

124. Determine the carrying capacities (*AREA* specifications) of the following columns. (*a*) 1200*c* dense shortleaf southern pine, 12 in. × 12 in. × 10 ft.-6 in., (*b*) 1100*c* oak, 14 in. × 16 in. × 17 ft.-2 in., (*c*) 1000*c* close-grained redwood, 6 in. × 6 in. × 11 ft.-3 in., (*d*) 800*c* western red cedar, 5 in. × 7 in. × 15 ft.-0 in. (Sizes are nominal.)

Ans. (*a*) 156,000 lb.
(*b*) 216,000 lb.
(*c*) 16,600 lb.
(*d*) 5,400 lb.

125. Compare the answers obtained from the data of Problem 124 if the design is made by use of equations (12), (13), and (14).

126. Select and splice a southern yellow pine timber to carry a tension stress of 70,000 lb. Take account of the reduction of allowable stress suggested by the footnote to Table 23. Use flanged connectors and steel side plates.

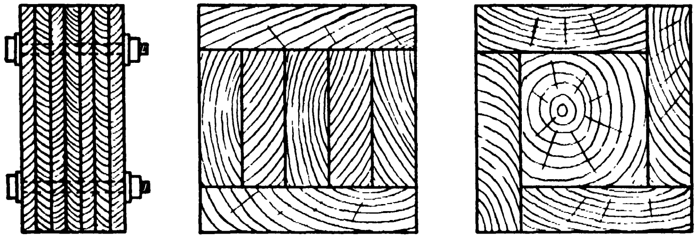
127. Repeat Problem 126 by choosing a built-up section composed of several planks. Splice by interlocking the planks, adding side planks, and using bulldog connectors.

97. Adequate Design of Timber Structures. Engineers have often taken a careless point of view with reference to timber design, particularly for temporary structures. A designer who will exercise extreme care in the design of steel or reinforced concrete structures often crudely approximates or simply guesses at timber sizes. This leads to waste since the designer certainly will not take a chance on his reputation by using slender members when he has not made proper design calculations. Thus, designers in general tend to overdesign wood structures, with the result that the actual economy which can be realized by a proper design in timber is not always achieved.

There has been a certain justification for the designer's attitude toward timber design. We realize, of course, that wood is not as uniform as

steel or reinforced concrete. However, modern methods of selection and classification have removed the guesswork from the choice of working stresses. There is little reason to question the fact that the specifications for 1400*f* close-grained redwood and 1100*c* Douglas fir (coast) will produce timbers fully capable of resisting the working stresses specified for such woods. This eliminates any justification that may have existed for the use of oversafe member sizes selected by "rule of thumb." Scientific methods of design are needed as much in timber as in steel.

Laminated Timber. A new wood material has received much study and some practical use during recent years. The reference is to plywood timber. Either as beams, columns, or gusset plates, the built-up material has a greater factor of safety than the plain timber. The reason is that planks can be oriented during assembly so that the weakening effects of



(a) Beam (b) Column (c) Column

FIG. 92. HEAVY LAMINATED TIMBER.

knots, shakes, splits, and wanes are reduced to a smaller factor in the built-up member than in a full sized timber. A few such possibilities are suggested by Fig. 92, but it seems likely that this idea will find considerable extension into built-up arches, rigid frames, continuous beams, etc. Units may be held together by nails, bolts, screws, or connectors, and by waterproof glue.

Careful scientific design, the use of modern connectors, and the construction of built-up members seem to suggest that timber may again find a wider usefulness than has existed in the recent past. It should be realized that timber lends itself to rustic architectural treatment thoroughly in harmony with the background of the countryside. Effective use was once made of that fact in Europe and in the eastern part of the United States.

CHAPTER 6

TENSION MEMBERS

98. Design of Tension Members and Connections. In the design of tension members it is usual to arrange the member and its connection in such a manner that there will be no bending in the member from eccentricity of the connection. When this arrangement can be made, the stress is considered to be distributed uniformly over the net section of the member, that is, at the root of a thread, across the section at a pin hole, or on a section taken through a group of rivet holes. The unit stress on the net section is the total force divided by the *net area*, and this stress must be kept within the allowable working stress in tension. Where an eccentric connection exists, the effect of the eccentricity may be provided for by specifications, or else the stress caused by the bending moment must be considered in the design. Only members where eccentricity is provided for by specification will be considered under the heading of simple tension members. For instance, a single-angle tension member connected by one leg has its net area reduced by specification to allow for eccentricity. Only 50 per cent of the unconnected leg is considered effective according to several specifications.

BARS AND RODS

99. Welded Tension Bar. The welded tension bar is a satisfactory member for use in the design of light structures such as stairway framing around industrial plants, foot bridges, electric sign supports, power line towers, and in other cases where the general specification requiring tension members to be capable of resisting compression does not apply. Since no holes need be punched through the bar, the full gross area may be used for carrying stress. The welding at the end of the bar should be arranged symmetrically in order that there will be no bending stresses caused by an eccentric connection.

EXAMPLE OF A TOWER DIAGONAL, *DP*30*a*. Probably the welded bar or *flat* is the most satisfactory tension diagonal for a light stairway tower such as the one illustrated. The gross area of the bar is effective, resulting in full efficiency. The member design to resist the direct stress is so simple that the calculations need no explanation. The joint detail is of some interest. The gravity axes of the four members meet at a point as shown in the illustration. The upper diagonal may be cut ¾ in. short of its line of contact with the lower diagonal to permit two ⅜-in. fillet welds to be placed between.

Usually, however, the gap is made only $\frac{1}{4}$ in. or $\frac{3}{8}$ in. wide so that a single rectangular butt weld serves the double purpose as indicated on the detail of the joint. The lower diagonal is connected by $5\frac{1}{4}$ in. of weld (the minimum required amount). The upper diagonal has an end connection of $5\frac{1}{2}$ in. of weld which is necessary because the minimum length of the side welds is usually set at $1\frac{1}{2}$ in.

100. Tension Rods. Tension rods are used only as secondary structural members for which the design stress is relatively small. One common use for a tension rod is as a sag rod in mill building construction. The purpose is to support the purlins between trusses in a direction parallel to the roof or to support the girts in a vertical direction between columns. This support is particularly necessary where the purlin or girt is a channel, which has a low flexural resistance about its axis of least radius of gyration. The maximum stress in a sag rod occurs at the ridge of the roof and is computed as the sum of its purlin reactions from the eave to the ridge produced by the component of dead load and snow load taken parallel to the roof. Other uses for tension rods are as hangers and as tie rods to resist the thrust of a floor arch or bridge arch for which fixed abutments are not available. For this latter use, large tie rods may be needed.

End Connections. Tension rods can be obtained either in round or square sizes. Large rods (1 in. or larger) usually are upset before threading so that the area at the root of the thread is about 20 per cent greater than the area of the bar. The cost of upsetting overbalances the saving for bars of smaller diameter. For non-upset bars the designer should increase the diameter at the *root of the thread* by $\frac{1}{16}$ in. over the actual required diameter to allow for *localized stress* at this point. Bent tension bars should be avoided, if possible, or else should be designed conservatively. Tension rods arranged for connecting to a pin are known as clevis rods or loop rods. These are shown in Fig. 93. Standard connections of these types are strong enough to develop the bar. They may be obtained with turnbuckles for the purpose of introducing initial tension.

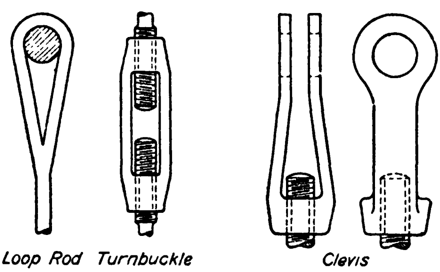


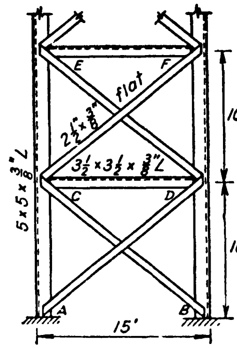
FIG. 93. PIN CONNECTED RODS.

Initial Tension in Rods. It is desirable to introduce initial tension into the diagonal wind bracing of towers and buildings, since sway is thereby reduced. Accordingly, tension rods used for this purpose should have an initial tension of at least 5000 lb. (often 5000 lb. per sq. in. for larger rods) introduced by turning up the end nuts or turnbuckles. (Spec. 53.) As long as any initial tension exists, the wind stress is divided equally between

DP30a. *Design and detail a welded bar as a tension diagonal CF of the stairway tower shown. Severe corrosion hazard; AISC and AWS spec.*

Data:

Design stress (wind) in diagonal = 20,200#T.



Member:

Allowable stress = 20,000 × 1.33 = 26,700#/□".

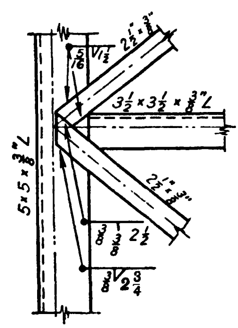
Area = 20,200 ÷ 26,700 = 0.76□".

Bar size. A $2\frac{1}{2}$" × $\frac{5}{16}$" flat furnishes 0.78□".

Corrosion. Choose a $2\frac{1}{2}$" × $\frac{3}{8}$" bar to allow for corrosion.

Slenderness ratio = (15 × 12) ÷ (0.29 × 2.5) = 248. (15' is the horizontal projection.)

Members of L/r < 250 do not sag.



Connection: *(Corrosion hazard; low weld value.)*

Value of member = 26,700 × 0.78 = 20,800#.

Value of $\frac{3}{8}$" fillet weld = 3000 × 1.33 = 4000#/".

Length of weld = 20,800 ÷ 4000 = 5.2".

Remarks: *The arrangement of the welds is shown on the detail. The diagonals join together on the near face of the column. The horizontal strut is welded to the back face of the column angle with 4" of $\frac{3}{8}$" fillet weld which is adequate to care for the horizontal component of the stress in the diagonal. The diagonals may be welded at the point of crossing, but this is not required. The connection is designed for the value of the member before its thickness is increased for corrosion.*

DP30b. *Select an upset rod diagonal to serve in place of the flat bar of DP30a. Make no allowance for stress concentration but add 20 per cent for corrosion.*

Net Area = (20,200 ÷ 26,700) 1.20 = 0.91□".

Net Area of a 1" sq. rod = 1.0□" (1.29□" at root of thread).

Net Area of a $1\frac{1}{8}$" round rod = 0.99□" (1.29□" at root of thread).

the crossing bars by reducing the tension in one and increasing the tension in the other. Accordingly, when the stress in one reduces from 5000 lb. to zero, the other will have a stress of 10,000 lb., which is the same stress that it would have if no initial tension had been introduced. It follows that *we should design heavy crossing diagonals with or without initial tension for exactly the same stress.* Initial tension may be introduced into riveted diagonals by detailing the members 1/16 in. short for each 20 ft. of length. The members are then pulled into place in the field with drift pins. Welded bars can be preheated to obtain the same result.

101. Tension Rod Design. EXAMPLES *DP*30*b* and *DP*31. The first example on tension rods is the selection of an upset rod as a diagonal for a tower. An upset rod is proper for this member because the diameter is greater than 1 in. The second example is properly the choice of a non-upset rod since the diameter is small. Some specifications have limited the minimum diameter of a tie rod in a building or bridge structure to 5/8 in. The reason is that a smaller non-upset rod has so little effective area that it furnishes almost no factor of safety against possible increase of stress. A 5/8-in. rod is therefore considered a minimum structural size in the same manner that a 2-in. angle is taken as the minimum section for a tension diagonal.

102. Eye-Bar Tension Members. The pin connected truss bridge with eye-bar tension members was at one time almost universally used. This type of truss is used today for long span bridges only, since the large dead weight of a long span structure is necessary to prevent rattling and excessive vibration under heavy traffic. Eye bars are also used to make up the chains of some suspension bridges, and they may be used as floor-beam hangers for large suspension or through arch bridges.

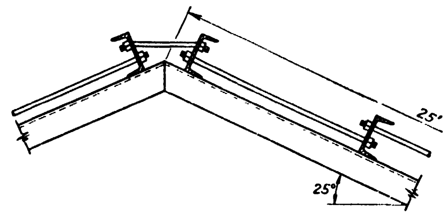
The use of eye bars offers the advantage that heat treated steel of high strength may be used with correspondingly high working stresses. For instance, in a recent suspension bridge design, the eye bars for the chain were heat treated to produce an elastic limit of 75,000 lb. per sq. in. and were stressed to nearly 50,000 lb. per sq. in. Even higher working stresses have been used.

The specifications of the *AREA* require that the eye bar have an excess net area of 35 per cent through the center of the pin over the body of the bar. (Spec. 185.) The pin diameter is required to be not less than 8/10 of the width of the widest bar attached. The thickness of the bar must be between 1 in. and 2 in., and it is commonly not less than 1/8 of the width. To meet these specifications, it is frequently necessary for the designer to use several bars in parallel. Bars are required to be packed on the pin so that adjacent bars will not touch.

EXAMPLES OF EYE-BAR SELECTION, *DP*32. Two main considerations in eye-bar selection are to meet the requirements of the specifications as to bar width and range of thickness. Then the bars must be arranged or packed on the pin. The packing may be complicated by too many bars. Hence, the designer may choose an alloy steel or heat treated eye bar to reduce the number of bars required. It is shown in *DP*32*b* that the choice of

DP31. *Design sag rods to support the purlins of an industrial building roof. Sag rods are placed midway between roof trusses which are spaced 16′ apart. Use AISC spec.*

Loads: *Roofing is corrugated asbestos at 3#/□′ and purlins weigh $3\frac{1}{2}$#/□′ of roof surface. Snow load is 20#/□′ of horizontal projection or 18.1#/□′ of roof surface. Total vertical load is therefore 24.6#/□′ of roof surface.*



Load component parallel to roof $= 24.6 \times \sin 25° = 24.6 \times 0.423 = 10.4$#/□′.

Rod Size:

Load carried by one sag rod $= 10.4 \times 25 \times 8 = 2080$#.

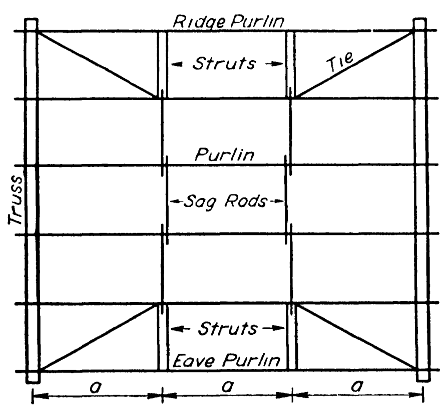
Required area of rod $= 2080 \div 20{,}000 = 0.104$□″.

Net diameter. A net diameter of $\frac{3}{8}$″ *will furnish the required area.*

Root diameter $= \frac{3}{8} + \frac{1}{16}$ *(for stress concentration)* $= \frac{7}{16}$″ *at root of thread.*

Minimum diameter $= \frac{5}{8}$″ *which furnishes a root diameter of 0.507″.*

Bent rod. The use of slant washers avoids bending the ridge rod.



Remarks: *Multiple sag rods (for example, those illustrated are at the one third points of the bay) are required for very wide bays. Diagonal ties and short struts, as shown, are needed to resist snow covering one side only of a large roof. For specifications regarding sag rod design, see Ketchum's "Steel Mill Buildings."*

DP32a. *Select an eye bar to act as a lower chord member of a 300′ railway bridge truss. AREA spec.*

Design stress = 1690 kips; pin size = 9″; panel length = 30′.

Required area of bars = 1,690,000 ÷ 18,000 = 93.8□″.

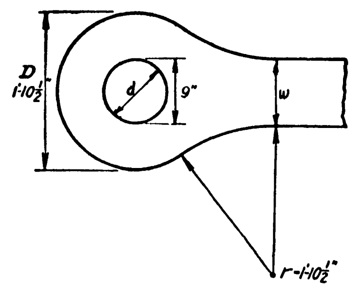
Choice of Bar: *Maximum bar width = (10/8)9″ = 11.25″. (Spec. 185.) Use 10″ bars.*

Minimum bar thickness = ⅛ (10″) = 1.25″.

For 6 parallel bars, thickness

$$= \frac{93.8}{6 \times 10} = 1\tfrac{9}{16}''.$$

Bearing stress = 1,690,000 ÷ (6 × 1.56 × 9) = 20,000#/□″.



Diameter of Head: *From a structural steel handbook, the head diameter (35% excess area) is found to be 22½″. This value may be checked by use of the formula*

$D = 1.35w + d = 1.35 \times 10 + 9 = 22.5''$.

DP32b. *Repeat the design of DP32a but use nickel steel eye bars to reduce the number required. See Spec. 165 for allowable stresses.*

Allowable stress for nickel steel eye bars by AREA spec. = 27,000#/□″.

Required area of bars = 1,690,000 ÷ 27,000 = 62.6□″.

Thickness of bars. For 4 bars of 10″ width the thickness is $\frac{62.6}{4 \times 10} = 1\tfrac{9}{16}''$.

Thickness for bearing on a carbon steel pin $= \frac{1{,}690{,}000}{4 \times 24{,}000 \times 9} = 1.96''$.

Use 4 nickel steel eye bars 10″ × 2″ drilled for a 9″ pin of carbon steel.

DP32c. *Repeat the design of DP32a for heat treated eye bars. Allowable tension after heat treatment = 35,000#/□″. Bearing on the pin at 24,000#/□″ still controls as in DP32b.*

nickel steel makes possible the use of 4 bars in place of the 6 needed of carbon steel, even though the carbon steel pin is retained. With a nickel steel pin, the eye bars could be reduced from a thickness of 2 in. to a thickness of 1$\frac{9}{16}$ in. Working stresses for alloy steels are given by Spec. 165.

PROBLEMS

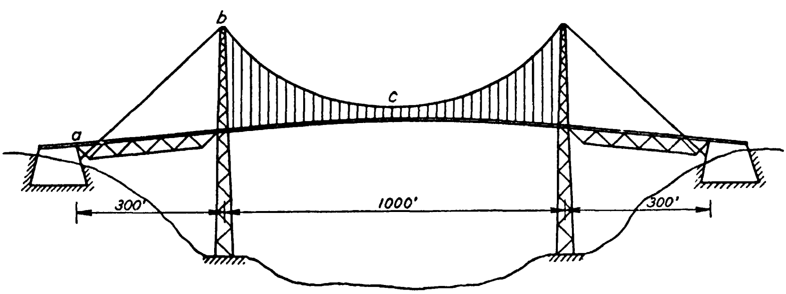
128. Redesign the welded bar of *DP*30 using a total stress of 26,000 lb. caused by wind. Detail the joint upon the assumption that the vertical members are 6 × 6 × $\frac{3}{8}$-in. angles. Use *AISC* weld stresses from § 55.

129. Design a welded bar as a hanger to carry a dead and live load stress of 75,000 lb. Detail its connection to the end of a 20-in., 65.4-lb. standard I-beam whose reaction it provides. Do not use overhead welds. *AISC* spec.; § 55 and § 215.

130. Repeat Problem 129 by using two tension bars attached to the sides of the beam.

131. Design sag rods to be placed at the one third points between roof trusses for a large industrial shed. Follow the details shown in *DS*31. Trusses are spaced at 20-ft. centers; roof slope, 30°; width of building, 65 ft.; eave overhang, 2 ft. Weight of roof covering, 4 lb. per sq. ft.; weight of purlins, 4 lb. per sq. ft.; snow load, 15 lb. per sq. ft. of roof surface. As a drafting room problem, sketch an elevation and plan of the roof showing details for placing the sag rods. *AISC* spec.

132. Design a sag rod for the side of a mill building to support the girts at their midpoints. Column spacing, 16 ft.; height to eave, 30 ft.; weight of side-wall covering, including windows, 7 lb. per sq. ft.; weight of girts, 2 lb. per sq. ft. Use *AISC* spec. and illustrate all details clearly.



PROBLEM 135.

133. A balcony around a gymnasium floor is to be supported at its outer edge by tension rods or hangers connected to the roof trusses. If the balcony is 14 ft. wide, and the roof trusses are on 15-ft. centers, design the hangers for a dead load of 75 lb. per sq. ft. and a live load of 100 lb. per sq. ft. Sketch a detail showing how to frame the floor system of the balcony in order to support it in this manner. *AISC* spec.

Ans. 1 in. sq. or 1$\frac{1}{8}$ in. rd. upset rod.

134. Design a diagonal for a 350-ft. railway truss where the design stress is 1440 kips and the pin size is 10 in. Use 4 bars and design by the *AREA* spec. The length of the member is 45 ft., 3 in. Compute the bending stress caused by the weight of the bar. The slope of the diagonal is 50° with the horizontal.

135. Design a chain for the suspension bridge shown. There are two chains. The design stress in the unloaded backstay *ab* is constant and is equal to 4800 kips in each chain. The stress from the top of the tower at *b* to the center of the bridge at *c* decreases uniformly to 4000 kips. Design the chain for a working stress of 50,000 lb. per sq. in.,

and change the chain section every 100 ft. from *b* to *c*. Detail a section of the chain showing how the bars are placed on the pin to reduce the bending stresses to a minimum; 11½-in. pins are to be used. Each bar is about 20 ft. long, which corresponds to the panel length of the floor system.

STRUCTURAL SHAPES

103. Single Angle Tension Members Connected by One Leg. Whenever the force transmitted from a connection into a member is not in line with the center of gravity of the member, this eccentric force produces bending in the member. As far as possible, it is wise to design the connection to reduce this bending to a minimum. The use of members which are symmetrical about one or both axes is therefore desirable. Clearly, any single angle connected by one leg must resist such an eccentric force. Single angle tension members are widely used for light diagonal bracing.

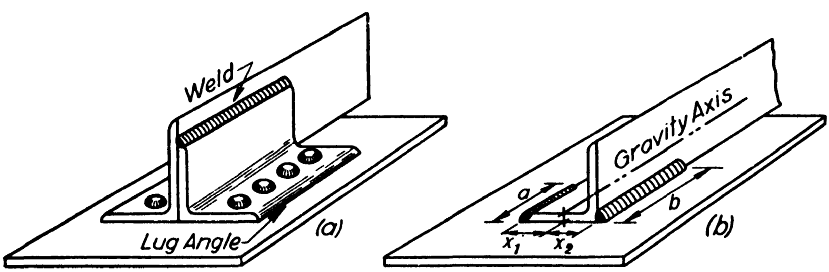


FIG. 94. SINGLE ANGLE CONNECTIONS.

The common specification is that the effective area of a single angle in tension shall be taken as *the net area of the connected leg plus one half the area of the unconnected leg.* The purpose of this specification is to allow empirically for the effect of the eccentric force. A lug angle connection, as shown in Fig. 94(*a*), is not to be considered as producing a connection to both legs, but it may be considered to transfer stress if it is welded to the main angle. It is far less effective for stress transfer if it is riveted to the main angle, because of slip between the two. Nevertheless, a lug angle is always useful in resisting moment of eccentricity.

Essentially the same specification applies to a single angle welded member as to a single angle riveted member. For the welded angle the gross area of the connected leg and one half of the gross area of the unconnected leg is effective. By use of a welded end connection, it is possible to make the connection resistance line up with the center of gravity of the angle in one plane (see § 57) although there will still be eccentricity in the perpendicular plane. The proper detail is shown in Fig. 94(*b*). Of

DP33. *Design a single angle web member of a light roof truss. Form a proper connection to a 5/16″ gusset with 3/4″ rivets. AISC spec.*

Angle Size:

Design stress = 15,600# caused by D.L. + L.L.

Net effective area = 15,600 ÷ 20,000 = 0.78□″.

Minimum angle. A 2½ × 2½ × 5/16″ angle is the minimum size.

Area furnished. Reduce area 25% to allow for unconnected leg and deduct for one 7/8″ rivet hole.

$$A = (1.47 \times 0.75) - (0.875 \times 0.312) = 0.83\square''.$$

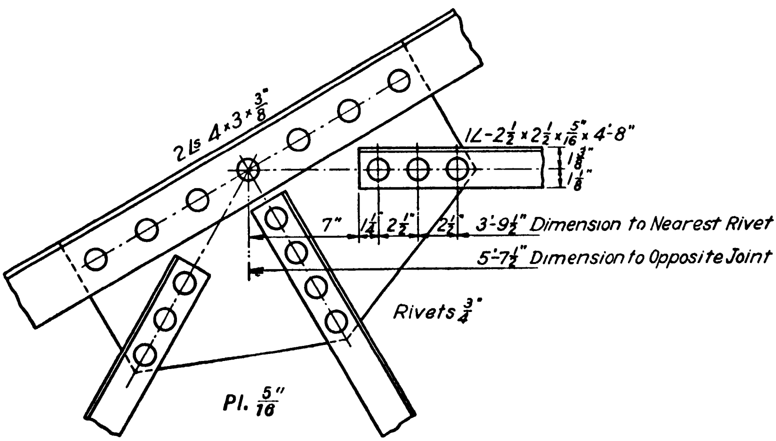
Rivets:

Rivet value in single shear = 0.44 × 15,000 = 6600#.

" " " bearing = 0.312 × 0.75 × 32,000 = 7500#.

Number of rivets = (20,000 × 0.83) ÷ 6600 = 2.5; use 3 rivets.

Remarks: *For light roof trusses this angle would ordinarily meet the strength requirement of most web members. However, it is common practice to use double angles to maintain symmetry.*



Detail: *The detail shown illustrates the proper way to dimension a member of a riveted joint. Information given on the angle controls its length and gage. Dimension lines show partial dimensions to locate the exact position of each rivet and the overall dimension for checking.*

course, the combined length of the two welds a and b must be sufficient to develop the effective area of the member. Also, the moment of the force resisted by the weld a, about the gravity axis of the angle, must be the same as the moment of the force resisted by the weld b. It follows that the lengths of the welds a and b must be directly proportional to the distances x_2 and x_1 or $a \div b = x_2 \div x_1$. From this we obtain a convenient working formula, $\frac{a}{a+b} = \frac{x_2}{x_1 + x_2}$.

104. Examples of Angles Connected by One Leg, *DP*33 and *DP*34. The riveted single angle tension member of the example *DP*33 is typical for a very light building truss. Trusses of standard weight usually have double angles of minimum section, even though they may not be needed for stress. Note that the end connection is designed to develop *the full value of the member* rather than its design stress. The resultant connection, 3 rivets, is the minimum connection usually accepted. A 2-rivet connection is regularly used for secondary members and by *AISC* specifications for light roof trusses.

The welded tension diagonal of the example *DP*34 illustrates a typical arrangement of members for a fairly heavy welded roof truss. A split beam section makes an excellent chord member for such a truss in that the 9-in. web furnishes an adequate depth for attachment of the web members. The use of this section is limited to trusses for which the chord areas must be at least 5 sq. in. so that a 12-in. I-beam may be split for the chords. The welded detail illustrates how the vertical and the diagonal are commonly welded to each other and to the chord by a single butt weld and by additional fillet welds.

Built-up Tension Members

105. Design of Riveted Tension Members other than Single Angles. The tension chords of riveted roof trusses and of highway bridge trusses often are formed of two or four angles, or, occasionally, of two channels. Roof trusses have single gusset plates, and the main tension members are usually two angles placed back to back on opposite sides of the gusset plate. Bridge trusses have double gussets. Light lower chord members may be double angles placed outside of the gusset plates. A joint detail at a lower chord panel point where two angles are used is shown in Fig. 96. The diagonals for most small highway bridge trusses are composed either of two angles with legs turned in, riveted inside of the gusset plates, or of a wide flange beam section. The vertical members are small plate-girders or beam sections riveted between the gusset plates. In addition to the vertical gusset plates, there is a horizontal plate riveted to the bottom of the lower chord angles at each joint. This plate serves the purpose of a connection plate for the lower laterals, and, if the lower chord is not continuous, it may also be a splice plate for the outstanding legs of the lower chord angles. The vertical legs of these angles are then connected to or spliced by the gusset plates. The chord splice may be placed at a joint unless the number of rivets required to splice the angles of the

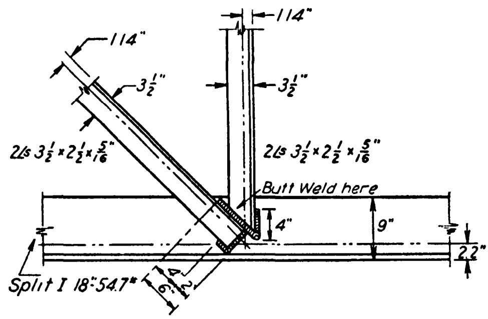
DP34. *Design a welded double angle tension member for a diagonal of a light building truss (Pratt type) where the chords are split I-beams (18″-54.7#). AISC spec.*

Size of Angles:

Maximum stress for the diagonal = 70,000#.

Gross area required = 70,000 ÷ 20,000 = 3.5□″ or 1.75□″ per ∠.

Angle size. A 3½ × 2½ × 5⁄16″ angle has a gross area of 1.78□″. The angles can be connected on the two sides of the I-beam web of the chord so that the entire gross area is considered effective.



Connection:

Strength of the connection. 1.78 × 20,000 = 35,600# per angle.

Value of weld. A 5⁄16″ fillet of a special shielded arc weld will be used which has a value of 1250# per ⅛″ of fillet leg.

Length of weld = 35,600 ÷ (1250 × 2.5) = 11.4″.

Division of weld. To shorten the side welds, a 3½″ weld can be placed across the end of the angle, leaving 7.9″ along the two sides. Then, by equations (11) and (12) from § 57

$$a = \frac{Lx}{c} - \frac{c}{2} = \frac{11.4 \times 1.14}{3.5} - \frac{3.5}{2} = 2.0''.$$

$$b = 7.9 - 2.0 = 5.9''; \text{ use } 6'' \text{ as shown.}$$

L is the total length of the weld or 11.4″.

c is the width of the angle leg or 3.5″.

x is the distance of the gravity axis from the back of the ∠.

Remarks: *The detail illustrates how the weld is to be placed on the diagonal. The use of a butt weld between the vertical and the diagonal can be avoided by separating the members sufficiently to allow space for two fillet welds.*

chord is so great that excessively large gussets are required. This fact, however, has led specification writers to recommend that it is better to splice the chord between panel points. *Severe gusset-plate stresses* are also avoided thereby. Since both legs of the chord angles are fully connected to stiff plates which tend to prevent the angles from bending in either direction, even though the pull is slightly eccentric, the full net area of the chord angles may be taken as effective area. (Spec. 85.)

All tension members that are composed of two or more separated pieces must have the parts connected by stay plates at intervals not exceeding 3 ft. Stay plates serve to develop a reasonably uniform distribution of stress between separate parts of the member and to prevent bending or vibration of the individual parts. (Spec. 103.) A stay plate holding 3 rivets as shown in Fig. 96 is the minimum allowable size. The lateral plates act as end stay plates for the lower chord, but all diagonals must have end stay plates 1½ times as long as the distance between end rivets or more than twice as long as the intermediate stay plate shown on the bottom chord in Fig. 96. The thickness of stay plates must not be less than $\frac{1}{50}$ of the distance between the connecting lines of the rivets. (Spec. 103.)

106. Chord Member Design. Example *DP*35. This design is worked out for a highway bridge chord member that is spliced at the joint. Hence, the gusset and the lateral plate serve as *splice plates*. Note that the slenderness ratio (L/r) is checked, which is important even for a tension member. The r-value used is taken about the horizontal axis because of vertical sagging and since stay plates shorten the free length in a horizontal direction.

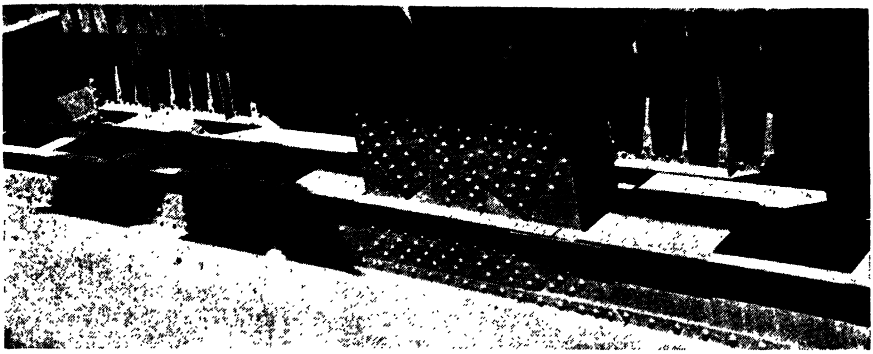
The net value of each leg must be determined in order to design the end connection. The calculation assumes net values to be proportional to net areas when a single rivet hole is deducted from each leg. In order to make this true, the first rivet spacing must be large enough so that the computed deduction by the *AASHO* formula ($s^2/4gh$) will be no greater than 2 rivet holes; the actual value obtained by computation is a deduction of 1.88 holes for a rivet spacing of 2½ in. Other rivet spaces will be set at 2 in. to shorten the gusset plate.

PROBLEMS

136. Redesign the tension member of *DP*33 for use in a welded structure. Design and detail the connection. Change the top chord to one half of an I-beam section equal to its present area. *AISC* and *AWS* spec. and highest *AWS* allowable stresses.

137. Redesign the welded tension member of *DP*34 for use in a riveted structure. Change the lower chord to two angles of equivalent section and introduce a $\frac{5}{16}$-in. gusset plate between them.

138. A single angle member is to act as a hanger to support a balcony of a small theatre. The hangers are spaced 10 ft. apart to carry the outer ends of channel floor beams which frame into the walls at their other ends. The floor-beam channels are 15-in., 45-lb. sections. The width of the balcony is 18 ft. Design the hanger and a riveted connection to the channel web if the D.L. is 75 lb. per sq. ft. and the L.L. is 100 lb. per sq. ft. of floor area. Detail the connection to reduce eccentricity. Use *AISC* spec.



Courtesy C. M. St. P. & P. R.R. Co.

FIG. 95. TENSION CHORD AND JOINT OF A RAILWAY BRIDGE TRUSS.

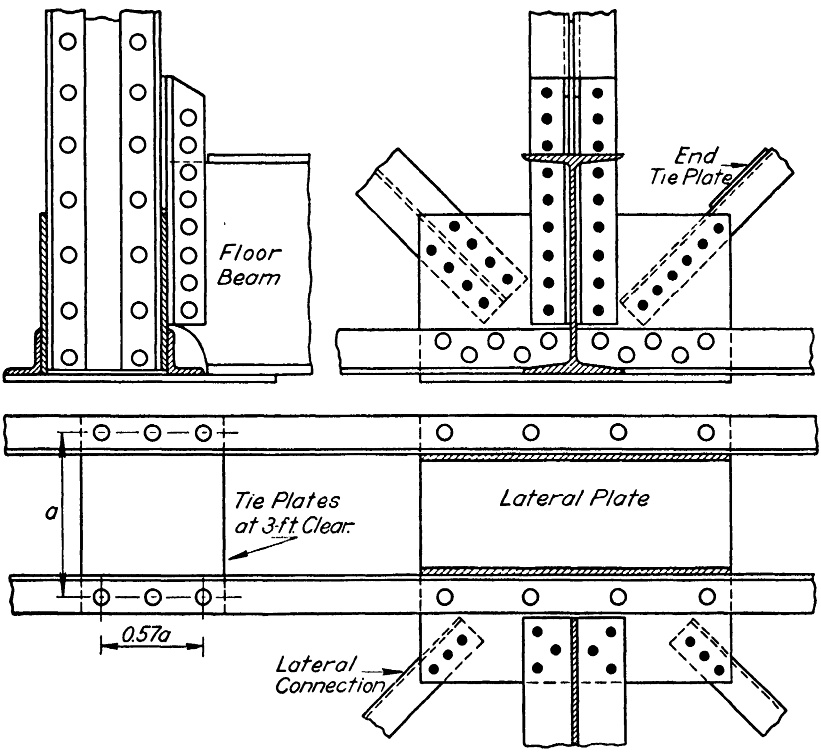
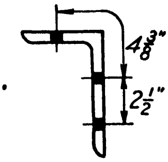


FIG. 96. DETAIL AT A LOWER CHORD JOINT OF A HIGHWAY BRIDGE TRUSS.

DP35. *Design a lower chord member for a highway bridge truss for a design stress from D.L. and L.L. of 103,000 lb. There is a splice at the joint. Panel lengths are 15′; gussets are ⅜″, and lateral plates are $\frac{5}{16}$″ thick. Use ¾″ rivets. AASHO spec.*

Member:



Net effective area required = 103,000 ÷ 18,000 = 5.73□″.
Gross area of 2 ∠s, 6 × 4 × ⅜″ = 7.22□″.
Net area = 7.22 − (4 × 0.875 × 0.375) = 5.91□″. (Deduction for 2 holes per ∠).
L/r = 15 × 12 ÷ 1.93 = 93.2. (Limited to 200, Spec. 84.)

Connection:

Shop rivet values. Single shear = 5970#; bearing on ⅜″ = 7580#.

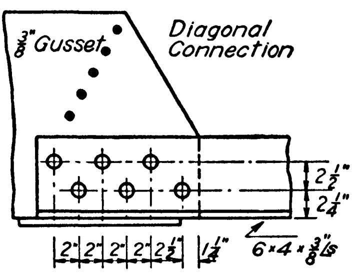
Field rivet values. Single shear = 4850#; bearing on $\frac{5}{16}$″ = 5280#.

Shop rivets through vertical legs = [(6/10 × 7.22) − 2 × ⅞ × ⅜] × 18,000 ÷ 5970 = 11.0.

Field rivets through horizontal legs = [(4/10 × 7.22) − 2 × ⅞ × ⅜] × 18,000 ÷ 4850 = 8.3.

Rivet spacing for deduction of 2 rivets (by $s^2/4gh$) is 2½″. Use successive values of g of 2½″ and 4⅜″; s = 2.5″, h = 0.87″ as follows: Spec. 106.

$$\text{Deduction} = 3 - \left(\frac{2.5^2}{4 \times 2.5 \times 0.87}\right) - \left(\frac{2.5^2}{4 \times 4.37 \times 0.87}\right) = 1.88 \text{ holes.}$$

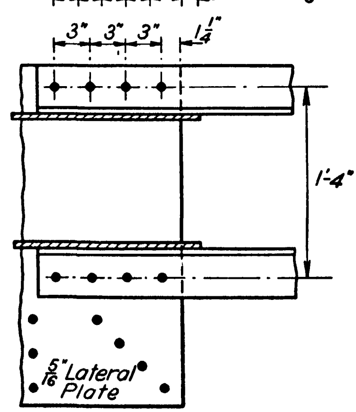


Stay Plate:

Distance between gage lines = 16″. (Spec. 103.)

Length of plate = (0.57 × 16) + 2.5″ = 12″.

Thickness = 16/50 = $\frac{5}{16}$″.



Comment: *For practical detailing it will be found that 12 rivets in place of 11 are used through the vertical legs, while 8 are used through the horizontal legs. Rivet spacing is shown as 2½ in. at the end of the gusset, but other rivet spaces are reduced to 2 in. to shorten the gusset.*

139. Redesign the hanger of Problem 138 if it is to have a welded connection. Design and sketch the connection detail. *AWS* spec.

140. Redesign the lower chord member of *DP*35 by making use of 5 × 3 or 5 × 3½ in. angles.

141. Design the lower chord of a Warren low truss bridge at a point where the stress is 170,000 lb. Use 4 angles 5 × 3 or 5 × 3½ in. and splice the member between joints to another member composed of angles of the same size but of 1⁄16-in. greater thickness. Turn the angle legs in any practical manner. Use *AASHO* spec.

142. Design the lower chord of a Fink roof truss for a stress of 62,000 lb. using *AISC* spec. This member is composed of 2 angles placed on opposite sides of a single 5⁄16-in. gusset plate. Design the connection of the member to the end gusset plate. Detail the placing of rivets so that only a single rivet hole need be deducted from each angle.

107. Choice of Cross-Section. As indicated by the variety of sections discussed in this chapter, the designer has considerable latitude in the choice of sections for tension members. The type of fabrication has much to do with a proper choice. Naturally, tension members composed of angles, channels, or beam sections are normally used in riveted structures, while plates or angles or channels are likely to be chosen for a welded structure. The structural eye bar is economical for long-span bridge structures and for heavy building trusses, but its use is probably diminishing despite the advantage it offers of economy through the choice of *alloy steels* of high strength. The ordinary tension rod has come into a certain amount of disfavor, probably because it has been seen too frequently in cheap structures designed improperly. Actually, it is a useful tension member. For greatest effectiveness, the tension rod should be *upset* and should be designed to operate under *initial tension.* Its end nuts are adequate for introducing such tension, although a turnbuckle is a convenience in maintenance. Certainly, the weakness in the design of tension rods has usually been their *end connections.* The use of bent plates and other weak details for end connections has frequently been the cause of failure. A pin hole for an end clevis connection, punched too close to the edge of a connection plate, has been another cause of failure. Tension rods should not be blamed for such disasters.

CHAPTER 7

COMPRESSION MEMBERS

108. Design of Columns and Compression Members. The design of a compression member differs from the design of a tension member in two major characteristics: (1) the full gross area of a compression member is considered to be capable of taking stress (rivets are assumed to fill their holes completely), (2) the allowable stress on a compression member must be reduced below that of a tension member to allow a margin of safety against buckling. The need for this reduction of stress is clear if we realize that a long column centrally loaded will buckle and collapse under a load producing an *average* direct stress considerably below the elastic limit.

COLUMN FORMULAS. Five types of column formulas are recommended for practical use.

(1) The *Straight-Line* formula (*AREA* before 1935 and widely used in building codes). Allowable $P/A = 15{,}000 - 50L/r$, but not more than 12,500 lb. per sq. in.

(2) The *Parabolic* formula (riveted end connections).

Allowable $P/A = 15{,}000 - \dfrac{1}{4}\dfrac{L^2}{r^2}$ (*AREA* and *AASHO*).

Allowable $P/A = 17{,}000 - 0.485\dfrac{L^2}{r^2}$ (*AISC* for $L/r < 120$).

(3) The *Rankine-Gordon* formula (*AISC* for special case of $L/r > 120$).

$$\text{Allowable } P/A = \frac{18{,}000}{1 + \dfrac{L^2}{18{,}000r^2}}.$$

(4) The *Secant* formula (*AREA* for $L/r > 140$ and riveted ends).

$$\text{Allowable } P/A = \frac{33{,}000/1.76}{1 + 0.25 \sec.\left(0.375\dfrac{L}{r}\sqrt{\dfrac{1.76P/A}{E}}\right)} \quad \text{(factor of safety of 1.76).}$$

(5) The *Euler* formula (riveted ends and L/r above accepted limits, i.e., > 200).

$$\text{Allowable } P/A = \frac{16E}{(L/r)^2} \div 2.2 \quad \text{(factor of safety of 2.2).}$$

Column Action

109. Generalization about Test Results. It is possible to derive theoretical column formulas. Unhappily, test results do not agree with derived formulas, so that, finally, corrective constants have to be intro-

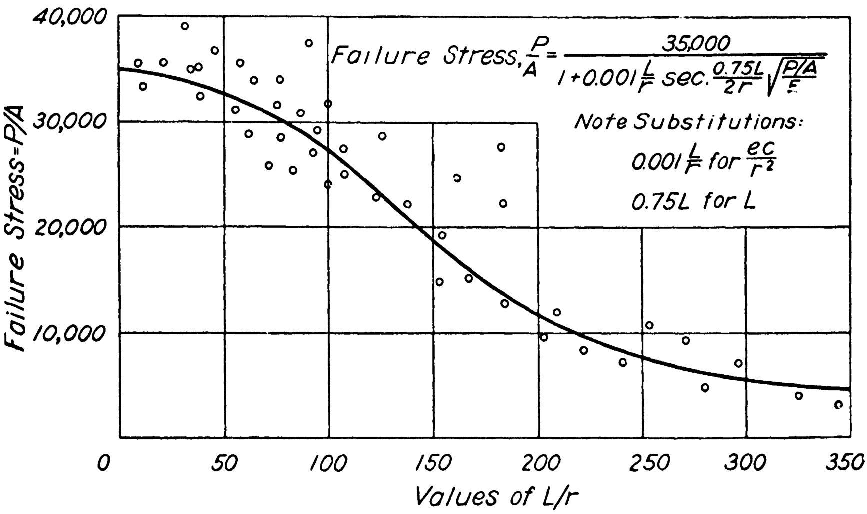


Fig. 97. Range of Test Results Compared with a Secant Curve for Built-up Steel Columns with Riveted Ends or Unlubricated Pins.

duced. Therefore, we will approach this subject by reviewing tests first and then by deriving a few useful expressions theoretically.

The plotting of column loads at failure in Fig. 97 indicates the general trend toward a reduction of strength with increase of slenderness, but, due to *non-homogeneity* or imperfections of materials, *inaccuracies* of manufacture, *differences of end restraint,* and *imperfect load application,* there is inevitably a considerable variation in test results obtained even in the same laboratory. Columns in actual structures unquestionably vary to an even greater extent. Hence, for a limited range of L/r (say from 40 to 120) either the straight line, the broken line, or the parabolic curve of Fig. 98 might represent the test results of Fig. 97 with about the same

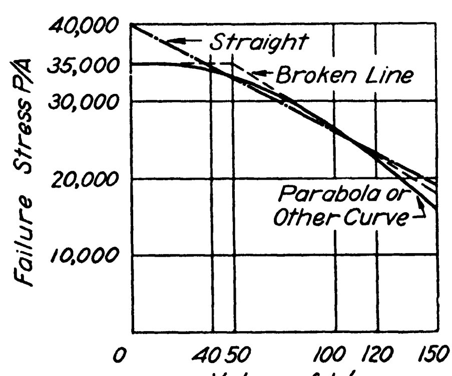


Fig. 98. Comparison of Column-Load Curves.

effectiveness. Thus we are brought to the realization that the several column formulas of § 108 may represent no greater differences of opinion as to column strengths than is inherent in the spread of test results. For the two decades before 1920, the straight-line formula was commonly recommended in standard specifications. Then the old Rankine-Gordon formula was resurrected and became popular. At the present time there is a marked tendency to revert to the parabolic formula, but, like the mode in ladies' hats, the cycle will probably repeat itself in a generation or so. The fact is that no one of these formulas does a sufficiently good job of representing all test results to justify its use to the exclusion of all others.

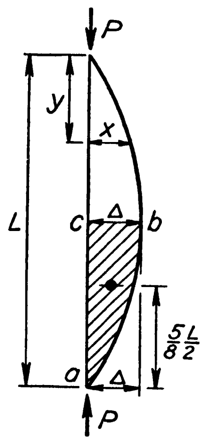


FIG. 99.
EULER DEFLECTION

110. The Euler Formula for Long Slender Columns. The only inescapable theoretical conclusion in column studies is that *long slender columns* with perfectly round ends, when centrally loaded, buckle and fail as described by the Euler formula. In Fig. 99 we have a deflected column with round ends supporting the load P. Evidently, the bending moment is Px, and the deflection curve has the same shape as the moment diagram. The stress just before failure is almost entirely flexural stress if we restrict our study to long slender columns or splines. Hence, the deflection Δ can be computed as P/EI times the statical moment of the deflection area abc (cross-hatched) about the point a.* It is reasonably accurate to take the deflection curve and therefore the moment diagram as a parabola. Thus, we obtain

$$\Delta = \frac{P\Delta}{EI}\left(\frac{2}{3}\,\frac{L}{2}\right)\frac{5}{8}\,\frac{L}{2} = \frac{5}{48}\,\frac{PL^2}{EI}\,\Delta. \tag{6}$$

Hence, we obtain for the breaking load

$$P = \frac{9.6EI}{L^2}.$$

For the breaking stress, we write

$$\frac{P}{A} = \frac{9.6E}{(L/r)^2} = \frac{\pi^2 E}{(L/r)^2} \text{ (nearly).} \tag{7}$$

The final form of this expression is the theoretical Euler formula.

Practical Revisions of Euler's Formula. First, the Euler equation should be divided by a factor of safety of about 2 because the load P is

* See *Theory of Modern Steel Structures*, Vol. 2, p. 43, Proposition (2). The deflection at A from a tangent drawn at B is equal to the statical moment of the M/EI area between A and B taken about the point A.

clearly the buckling load that produces the severe deflection and heavy flexure occurring just before failure. Then, the deflected curve used is theoretically correct only for a perfectly round-end column. Both T. H. Johnson and J. B. Johnson, early experimenters in this field, agreed that the breaking loads for slender columns with practical end connections were very different from those given by the original Euler formula. Their test results were expressed approximately by the following formulas.

$$\text{(8)} \qquad \text{Breaking stress } \frac{P}{A} = \frac{5}{3}\frac{\pi^2 E}{(L/r)^2} = \frac{16E}{(L/r)^2} \text{ (nearly) for typical pin connected ends,}$$

and

$$\text{(9)} \qquad \text{Breaking stress } \frac{P}{A} = \frac{5}{2}\frac{\pi^2 E}{(L/r)^2} = \frac{25E}{(L/r)^2} \text{ (nearly) for flat or nearly fixed ends.}$$

The first case is accepted to represent common end conditions either due to the use of ordinary unlubricated pins or of riveted or welded ends that are slightly restrained but *far from fixed.* The revised Euler formulas, equations (8) or (9), are applicable, of course, to long slender columns only. This will be evident since these formulas give an infinite breaking load when $L/r = 0$. Actually, the Euler formula only applies to columns above the range of slenderness permitted by ordinary design specifications. However, such slender compression members are not uncommon in electric sign supports, industrial stairway towers, etc.

111. The Straight-Line Formula for Short Columns. It is accepted that the reasonable ultimate value of a short column must not be considered to be greater than its elastic limit. However, tests show occasional perfect columns (short and almost theoretically straight) that fail above this stress which the *ASTM* sets at a minimum of 33,000 lb. per sq. in. for structural carbon steel. Tests do not show any appreciable reduction of strength with increase of slenderness until L/r exceeds about 50. If, therefore, 15,000 is to be taken as the allowable stress on stocky columns ($L/r < 50$) we might set up a linear relationship for use where L/r exceeds 50. The 1920 *AREA* formula was of this type.

$$\text{(10)} \qquad \text{Allowable } \frac{P}{A} = 15{,}000 - 50\frac{L}{r}.$$

There is no theoretical justification for this formula. Its use was established largely because *the simplest way to reduce one number is to subtract another from it* and the simplest reduction factor, based upon slenderness, is one that depends upon the first power of L/r. T. H. Johnson formally proposed this type of formula in 1886,* although it had probably been used earlier.

* Transactions, *ASCE* for 1886, p. 517.

112. The Parabolic Formula for Short Columns. The next step in elaboration would be to change the power of L/r in equation (10). J. B. Johnson advocated the adoption of a parabolic formula, that is, one of the type

$$\text{Allowable } \frac{P}{A} = 15{,}000 - \frac{1}{4}\left(\frac{L}{r}\right)^2. \tag{11}$$

This parabolic formula has no theoretical justification, but it does have one rather intangible connection with column theory. By varying the coefficient of $(L/r)^2$ we can make the parabola tangent to the Euler column curve or to some revised Euler curve such as equation (8) or (9).* At the point of tangency, Johnson showed the y coordinate to be one half of the maximum unit stress f reached by the parabola at $L/r = 0$. The corresponding x coordinate in terms of L/r is $2k/f$ where k is π^2, 16, or 25 to be taken from the corresponding Euler equation (7), (8) or (9). The importance of locating this point of tangency is reduced by the fact that the parabola may become tangent to the Euler curve below the minimum value of L/r (about 200) for which the Euler formula is considered useful. To combat this difficulty, the *AISC* code lists a parabolic formula to cover the range from 0 to 120 for L/r and then recommends a Rankine-Gordon formula from 120 to 200 for L/r. Beyond 200 a revised Euler formula such as (8) could be used with a factor of safety of about 2.2. Of course, slender columns where $L/r > 200$ have a very limited use.

113. The Rankine-Gordon† Formula. Evidently the second way to reduce a working stress is *to divide it by a number larger than unity.* The Rankine type of formula follows this procedure. For example, the *AISC* formula for L/r above 120 is

$$\text{Allowable } \frac{P}{A} = \frac{18{,}000}{1 + \dfrac{(L/r)^2}{18{,}000}}. \tag{12}$$

This form of the Rankine-Gordon formula is rational. If we write the expression for fiber stress in an eccentrically loaded section as

$$f = \frac{P}{A} + \frac{Mc}{I} = \frac{P}{A}\left(1 + \frac{ec}{r^2}\right), \tag{13}$$

* See *Modern Framed Structures,* Johnson, Bryan and Turneaure, chapter on compression members.

† Lewis Gordon, an English engineer, proposed this general type of formula which had also been suggested even earlier by Thomas Treadgold, another Englishman. However, the greatest of all early engineering teachers, William J. Rankine (1820–1872), professor at Glasgow, Scotland, proposed and used the formula in its present form and it commonly carries his name.

we can obtain the expression

$$(14)\qquad \frac{P}{A} = \frac{f}{1 + \frac{ec}{r^2}}.$$

This rational formula is of the same form as the Rankine-Gordon formula. In fact, it is only necessary to assume that the depth of the column $2c$ and the eccentricity of load or side deflection e are each proportional to the length L, ($ec = kL^2$) to obtain the generalized Rankine-Gordon formula

$$(15)\qquad \frac{P}{A} = \frac{f}{1 + k\left(\frac{L}{r}\right)^2}.$$

The constant k is determined to make the formula agree with tests.*

114. The Secant Formula for All Column Lengths. If the Euler formula is rederived for a column with an initial eccentricity of loading, e, which may be merely an initial crookedness (Fig. 100), the result is as follows:

$$(18)\qquad \text{Breaking Stress } \frac{P}{A} = \frac{\text{Elastic Limit}}{1 + \frac{ec}{r^2}\sec\frac{L}{2r}\sqrt{\frac{P}{A}\frac{1}{E}}}.$$

According to the *AREA* specifications this formula becomes

$$(19)\qquad \text{Allowable } \frac{P}{A} = \frac{33{,}000/1.76}{1 + 0.25\sec\frac{0.75L}{2r}\sqrt{\frac{1.76}{E}\frac{P}{A}}}.$$

This formula is intended to control columns with riveted ends above $L/r = 140$. (Spec. 166.)

* The constant k may also be determined rationally by equating the Euler and the Rankine-Gordon formulas for *some large value of* L/r. To obtain the value of k in equation (15), assume that the Rankine-Gordon formula is expected to give the same result as the revised Euler formula (8). (Use a factor of safety of 2.2 to reduce an elastic limit of 37,000 to a working stress of 17,000 for $L/r = 0$, *AISC* specifications.)

$$(16)\qquad \frac{16E}{2.2\left(\frac{L}{r}\right)^2} = \frac{18{,}000}{1 + k\left(\frac{L}{r}\right)^2}.$$

Neglect the number unity which is negligibly small as compared to $k\left(\frac{L}{r}\right)^2$ when L/r is large, cancel $\left(\frac{L}{r}\right)^2$ from each side of the equation, and solve for k. Thus we obtain

$$(17)\qquad k = \frac{18{,}000 \times 2.2}{16 \times 30{,}000{,}000} = \frac{1}{12{,}000}.$$

This is not very close to the value of $\frac{1}{18{,}000}$ used in equation (12). Therefore, we conclude that although the Rankine-Gordon formula is of a rational form, its actual constants must be chosen to agree with test results.

Choice of Constants. Note that the factor ec/r^2, which is unknown for a centrally loaded column, has been given a minimum value of 0.25 to care for eccentricity caused by slight initial crookedness. The length L in the denominator has been reduced to $0.75L$ to account for end restraint of the riveted end connection. The factor of safety, 1.76, is introduced under the radical in order to permit the load P to be consistently the *allowable load* instead of the *breaking load*, as it was defined in equation (18). Another possible value* for ec/r^2 is $0.001\ L/r$, which is used to draw the curve of Fig. 97. Since r varies as c, ec/r^2 can vary as L/r if crookedness is proportional to length. However, this assumption neglects the fact that *crookedness is only initial eccentricity* while final eccentricity is properly a deflection.

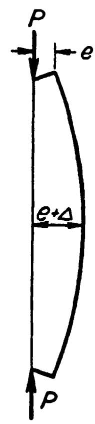


FIG. 100. ECCENTRIC LOADING.

The secant formula can be made to fit test results for all values of L/r with good accuracy by proper choice of constants. (See Fig. 97 where a revised secant curve is used.) On the other hand, the fact that the working stress P/A, which we are trying to determine, also appears in the secant formula makes it so awkward to use that it will probably never be widely accepted. For a known eccentricity e, which produces both direct stress and flexure, it becomes the only column formula that logically attempts to express a working stress different from the one used for a centrally loaded column. The *AREA* specifications permit us to use equation (19) for all values of L/r when the eccentricity e is known; the factor 0.25 then is revised to become $(ec/r^2) + 0.25$. (Spec. 166.)

115. Choice of a Column Formula. Usually the column formula is given by the specifications, but instances arise when the engineer must choose for himself or even write specifications, to govern a particular design. He should therefore check these observations:

1. If the straight-line and Rankine-Gordon formulas are chosen to represent correctly test results from $L/r = 50$ to $L/r = 120$, they will give excessive working stresses below $L/r = 50$. Usually, the formula is not used for low values of L/r but, instead, a constant working stress is specified for stocky columns. For example, the 1920 *AREA* formula, $15{,}000\text{-}50L/r$, was not used below $L/r = 50$; the working stress was fixed at 12,500 for L/r values from 0 to 50.

2. The straight-line formula gives values of working stress that are too low for slender columns where L/r exceeds about 120. The Rankine-Gordon formula may be used for values of L/r up to 200 or even more.

* See *Modern Framed Structures*, Johnson, Bryan, and Turneaure, Vol. III, chapter on compression members.

However, for such slender columns this formula tends to give working stresses that are too high.

3. The parabolic formula may be made to represent working stresses for the range of L/r between 0 and 140 quite satisfactorily. Above this range, the parabolic formula is seriously in error on the conservative side.

4. The secant formula may be made to cover the entire range of L/r values with reasonable accuracy. It is cumbersome to use because the working stress to be computed also occurs in the formula, and it is extremely sensitive because of the great variation of the secant for angles

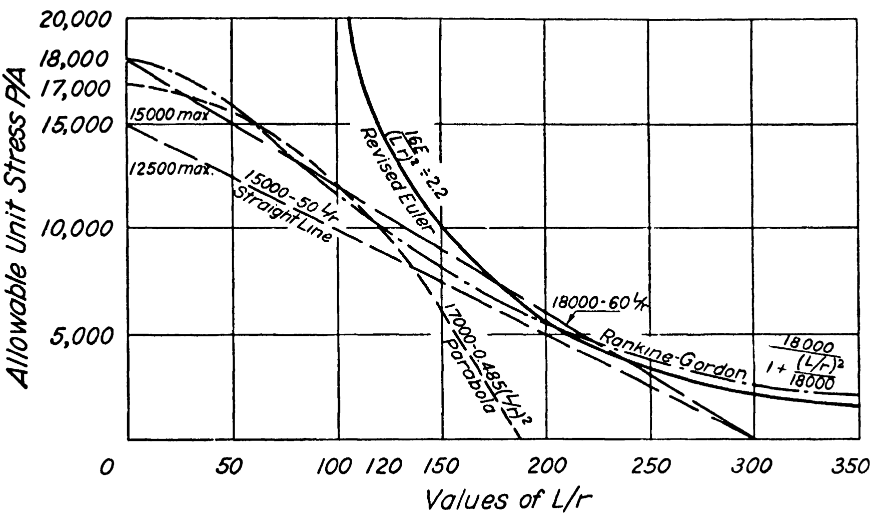


FIG. 101. COMPARISON OF COLUMN FORMULAS.

near $\pi/2$. It is the only common formula, however, that offers a means of introducing a known eccentricity into the calculation of the working stress. Since the working stress for pure flexure is greater than the working stress for column action, it is logical that the working stress should be varied with changes in the eccentricity of the load.

5. The Euler formula, which is the special case of the secant formula for zero eccentricity, should be used in studying very slender columns under centric loading (certainly above $L/r = 200$). Remember that a factor of safety of about 2 should be introduced and remember to choose a proper coefficient to represent the best possible assumption about end conditions.

6. The straight-line formula and the parabolic formula are not rational. If these formulas are solved for the fiber stress f, the result is $P/A + kL/r$ or $P/A + k(L/r)^2$. The second term in each case represents the flexural

stress which should depend upon the load P. For the Rankine-Gordon formula, the fiber stress is $P/A\ [1 + k\,(L/r)^2]$ which is of more rational form. The secant formula is rational and the fiber stress is given by

$$f = \frac{P}{A}\left(1 + \frac{ec}{r^2}\sec\frac{L}{2r}\sqrt{\frac{P}{A}\frac{1}{E}}\right). \tag{20}$$

Rationality need not be considered very important as long as a formula is set up to serve a limited function. It becomes important when we try to extend the scope of usefulness of a formula.

7. Plotted column formulas as shown by Fig. 101 may be compared with the range of test results from Fig. 97 to determine the range of slenderness ratio for which each column formula is applicable. Any formula becomes a good one that represents test results accurately for the range of L/r values under consideration.

Column Design

116. Instructions to be Followed in Designing Compression Members.

1. Guess at the allowable unit stress, making it not more than the practical or stated upper limit for the column formula specified.

2. Divide the load by the assumed allowable stress to find the approximate gross area required.

3. Select a column cross-section that will provide at least this required gross area, and compute the minimum radius of gyration for this section.

4. Calculate the maximum allowable unit stress for this section from the column formula using the least radius of gyration (minimum value of L/r) of the section selected.

5. If the calculated allowable stress does not exceed the actual average unit stress (total load divided by gross area of section selected) by more than 2 or 3 per cent, the section represents a good choice.

6. If the calculated allowable stress exceeds the actual unit stress by more than 5 per cent, the section is oversafe and uneconomical, and its area can usually be reduced.

7. If the calculated allowable stress is smaller than the actual unit stress, the section is overstressed, and its area should be increased. It may be possible to make the column safe merely by spreading its parts further apart so that its least radius of gyration will be increased. This change increases the allowable stress and therefore the allowable load without increasing the area of the section.

The above instructions are applicable only to the design of a compression member for which the applied load is centric, that is, when there is no bending moment applied to the column.

117. Rolled Column Selection. EXAMPLES *DP*36*a*, *DP*36*b*. Columns of rolled beam sections cost less per pound than built-up columns of angles and channels. They are therefore preferred unless a different shape of cross-section is needed for convenient attachment of floor joists and girders. The illustrative problems *DP*36*a* and *DP*36*b*

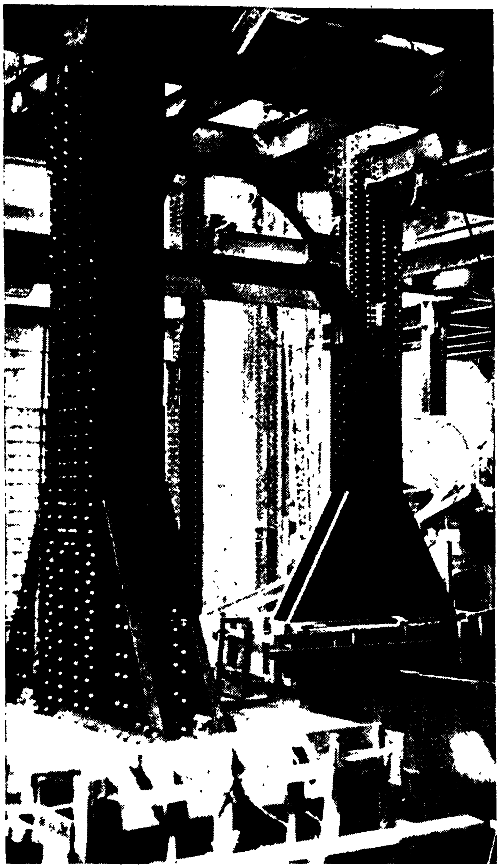


FIG. 102. COLUMN BASES, EMPIRE STATE BUILDING.

show the selection of a typical building column for a story height of 20 ft. and for a column load of 300,000 lb. The section selected is a 12*WF*79 beam section for which the radii of gyration are 5.34 and 3.05. Since the least radius of gyration controls the allowable stress, a built-up section with equal radii about the two principal axes might weigh less. However, the fabrication cost would probably overbalance this possible saving.

Design of the Column Base. A column base will be designed to permit a bearing stress of 500 lb. per sq. in. on the concrete footing.

Area of base plate required = 300,000 ÷ 500 = 600 sq. in. Use a base size 26 × 26 in. which produces a bearing pressure of 444 lb. per sq. in.

Bending moment in base plate. The base plate extends as a cantilever beam beyond the face of the column. The smaller dimension of the column is 12.08 in. The projection

DP36a. *Select a rolled column section to support a balcony load in an industrial building. The column carries no wind stress. Column length is 20′-0″; total dead load and live load is 300,000#. AISC spec.*

Column formula. $17{,}000 - 0.485\left(\frac{L}{r}\right)^2$.

Assume allowable unit stress at 13,000#/□″.

Approximate area required = 300,000 ÷ 13,000 = 23.1□″.

Tentative Section:

A 12WF79 section has an area of 23.2□″.

Slenderness ratio. $L/r = 20 \times 12 \div 3.05 = 78.8$.

Allowable unit stress by column formula $= 17{,}000 - 0.485 \times 78.8^2 = 14{,}000$#/□″.

Revised Section:

The area can be reduced about 7% if L/r is not changed appreciably. Try the 12WF72 section which has practically the same r-value as the 79# section.

Actual unit stress. $P/A = 300{,}000 \div 21.2 = 14{,}200$#/□″.

Remarks: *The 12WF72 section is overstressed by 200#/□″. Hence, we will choose the 12WF79 section since the overstress exceeded 1%, a commonly accepted standard.*

DP36b. *Repeat DP36a by use of a Rankine formula.*

Column formula. $\dfrac{18{,}000}{1 + \dfrac{(L/r)^2}{18{,}000}}$ *(not to exceed 15,000#/□″).*

Assume allowable unit stress at 13,000#/□″.

Approximate area required = 300,000 ÷ 13,000 = 23.1□″.

Tentative section. A 12WF79 section has an area of 23.2□″.

Slenderness ratio. $L/r = 20 \times 12 \div 3.05 = 78.8$.

Allowable unit stress $= \dfrac{18{,}000}{1 + \dfrac{78.8^2}{18{,}000}} = 13{,}300$#/□″.

Remarks: *The section is understressed about 2%, which is considered to be a satisfactory design.*

is (26 − 12.08) ÷ 2 = 7 in. approximately. The bending moment *per inch width* of plate is $\frac{444 \times 7^2}{2} = 10{,}900$ in-lb.

Section modulus required = 10,900 ÷ 20,000 = 0.55. Therefore, $d^2 = 6 \times 0.55$, or $d = 1.82$ in. Use a slab 26 × 26 × 2 in. made of rolled structural steel. A thickness of 1⅞ in. would be satisfactory if it could be obtained.

Detail of base. The base usually is riveted to the column in the field by use of a pair of clip angles to the column web. The face of the column is planed flat to produce a perfect bearing on the rolled slab. The clip-angle connections may be welded to the base plate in the shop. Angles 4 × 4 × ⅜ in. are adequate since their only purpose is to keep the column in its proper position. Two anchor bolts of ¾-in. diameter are used to locate the base plate accurately on the footing. (See Fig. 103.)

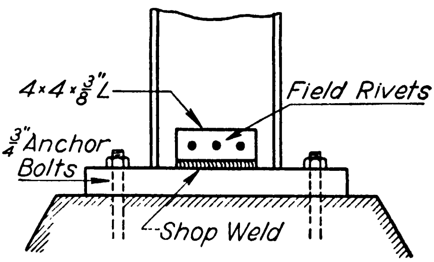


FIG. 103. COLUMN-BASE DETAIL.

REMARKS. If any possibility existed that the column might be struck by a moving crane load or by a moving truck, it would be necessary to use heavier clip angles and to anchor the base plate to the footing with larger anchor bolts. Otherwise, a reasonably light blow might knock the column loose from the footing and cause an unnecessary failure. Some designers prefer to use clip angles on both web and flange.

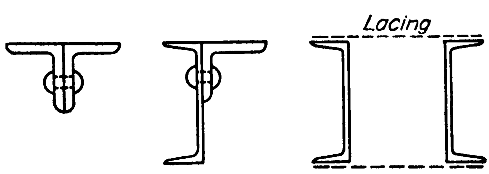


FIG. 104. SECTIONS FOR LIGHT STRUTS.

118. Struts and Light Compression Members. Stiff struts frequently are required in steel structures where the actual compressive stress to be carried is exceedingly small. For instance, the eave strut in a mill building may have to carry a total stress of only 2000 or 3000 lb. and still have to act as a horizontal column with an unsupported length of 16 or 18 ft. Any column which will meet the requirement that the maximum L/r must not exceed 200, will have several times the required strength. Where the span is relatively short, two angles placed back to back form a fair strut. A channel and angle placed back to back provide a section that may be used for longer spans. However, the most satisfactory strut, for important uses, is formed by two channels with flanges turned out and laced on both sides. These sections are illustrated in Fig. 104.

Design of a Mill Building Strut, *DP37*. Light eave struts and similar bracing members seldom carry enough stress to make an economical design possible. The strut of problem *DP*37 has a load of only 4000 lb. while the capacity of the minimum section is 32,400 lb. Since the member has to act as a beam of 20-ft. length to carry its own weight, which produces a fiber stress in flexure of about 2500 lb. per sq. in., the actual capacity of the member is considerably less than 32,400 lb. The design of a member to resist both direct stress and flexure will be discussed later, but it is evident that this member is amply strong and that a further analysis is unnecessary.

Lacing bars were selected to meet the requirement as to L/r value of 140 for single lacing (Spec. 50) but they were not checked to determine their capacity to resist a lateral shear. The requirement that they shall resist a lateral shear of 2 per cent of the direct stress would set up a compressive design stress of $(0.02 \times 4000) \div \sin 60° = 93$ lb. The unit stress in one bar would be $93 \div 2(0.44 \times 1.5) = 70$ lb. per sq. in. Evidently, this *specification stress* is insignificant for the lacing of light struts and would only influence the design of lacing bars for short columns that were heavily loaded.

It is also worth mentioning that the end tie plates selected do not meet the requirements for end tie plates from Spec. 47. They were chosen to meet the requirements for intermediate tie plates. The explanation is that this member is not in any sense a *main* compression member for which the end tie plates perform a very important function in distributing stresses uniformly. To meet the requirements for the end tie plate of a main compression member (Spec. 47) would overbalance the design. The end tie plate shown is adequate. This procedure is authorized for highway bridges by Spec. 103.

PROBLEMS

143. Select a rolled column section to carry a direct load of 500,000 lb. The length is 18 ft.-0 in. Use the *AISC* column formula. (Spec. 10.) Design a rolled base plate for this column, assuming an allowable bearing pressure of 600 lb. per sq. in. Detail the base connection.

144. A railroad trestle is built of rolled column sections with the proper bracing. The column length is 16 ft. The direct stress per column for D.L., L.L. and impact is 160,000 lb. Determine the required size by use of the *AREA* column formula. (Spec. 164.) Allow 600 lb. per sq. in. bearing pressure on the masonry and design a proper base plate using the *AREA* working stress for flexure of 18,000 lb. per sq. in. Detail a base connection using $4 \times 4 \times \frac{3}{8}$-in. clip angles to both flange and web.

145. Determine the allowable load on the following columns as controlled by the *AISC* specifications.

12-in. *WF*, 99 lb. per ft., length 18 ft.
14-in. *WF*, 158 lb. per ft., length 14 ft.
14-in. *WF*, 426 lb. per ft., length 16 ft.

146. Determine the allowable load on the columns of Problem 145 as controlled by the *AREA* specifications. (Pin ends, Spec. 164.)

147. Redesign the eave strut from *DS*37 by making use of a cross-section composed of a channel and angle placed back to back. Design the connection of the strut to the column. Suggestion — try a 5-in. channel at 6.7 lb. and a $4 \times 3 \times \frac{5}{16}$-in. angle with the 4-in. leg outstanding.

148. Design by *AISC* specifications a two-angle strut with angles placed back to back to act as stiff bracing between the lower chords of mill building trusses. The length is 15 ft. The stress to be carried may be considered negligible. Compute the flexural stress caused by the dead weight of the member.

DP37. *Design an eave strut for a large mill building where the width of bay (strut length) is 20′-0″. The design stress is 4000# caused by wind. AISC spec.*

Section: *Two channels separated by the 12″ width of the vertical column and riveted to the faces of the column will be used. Channels will be laced top and bottom.*

Minimum radius of gyration.

Max. $L/r = 200$; *hence* $r_{min.} = 20 \times 12 \div 200 = 1.20''$.

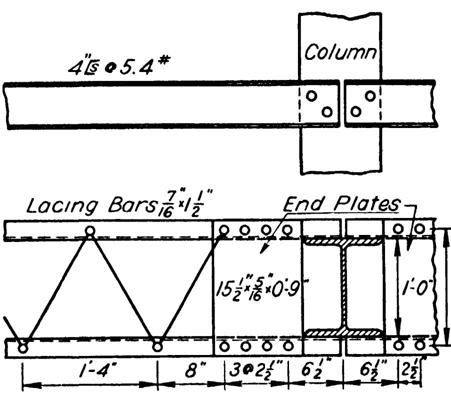
Trial section. The lightest 4″ channel has a min. r of 0.45 and a max. r of 1.56.

Value of Section: *Area* = 3.12□″. (4″[s @ 5.4#.)

$$\text{Allowable stress} = \frac{18{,}000}{1 + \dfrac{240^2}{18{,}000 \times 1.56^2}} = 7800\#/\square''.$$

Allowable load = 7800 × 3.12 = 24,300#.

The allowable wind load is 1.33 × 24,300 = 32,400#. (*Spec.* 7.)



Lacing Bars: *60° single lacing is adequate since L/r between rivets* = 16/0.45 = 36.

Length of bar between rivets = $\sqrt{14^2 + 8^2} = 16\frac{1}{8}''$.

$$\text{Min. thickness of bar} = \frac{1}{0.288}\left(\frac{16.12}{140}\right) = 0.40''. \quad (\text{Spec. } 50.)$$

Use a bar $1\frac{1}{2}'' \times \frac{7}{16}''$ *punched for* $\frac{1}{2}''$ *rivets.*

Tie plates. Select to fulfill Spec. 47 for intermediate tie plates.

End Connection: *Value of* $\frac{3}{4}''$ *rivets in bearing on channel web* = (0.75 × 0.18 × 32,000) 1.33 = 5730#.

Number of rivets to develop member = 32,400 ÷ 5730 = 5.7.

Since the actual stress is small, 4 rivets are used.

149. Design a vertical member of a highway bridge to carry a direct load of 100,000 lb. in compression. The length of the member is 22 ft.-6 in. Use two channels with flanges turned in, laced. Determine the required distance back to back of channels. Use the *AASHO* specifications for highway bridges.

150. Design a strut to act as sway bracing in a railway viaduct. Use two angles placed 3/8 in. back to back, of 22-ft. length. The actual stress to be carried is only 10,000 lb. Use the *AREA* specifications. The minimum thickness of metal allowed is 3/8 in.

Truss Members

119. Compression Members for Bridge Trusses. The typical section for the upper chord and end post of an ordinary bridge truss consists of two channels with flanges turned out and webs vertical, a top cover plate, and bottom lacing. The gusset plates are riveted inside of the channel webs and are used to splice the channels at joints where the upper chord is not continuous. The cover plate is spliced by a top splice plate. *At the hip joint it is necessary to carry the entire stress through the gusset plates* since the splice plate across the cover plates has to be bent and is therefore unsatisfactory as a compression plate. A typical top chord section is shown in Fig. 105.

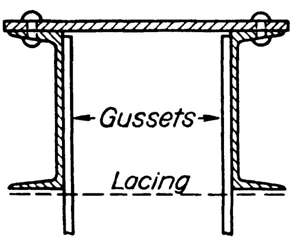


Fig. 105.
Upper Chord Section.

When designing the end connections for an upper chord member of a highway bridge, one must know whether the rivets are field or shop rivets. Usually the gusset is riveted to one top chord member in the shop and the connection to the other member must be made in the field. Field rivets have a lower working stress by the 1935 *AASHO* specifications than shop rivets. (Spec. 70.) In many cases it will be found that the bearing value of the rivets on the thin webs of the channels, rather than the single shear value, controls. For railway bridges, the *AREA* specifications make no distinction between shop and field rivets. This seems a reasonable procedure because all rivets are now power driven. This is also true of the 1941 *AASHO* specifications.

Lacing. The design of lacing bars for compression members is made dependent upon the regular column formula. The stress in the bar is determined from a specified shear, such as, for example, 300 times the gross sectional area of the chord in inches, or $300A$. (The requirements of the *AASHO* and *AREA* are given by Spec. 104 and 183.) This total specified shear is considered to be divided equally between the top cover plate and the lacing. Then, if double lacing is used, the shear per bar is obtained by dividing this total shear of $300A$ by four. Assuming the bars to be at 45°, we compute the stress per bar to be $\left(\frac{300A}{4}\right) \times 1.41$.

The bar usually is selected of minimum width (Spec. 104) and the thickness is calculated on the basis that the bar is a column carrying the stress determined above. The minimum thickness for single and double lacing is specified as 1/40 and 1/60 respectively of the distance along the bar between end rivets. (Spec. 104 and 183.)

Pin-End Connection. At its lower end, the end post frequently rests upon a pin. The usual detail is shown in Fig. 106. The pin is required to allow the end of the truss to rotate through a small angle as the truss deflects. The total stress in the end post must be resisted by bearing on

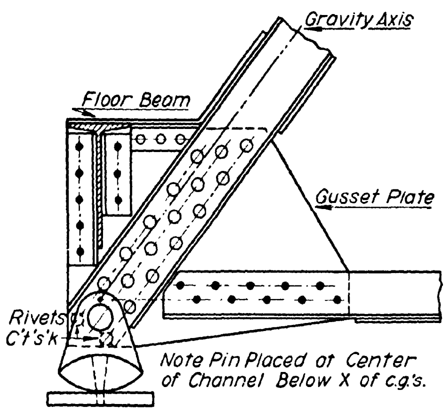


FIG. 106. ROCKER END OF A TRUSS BRIDGE.

the pin. The gusset plate is riveted to the channel web so that it acts with the web in furnishing bearing area. If the combined thickness of two channel webs and two gussets still does not furnish sufficient area in bearing on the pin, it is necessary to rivet pin plates to the outside faces of the channels. The rivets through the gusset plate, pin plate, and channel web must be adequate in shear and bearing to transfer the bearing stress out of the plates and into the channel. The design of the pin and of the cast rocker are specialized procedures that will not be considered here.

120. Design of the End Post of a Bridge Truss and Its Connections.

PROBLEM. Design the end post and connections for a low truss highway bridge of ordinary Warren type for use in city street service. (See Fig. 107.)

Data.

Length of member = 10 ft.-11 3/8 in.
Design stress = 157,300 lb. from dead and live loading.

Working stresses from City Building Code.

Compression = $15,000 - 50\frac{L}{r}$, not to exceed 13,500 lb. per sq. in.

Bearing on pins = 24,000 lb. per sq. in.
Rivets, single shear; shop, 12,000; field, 10,000 lb. per sq. in.
Rivets, bearing; shop, 24,000; field, 20,000 lb. per sq. in.

Specifications. *AASHO* specifications for highway bridges as given in § 216 except for low working stresses as given above.

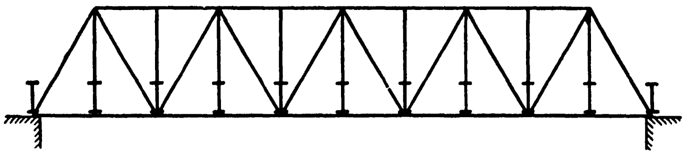


FIG. 107. WARREN HIGHWAY TRUSS.

Selection of Cross-Section.

Assumed allowable stress = 13,000 lb. per sq. in.
Approximate required area = 157,300 ÷ 13,000 = 12.1 sq. in.
Cover plate = 16 × ⅜ in.; gross area, 6 sq. in.
Edge distance of plate for ¾-in. rivets = 1¼ in.

$$\text{Thickness of plate} = \frac{13.5}{40} = 0.34. \quad \text{Use } \tfrac{3}{8} \text{ in. (Spec. 87.)}$$

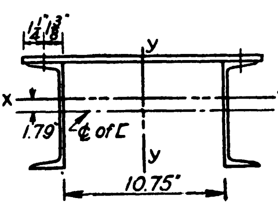


FIG. 108. END POST SECTION.

Channels. The remainder of the area or 6.1 sq. in. must be furnished by the channels. It is customary to use 9-in. channels with a 16-in. cover. The lightest 9-in. channel is 13.4 lb. per ft. and the area is 3.89 sq. in. The web thickness is but 0.23 in. A thicker web is desirable and an 8-in., 13.75-lb. channel is selected, for which the web thickness is 0.303 in. (Under some specifications the webs of rolled sections are not limited by a minimum thickness, and the 9-in., 13.4-lb. channel would be used.) This web is nearly $^5/_{16}$ in. which is satisfactory. The area per channel is 4.02 sq. in. The gross area of the entire section is 14.04 sq. in.

Center of gravity. Take statical moments of the areas about the centers of the channels.

Channels	8.04 × 0	= 0.00
Cover	6.0 × 4.19	= 25.14
	Total	= 25.14

Eccentric distance of C.G. above center of web is 25.14 ÷ 14.04 = 1.79 in.

Moment of inertia about the x-x axis.

$$\begin{array}{lll} \text{Channels} \begin{cases} I_0 & \\ Ay^2 = 8.04 \times 1.79^2 = \end{cases} & & \begin{array}{r} 71.6 \\ 25.8 \end{array} \\ \text{Plate} \quad Ay^2 = 6.0 \times 2.40^2 = & & \underline{34.6} \\ \qquad\qquad\qquad \text{Total} = & & 132.0 \text{ in.}^4 \end{array}$$

Moment of inertia about the y-y axis.

$$\begin{array}{lll} \text{Channels} \begin{cases} I_0 & \\ Ax^2 = 8.04 \times 5.93^2 = \end{cases} & & \begin{array}{r} 3.0 \\ 282.0 \end{array} \\ \text{Plate} \quad \tfrac{1}{12} \times 0.375 \times 16^3 = & & \underline{128.0} \\ \qquad\qquad\qquad \text{Total} = & & 413.0 \text{ in.}^4 \end{array}$$

Radii of gyration.

$$r_{xx} = \sqrt{\frac{I_{xx}}{A}} = \sqrt{\frac{132.0}{14.04}} = 3.07;$$

$$r_{yy} = \sqrt{\frac{I_{yy}}{A}} = \sqrt{\frac{413.0}{14.04}} = 5.42.$$

These radii of gyration meet the specification for low truss bridges that r_{yy} must be at least 1.5 times r_{xx}. (Spec. 117.)

Allowable stress $= 15{,}000 - 50\dfrac{131.37}{3.07} = 12{,}800$ lb. per sq. in.

Actual stress $= 157{,}300 \div 14.04 = 11{,}200$ lb. per sq. in.

REMARKS. This section is not an economical one but the normal revision to a 14-in. cover plate would entail an undesirable sacrifice of lateral stiffness. We might take advantage of Spec. 87 and use a $16 \times \frac{5}{16}$-in. cover. The effective width then becomes $40 \times \frac{5}{16} + 2 \times 1\frac{1}{4} = 15$ in. The effective area of the section becomes $15 \times 0.312 + 8.04 = 12.7$ sq. in. The actual stress is $157{,}300 \div 12.7 = 12{,}400$ lb. per sq. in. This is a satisfactory and economical design since the radius of gyration will be practically unchanged and the allowable stress is still 12,800 lb. per sq. in.

Design of Lacing Bars and Stay Plates.

Stay plates. The end stay plates must be $1\frac{1}{4}$ times as long as the distance between rivets. (Spec. 103.) $1.25 \times (16 - (2 \times 1.25)) = 17$ in. The thickness must be $\frac{1}{50}$ of the distance between lines of rivets or $13.5 \div 50 = 0.27$ in. (Spec. 103.) Use an end stay plate $16 \times \frac{5}{16} \times 1$ ft.-6 in.

Lacing bars. The minimum lacing bar of $2\frac{1}{4}$-in. width for $\frac{3}{4}$-in. rivets will be used. (Spec. 104.) The length of bar between rivets is $1.41 \times 13.5 = 19$ in. Double lacing at 45° will be used. The shear per bar for section with $\frac{5}{16}''$ cover plate is $300A/4 = 300 \times 12.7 \div 4 = 950$ lb. The stress is $1.41 \times 950 = 1350$ lb. This method of computing the shear is somewhat more severe than the *AASHO* requirement. (Spec. 104.)

Thickness of lacing bars. The lacing bar must carry a stress of 1350 lb. in direct compression. The least radius of gyration is $0.29t$. Take t at its minimum value of $\frac{19}{60} = \frac{5}{16}$ in. approx. (Spec. 104.) $r = 0.29 \times 0.312 = 0.09$ in. The slenderness ratio is $0.7 \times 19/0.09 = 148$. (The free length of bar is reduced 30 per cent for cross-riveted double lacing.) The allowable unit stress is $15{,}000 - 50 \times 148 = 7600$ lb. per sq. in. The allowable load is $2.25 \times 0.312 \times 7600 = 5300$ lb. which is more than adequate.

Design of the Riveted Connection at the Hip Joint.

Value of the member with $\frac{5}{16}$-in. cover plate $= 12.7 \times 12{,}800 = 163{,}000$ lb.

Value of the rivets. These rivets, which pass through the channel web, are in single shear and in bearing on metal that is 0.303 in. thick. Assume that these are $\frac{7}{8}$-in. field driven rivets. The single shear value per rivet is 6010 lb. The bearing value is $0.875 \times 0.303 \times 20{,}000 = 5300$ lb. (bearing controls).

Number of rivets. $163{,}000 \div 5300 = 31$ rivets. Use 16 through each channel. Three rows of rivets can be used in an 8-in. channel. The detail is shown in Fig. 109.

Design of the Connection at the Shoe.

Bearing on pin.* The required thickness for bearing on a $4\frac{1}{2}$-in. pin is $163{,}000 \div (24{,}000 \times 4.5) = 1.51$ in.

* Since no information is given in this problem as to the actual slope of the end post, the pin is designed for bearing equal to the value of the member. The force acting on the pin is actually the vertical end reaction when a gusset plate is used as in Fig. 109.

Thickness of pin plates. The combined thickness of two gusset plates and two channel webs is $2(0.375 + 0.303) = 1.35$ in. One $\frac{5}{16}$-in. pin plate will be added on the outside of each channel. The total thickness then becomes 1.97 in.

Bearing on webs of channels. The part of the load carried directly into the channel webs by bearing is $\frac{0.606}{1.97} \times 163{,}000 = 50{,}000$ lb. The remainder, or 113,000 lb., must be transferred by the rivets between the plates and channel webs.

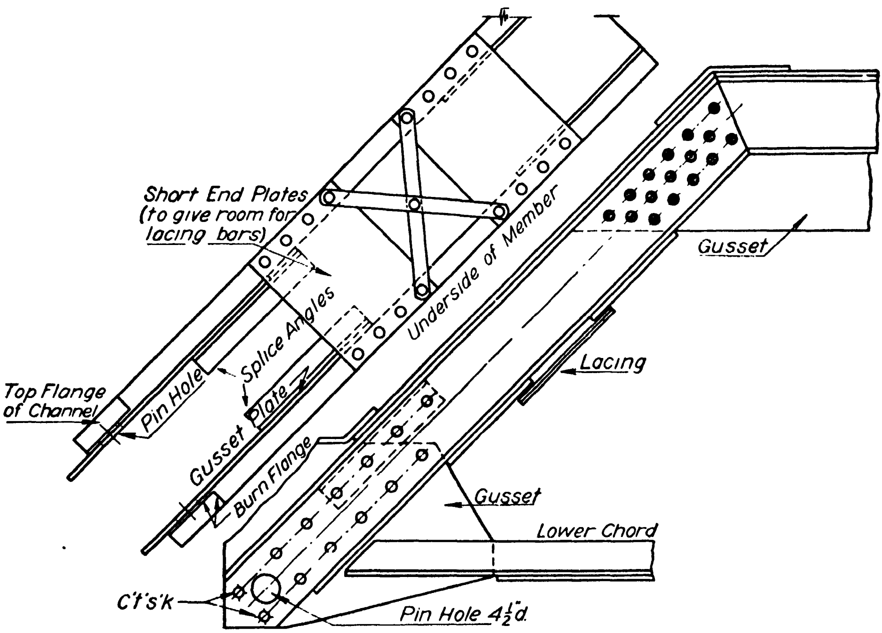


FIG. 109. END POST OF A BRIDGE TRUSS.

Value of the rivets. The rivets act in double shear and in bearing on the channel web. Bearing controls and the rivet value is $0.303 \times 0.875 \times 24{,}000 = 6360$ lb. This value is for $\frac{7}{8}$-in. shop driven rivets. The plates are shop riveted because the pin hole must be drilled in the shop.

Number of rivets. $113{,}000 \div 6360 = 17.8$ rivets.

The detail (Fig. 109) shows 11 rivets through each channel but 2 are countersunk rivets that are not considered to offer full value.

REMARKS. As there is considerable excess area available for bearing on the pin, the designer might decide to reduce the size of the pin below $4\frac{1}{2}$ in. if this could be done without overstressing the pin in shear or flexure. On the other hand, a pin slightly larger than $4\frac{1}{2}$ in. could be used, and the pin plates might be eliminated altogether.

121. Design of Compression Chords for Roof Trusses. The compression chord of a riveted roof truss is composed usually of two angles so that a single gusset plate can be placed between the angles for the joint

connection. A welded roof truss, which needs no gussets, can be formed with a tee-shaped compression member obtained by splitting a wide flange beam section. The type of roof construction has much to do with the design of such a chord member. The free length for buckling is controlled by the connections for the diagonal roof bracing and is usually taken as the longest distance between such connections. Some *lateral stability* will be obtained from the purlins but they are not ordinarily considered to be fully effective for this purpose.

Design of a Split-Beam Chord Member, *DP*38. This example illustrates the design of a split-beam compression section by use of the Rankine-Gordon type of column formula. The final allowable stress is shown to be about 500 lb. per sq. in. below the actual stress. This brings up the question as to whether such a design is ever satisfactory. The proper point of view is that the excess stress, which is about 4 per cent above the allowable, should be eliminated if it does not require an *unreasonable* increase in cost. Design calculations are seldom carried to a greater degree of precision than *1 per cent*, and there are unavoidable approximations introduced into the calculations. Hence, the magnitude of the overstress considered here is not an extremely serious matter, but it need not be permitted, since it will usually be possible to reduce such an overstress without having to choose a seriously uneconomical section. For the case considered, it is possible to choose a section that weighs but 6 per cent more, which will be 2 per cent understressed. The heavier section should be chosen.

PROBLEMS

151. Redesign the end post designed in § 120 by making use of a 14-in. cover plate Determine the required diameter of pin so that the pin plates may be eliminated. Use *AASHO* working stresses.

152. Design an end post for a low-truss highway bridge where the design stress is 210,000 lb. and the length of the end post is 11 ft.-0 in. Use the *AASHO* specifications.

153. Design an upper chord member of a bridge truss for a stress of 300,000 lb. The panel length is 20 ft. Detail the end connection at a splice. Use the *AREA* specifications.

154. Determine the allowable stress on a chord section composed of an 18 × 7⁄16-in. cover and two 10-in., 20-lb. channels. The edge distance of the rivet holes through the cover is 1½ in. The panel length is 22 ft. Use *AREA* specifications.

155. Redesign the member from *DS*38 by making use of a section composed of two angles with legs turned in and tack welded together.

156. Design a member similar to the one designed on *DS*38 but where the total D.L. plus L.L. stress is 50,000 lb. and the wind stress is 20,000 lb. Use *AISC* specifications and working stresses.

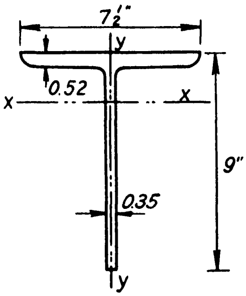
157. Select the proper section for a top chord member of a welded roof truss where the design stress is 16,000 lb., of which 10,000 lb. is caused by wind. The panel length and the distance between purlins are both 5 ft. If a split I-beam is used, remember that the depth must be great enough to hold the welded connections for the web members. This requires at least 5 in. Use *AISC* specifications. Length for buckling is 10 ft.

DP38. *Design a split I-beam section to act as the compression chord of a quadrangular roof truss. There is lateral support provided at each panel point. AISC spec.*

Data:

Panel length is 12′-0″.

Design stress = 90,000# for D.L. + L.L. and 15,000# for wind.



Working Stress:

Compression $= \dfrac{18{,}000}{1 + \dfrac{L^2}{18{,}000r^2}}$ *but not to exceed* $15{,}000\#/\square''$.

Allowable increase for wind $= 33\frac{1}{3}\%$.

Selection of Cross-Section:

Assumed allowable stress $= 13{,}000\#/\square''$.

Approx. area reqd. $= 90{,}000 \div 13{,}000 = 6.9\square''$.

Wind stress is neglected since it is less than $\frac{1}{3}$ *of D.L. + L.L. stress.*

Trial section. An 18WF47 beam section furnishes $13.8\square''$ *or* $6.9\square''$ *for each split tee.*

Equivalent section. As shown by the sketch, the tee is equivalent to a horizontal plate $7.5'' \times 0.52''$ *and a vertical plate* $8.48'' \times 0.35''$.

Moment of inertia about axis of symmetry (minimum I).

Horizontal plate. $\dfrac{1}{12} \times 0.52 \times 7.5^3 = 18.27$

Vertical plate. $\dfrac{1}{12} \times 8.48 \times 0.35^3 = \underline{0.03}$

Total $I = 18.30$

Radius of gyration.

$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{18.3}{6.9}} = 1.63''.$$

Allowable stress.

$$f = \frac{18{,}000}{1 + \dfrac{144^2}{18{,}000 \times 1.63^2}} = 12{,}500\#/\square''.$$

Actual stress $= \dfrac{90{,}000}{6.9} = 13{,}000\#/\square''$.

Remarks: *This section is overstressed by 4%. An 18WF50 section will provide 6% more area and will be understressed by 2%. It is therefore preferred.*

Special Problems in Column Design

122. Design of Members that Undergo Reversal. The diagonals of a Warren bridge truss and certain members of other trusses may reverse from tension to compression or *vice versa* during the passage of the live load. The usual specification is that the maximum tensile and maximum compressive stresses shall be calculated and that each shall be increased by 50 per cent of the smaller stress; then the member shall be proportioned so that it will be able to resist each increased stress. (Spec. 82.) Depending upon the ratio between these increased stresses, the member will be designed for tension and checked for compression or else the process will be reversed. In all cases the connections are designed to resist the sum of the original unincreased stresses. (Spec. 82.)

Design of a Chord Member of a Warren Bridge Truss, *DP*39. In this member, which is at the center of the span, the maximum tension and the maximum compression are identically equal, the dead load stress being zero. The section is therefore selected as a compression member and checked for tension with one rivet hole removed from each angle leg. The connection must be able to resist the sum of the two maximum stresses since continued reversal of stress tends to loosen up the rivets of the end connections.

123. Two-Story Column. It is occasionally necessary to design a column two stories high that is unsupported in one direction at the top of the first story, as in Fig. 110. The problem is to determine how to apply the usual column formulas to limit the working stress properly for such a column. In using the standard formulas, the only variable under our control in this instance is the column length L. Evidently, it would be very conservative to take L as the full height of two stories and overly liberal to accept L as one story height. As a start toward a solution, it might be well to see how the Euler formula would need to be revised to apply to a very slender column that was loaded both at the upper end and at the mid-height.

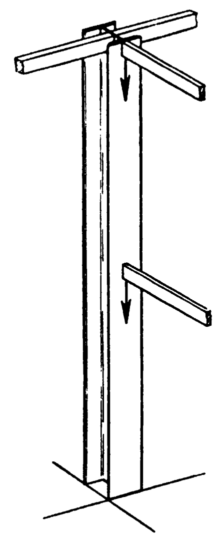


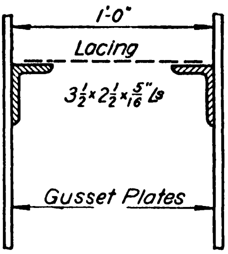
Fig. 110. Two-Story Column, Doubly Loaded.

Euler Formula Revision. The buckled column in its deflected position is illustrated by Fig. 112. The tangent at the mid-height is not vertical but slopes as shown. Hence, it will be most convenient to compute the center deflection Δ (which is nearly the maximum) as the average of the two end deflections from the tangent, or as $\frac{1}{2}(\Delta_1 + \Delta_2)$.

The bending moment at any point above the mid-height is $kPx + \dfrac{(1-k)P\Delta}{L}y$. Coordinates of the deflected curve

DP39. *Design the center diagonal of a Warren bridge truss, which undergoes reversal. The member is 11'-3" long. Use two angles with legs turned in. Use AASHO spec.*

Compression Member:



Maximum stresses are ± 26,000#.

Design stresses = ± 26,000 × 1.5 = ± 39,000#. (Spec. 82.)

A section will be selected for compression and checked for tension.

Column formula. $P/A = 15{,}000 - \frac{1}{4}\left(\frac{L}{r}\right)^2.$

Estimated allowable stress = 11,000#/□".

Approx. area reqd. = 39,000 ÷ 11,000 = 3.6 sq. in.

Trial section. 2 angles 3½ × 2½ × 5⁄16" furnish a gross area of 3.56□".

The controlling value of r is 1.11.

Allowable unit stress $= 15{,}000 - \frac{1}{4}\left(\frac{135}{1.11}\right)^2 = 11{,}300$*#/□".*

The section is therefore satisfactory for compression.

Check on Tensile Stress:

Net area. Assume one ⅞" hole to be deducted from each leg. The net area is 3.56 − 2 × 0.875 × 0.31 = 3.0□".

Effective area. For this type of member only 80% of the net area may be effective. (Spec. 85.) 0.80 × 3.0 = 2.4□".

Tensile stress = 39,000 ÷ 2.4 = 16,300#/□". (Allowable f = 18,000.)

End Connection; Field Rivets:

Connection value = 2 × 26,000 = 52,000#. (Spec. 82.)

Rivet value in single shear = 0.44 × 11,000 = 4840#.
" " " bearing = 0.312 × 0.75 × 22,500 = 5270#.

Number of rivets = 52,000 ÷ 4840 = 10.8. Use 6 in each ∠.

Controlling Specification: *If the alternate stresses occur in succession during one passage of the live load, each shall be increased by 50% of the smaller. The connections of such members shall be proportioned for the sum of the net alternate stresses not so increased. (Spec. 82.)*

are measured from the upper end. Similarly, the moment at any point below the mid-height is $Px - \frac{(1-k)P\Delta}{L}y$. In this formula coordinates are measured from the lower end of the column.

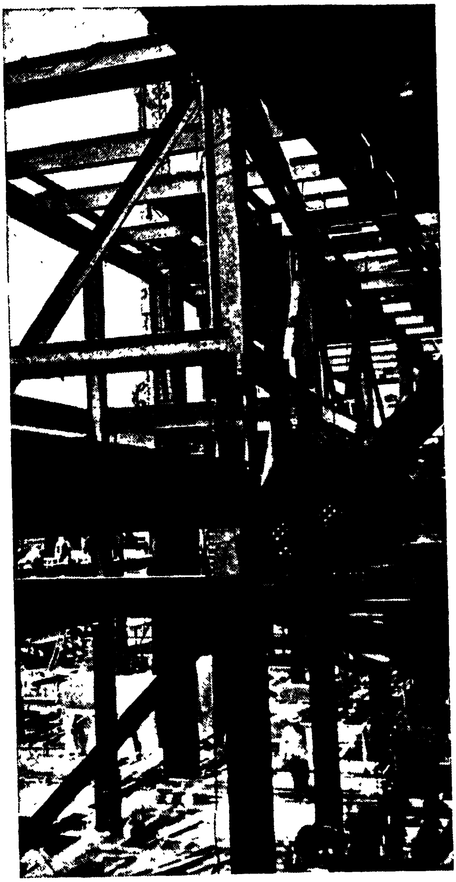


FIG. 111. TWO-STORY COLUMN WITH BEAMS AND STRUTS.

We may apply the rule of area moments to the calculation of Δ_1 and Δ_2, that is, the deflection at A from a tangent at B is equal to the statical moment of the M/EI area from A to B taken about A, *the point where the deflection is wanted.* Either half of the curve of deflection will be assumed to be a parabola (an evident approximation) so that there will be a parabolic moment area caused by the vertical load. The horizontal end force

produces a straight-line moment diagram. Thus we obtain

$$EI\Delta_1 = \underbrace{\frac{2}{3}\left[kP\Delta\right]\frac{L}{2}\left(\frac{5}{8}\,\frac{L}{2}\right)}_{\text{parabolic area}} + \underbrace{\frac{1}{2}\left[\frac{(1-k)P\Delta}{L}\,\frac{L}{2}\right]\frac{L}{2}\left(\frac{2}{3}\,\frac{L}{2}\right)}_{\text{triangular area}} \tag{21}$$

$$= \frac{kP\Delta L^2}{9.6} + \frac{(1-k)P\Delta L^2}{24},$$

and

$$EI\Delta_2 = \underbrace{\left[\frac{2}{3}P\Delta\right]\frac{L}{2}\left(\frac{5}{8}\,\frac{L}{2}\right)}_{\text{parabolic area}} - \underbrace{\frac{1}{2}\left[\frac{(1-k)P\Delta}{L}\,\frac{L}{2}\right]\frac{L}{2}\left(\frac{2}{3}\,\frac{L}{2}\right)}_{\text{triangular area}} \tag{22}$$

$$= \frac{P\Delta L^2}{9.6} - \frac{(1-k)P\Delta L^2}{24}.$$

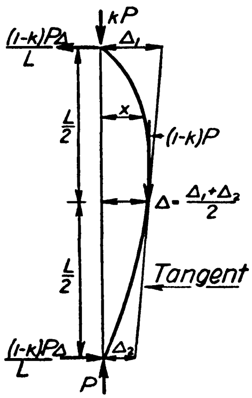


FIG. 112. EULER DEFLECTION CURVE.

Therefore, since $\Delta = \dfrac{\Delta_1 + \Delta_2}{2}$, we may write

$$EI\Delta = \left(\frac{1+k}{2}\right)\frac{PL^2}{9.6}\Delta\,; \tag{23}$$

or

$$P = \frac{9.6EI}{L^2}\left(\frac{2}{1+k}\right). \tag{24}$$

From which, by substitution of π^2 for 9.6, we may write,

$$\frac{P}{A} = \frac{\pi^2 E}{(L/r)^2}\left(\frac{2}{1+k}\right). \tag{25}$$

The *breaking load* from equation (25) is $2/(1+k)$ times the usual expression for Euler's formula. In other words, the placing of only a fraction (k) of the total load at the top of the column, the remainder being applied at the mid-height, strengthens the column in the ratio $2/(1+k)$.

Revision of Working Formulas. Of course, the Euler formula is not the common one used for the design of a practical structure, but it is the only formula from which basic theory can be set up. If, for a given end condition or type of loading, a column is doubled in strength according to the Euler formula, it becomes logical to decrease by one half the reduction term for buckling allowance in the column design formula. Thus, we will introduce the factor $(1+k)/2$ into the column formulas of the *AISC* specifications as follows:

$$\frac{P}{A} = 17{,}000 - 0.485\,\frac{L^2}{r^2}\left(\frac{1+k}{2}\right), \tag{26}$$

or

$$\frac{P}{A} = \frac{18{,}000}{1 + \left(\dfrac{1+k}{2}\right)\dfrac{L^2}{18{,}000r^2}}. \tag{27}$$

(Also see *Structural Design in Steel*, Shedd.)

In this expression k is the ratio of the load applied at the top of the column to the total applied load, and L *is the two-story height*. The formulas given above apply to the case where both stories are of the same height, but they may be used with confidence for practical cases where the variation between stories is not great. The influence of two or three feet of variation in story heights is not important.

DESIGN OF A TWO-STORY COLUMN, *DP*40. The interesting feature of this design problem is that the buckling tendency is greater in the lower story than for the column as a whole. This is due to the fact that the smaller radius of gyration controls buckling in the lower story while the column must buckle in the direction of the greater radius of gyration if it buckles over the two-story length. This will be a rather common experience in the design of two-story columns.

PROBLEMS

158. Design a member for reversal where the maximum and minimum stresses are each 14,000 lb. All other data are the same as those for the example worked out in *DP*39. Design the end connection and select the end plate and size of lacing bar if the distance back to back of angles is 10 in.

159. Design a diagonal of a Warren truss for a maximum tensile stress of 40,000 lb. and a maximum compressive stress of 10,000 lb. The length of the member is 13 ft. Use two angles laced with flanges turned in. Follow *AASHO* specifications. Design the end connection and select the end tie plate and size of lacing bar if the distance back to back of angles is 12 in.

160. Design a diagonal of a deck Warren truss for a maximum stress of +60,000 lb. and for a minimum stress of −60,000 lb. The length of the member is 16 ft.-3 in. Follow *AASHO* specifications. Use two channels laced on both sides with flanges turned in. Select channels with web thickness at least $\frac{5}{16}$ in. Redesign this member using two angles laced. Which is the better design?

161. Redesign the two-story column of *DP*40 by use of *AASHO* specifications. Double the loads.

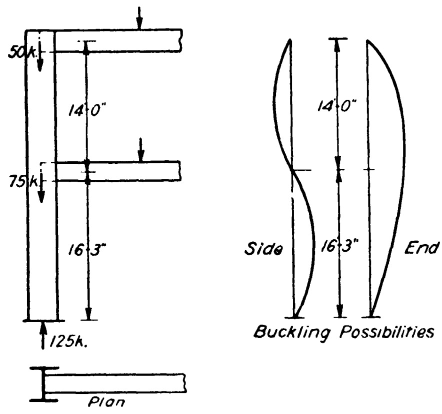
162. Design by *AISC* specifications a two-story column of 40 ft. total height where the load of 200,000 lb. is applied equally to the top and the mid-height of the column. Loads are centric and the column is braced in one direction only at the mid-height.

124. The Importance of Careful Design of Compression Members. The design of all members in a structure is important, but there are reasons why the design of the compression members is critical. Failure of a tension member starts taking place when the member passes the elastic limit, but a final failure will not occur until the tension stress reaches the ultimate strength which, for structural steel, is nearly twice the elastic limit. In contrast, a compression member of ordinary proportions buckles and collapses at a load about equal to the elastic limit, while a slender strut may fail at one half of this unit stress. Evidently then, we do not really have as great an *ultimate factor of safety against complete collapse* in the compression members as in the tension members of our structures.

DP40. *Design a two-story column of total height 30′-3″ with 3/5 of the 125,000# load applied at 16′-3″ above the base. AISC spec.*

Assumed working stress = 14,000#/□″.

Approx. area = 125,000 ÷ 14,000 = 8.92□″.



Trial Section:

An 8WF31 section furnishes 9.12□″ and will be tried out.

Radii of gyration are 3.47 and 2.01.

Special buckling factor $= \dfrac{1+k}{2} = \dfrac{1+0.4}{2} = 0.7$. *(Equation 26.)*

Allowable working stress for buckling over entire length $= 17{,}000 - 0.485\left(\dfrac{363}{3.47}\right)^2 0.7 = 13{,}300$#/□″.

Actual average unit stress $= \dfrac{125{,}000}{9.12} = 13{,}700$#/□″.

Allowable working stress for buckling in lower story only $= 17{,}000 - 0.485\left(\dfrac{195}{2.01}\right)^2 = 12{,}450$#/□″.

Revision of Section: *An 8WF35 section furnishes 10.3□″ of area and will be stressed to 12,150#/□″. Its radii of gyration are 3.50 and 2.03 so that it is safe from buckling in the lower story and also for the column as a whole.*

Another consideration should be the influence of eccentricity of load which may be caused either by load position, by initial crookedness, or from eccentric end connections. The deflection of an eccentrically loaded tension member decreases as the load is increased and is greatly reduced by the change in shape that takes place beyond the elastic limit. A column, on the other hand, changes shape in a direction that increases the initial eccentricity when the elastic limit is passed. Hence, the compression members are an unquestioned source of weakness for structures where the loads may become eccentric to the axes of the members. Even the possibility of a collision with resultant bending of a member is a much more serious consideration in the design of compression posts than for tension bars.

These thoughts simply clarify the need for careful analysis of all loads that may come onto the compression members of a structure and the importance of adequate design. Localized buckling needs study as much as buckling of the member as a whole. Designing such details as lacing bars and end tie plates in a conservative manner is a wise precautionary measure. The weakness of a lacing bar connection was believed to have caused one failure of the great Quebec cantilever bridge.

CHAPTER 8

ROLLED BEAMS AND GIRDERS

125. Functions of Beams and Girders. The most important use for beams and girders is in supporting floors for buildings and bridges. The main carrying members are called girders while the smaller members of shorter span are known as beams. The proper distinction is found in the manner of loading. A beam ordinarily receives its load directly from a floor slab resting upon its upper flange, while a girder receives a major part of its load from the *reactions* of smaller beams that it supports. An exception to this definition is the floor beam of a bridge which may receive a considerable part of its load from smaller beams, the stringers. The beams in building floors that serve the same purpose as the stringers of a bridge are known as joists.

FUNDAMENTAL THEORY

126. Beam Formulas. Continued use will be made of the common beam-flexure formula and of the beam-shear formula. The beam-flexure formula is

$$(1) \qquad f = \frac{My}{I}, \quad \text{or} \quad f_{\text{max.}} = \frac{Mc}{I},$$

where f is the fiber stress at the distance y from the neutral axis,
y is the distance from the neutral axis to any fiber,
c is the maximum value of y (at the extreme fiber),
I is the moment of inertia of the effective cross-section about the neutral axis.

The beam-shear formula is

$$(2) \qquad s_s = \frac{VQ}{It},$$

where s_s is the unit shear in the beam web at a distance y from the neutral axis,
V is the total vertical shear at the cross-section,
Q is the statical moment about the neutral axis of the cross-sectional area *outside* of the section on which shear is being calculated,
t is the thickness of metal at the section where the unit shear is desired,
I is the moment of inertia of the effective cross-section (gross for shear).

The limitations upon the application of the beam theory are that sections which are plane before bending must remain plane after bending and that

stress must be proportional to strain. This means that all stresses must be below the elastic limit.

127. The Section Modulus. The usual way to select a beam is by use of the section modulus. The beam-flexure formula may be rewritten as

$$\frac{M}{f} = \frac{I}{c} = S. \qquad (3)$$

The factor S is the section modulus defined as the moment of inertia divided by the distance to the extreme fiber or by one half of the depth for an ordinary rolled beam section. The procedure for use of the section modulus in design is therefore as follows:

(*a*) Divide the controlling bending moment by the allowable fiber stress to obtain the required value of the section modulus.

(*b*) Select by use of the structural handbook a rolled beam section that will provide at least this required modulus. The criterion of economy is weight rather than modulus and it is desirable therefore to select the beam of lightest weight that will provide the required section modulus.

Selection of Standard Sections

128. Economy in Rolled Beam Selection. Example *DP*41*a*. It is shown that the wide flange beam sections are the most economical for the case studied. This is usually true, and for this reason they have largely replaced the use of channels and standard I-beams.

Section Selection with Flange Hole, *DP*41*b*. The procedure followed is to guess at a satisfactory section guided by the requirements for the beam without a flange hole (*DP*41*a*). Then, the section modulus for a hole in *each* flange is computed and deducted from the gross section modulus to arrive at the *effective section modulus*. The procedure of allowing for a hole in each flange may appear strange since it is a compromise between theory and test results. Recent tests have not indicated any appreciable shift in the neutral axis due to a flange hole. Hence, there has been some thought that the influence of a hole might be neglected altogether. However, as long as holes in tension members are known to reduce the ultimate strengths of such members, there is reason to believe that beams will be similarly weakened by holes in the tension flange. A compromise is to retain the neutral axis at the mid-height by deducting equivalent areas from both flanges. This is undoubtedly conservative, but the alternative procedure of calculating a displacement of the neutral axis is probably no more representative of actual conditions. The example *DP*41*c* gives similar calculations for a beam with holes in the web.

129. Deflection Limitation upon Beam Design. There are two major reasons for limiting the deflections of beams. First, there is the problem of cracking plaster which can unquestionably be avoided if the calculated deflection is less than 1/360 times the span. Secondly, beams of different sizes often have to deflect alike, as for example, when they are "bricked in" together. Such beams must have identical calculated deflections.

DP41a. *Select a rolled beam section to support a load of 1000#/′ on a span of 30′. Allow a fiber stress of 20,000#/□″ and estimate the weight of beam to be 50#/′.*

Bending moment $= \frac{1}{8} \times 1050 \times 30^2 \times 12 = 1{,}420{,}000''\#$.

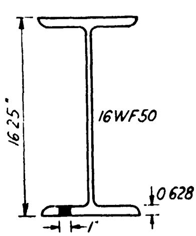
Required section modulus $= \dfrac{1{,}420{,}000}{20{,}000} = 71.0$.

First choice is a 16WF45 section which provides a modulus of 72.4.

Second choice is a 14WF48 section which provides a modulus of 70.2.

Other sections of sufficient strengths but greater weights are an 18″–58.0# channel or an 18″–54.7# I-beam.

DP41b. *Redesign the rolled beam of DP41a on the supposition that it is necessary to punch a hole of 1″ diameter through the tension flange.*



Assumption. Since tests do not show an influence upon the location of the neutral axis, the usual conservative design practice is to reduce the section modulus for a hole in each flange.

Try a 16WF50 section.

Section modulus of rectangular holes $= 0.628 \times 1.0 \times 2 \times (8.12 - 0.314)^2 \div 8.12 = 9.4$.

Net modulus $= 80.7 - 9.4 = 71.3$. *(71.0 reqd.)*

DP41c. *Redesign the beam of DP41a to resist the moment of 1,420,000″# when a row of ¾″ holes is punched in the web at 1″, 3″, 5″, and 7″ from the neutral axis. The required modulus is 71.0. Try a 16WF45 section, web thickness = 0.35″.*

Section modulus of rectangular holes $= 0.35 \times 0.75 \times 2 \times (1^2 + 3^2 + 5^2 + 7^2) \div 8.06 = 5.4$.

Net modulus $= 72.4 - 5.4 = 67.0$ ***(acceptable).***

DP41d. *Limit the deflection of the beam of DP41a to 1/360 times the span.*

Maximum $\Delta = \dfrac{1}{360} \times 30 \times 12 = 1.0''$.

Hence, $1.0 = \dfrac{5}{384}\,\dfrac{wL^4}{EI} = \dfrac{5 \times 1050 \times 360^4}{384 \times 12 \times 30{,}000{,}000 \times I}$, *or* $I = 638$.

Minimum weight section to provide this moment of inertia is 18WF47.

Remarks: ***The limitation of ¹⁄₃₆₀ times the span was set originally as the practical maximum deflection that could be permitted for plaster on wood lath. Modern plaster ceilings are placed on metal lath and there is reason to believe that heavier deflections are permissible.***

The deflection of a uniformly loaded beam with simple supports is

(4) $$\Delta = \frac{5}{384}\frac{wL^4}{EI}.$$

In this formula, w is the uniform load per unit length of span and the other terms have their usual significance. This formula is used in *DP*41*d*.

EXAMPLES. The example *DP*42*a* shows how the selection of a beam may need to be revised to reduce the deflection.

The example *DP*42*b* illustrates the calculations necessary to adjust the design of a beam so that it may be " bricked in " with another beam and still serve its intended function.

The two beams must have equal deflections, which may be expressed as follows:

$$\frac{5}{384}\frac{w_1L_1^4}{EI_1} = \frac{5}{384}\frac{w_2L_2^4}{EI_2}.$$

Hence, we may cancel out identical quantities for beams of the same span and of like materials to obtain

(5) $$\frac{w_1}{I_1} = \frac{w_2}{I_2}, \text{ or } \frac{w_1}{w_2} = \frac{I_1}{I_2}.$$

This relationship is used in the example *DP*42*b* to obtain beams that will work together. The example *DP*42*c* concerns the deflection of an aluminum alloy beam.

BUCKLING RESISTANCE OF BEAMS

130. Flange Buckling. The usual working stresses for beams are identical in tension and compression. It is understood that such working stresses presuppose restraint of the compression flange of the beam against *lateral buckling*, for, otherwise, failure would occur at or even below the elastic limit by lateral deflection of the compression flange. Evidently, the compression flange acts as a column, and its allowable stress should be controlled by a column formula. Accordingly, in most specifications, particularly where the column formula is of the Rankine-Gordon type, we usually find a similar formula set up to control the stress in the compression flanges of beams and girders. Some formulas in common use are:

(6) $$f_c = 16{,}000 - 150\frac{L}{b} \quad \text{(straight-line type; } \textit{AREA} \text{ 1931)}$$

(7) $$f_c = 18{,}000 - 5\frac{L^2}{b^2} \quad \text{(parabolic type; } \textit{AREA} \text{ 1935)}$$

(8) $$f_c = \frac{22{,}500}{1 + \dfrac{L^2}{1800b^2}} \quad \text{(Rankine-Gordon type; } \textit{AISC} \text{ 1937 — not to exceed 20,000 lb. per sq. in.)}$$

In each case the maximum value of L/b, the ratio of unsupported length to width for the compression flange, is limited to a maximum value of 40.

DP42a. *Find the minimum depth of rolled beam to carry the load specified in DP41a at a maximum weight of about 75#/′ and with a deflection of 1/360 times the span.*

Maximum deflection $= 30 \times 12 \div 360 = 1.0''$.

Hence, $1.0 = \dfrac{5wL^4}{384EI} = \dfrac{5 \times 1075 \times 360^4}{384 \times 12 \times 30{,}000{,}000 \times I}$, *or* $I_{min.} = 653$.

The 12WF79 section will provide this moment of inertia. Shallower beams can be obtained but they will weigh considerably more.

DP42b. *A beam is to span 15′-0″ and carry a heavy uniform load of 5500#/′. It will be "bricked in" with a floor joist of the same span that is stressed to 19,200#/□″ by a uniform load. The joist is a 12WF36 section. Provide a beam that will work with the standard floor joist.*

Assumed weight of beam $= 65\#/'$.

Maximum bending moment $= \frac{1}{8} \times 5565 \times 15^2 \times 12 = 1{,}880{,}000''\#$.

Required modulus (min.) $= 1{,}880{,}000 \div 20{,}000 = 94$.

Modulus of floor joist $= 45.9$; $I = 280.8$.

Bending moment in floor joist $= 19{,}200 \times 45.9 = 880{,}000''\#$.

Load on floor joist is obtained from the equation $M = \frac{1}{8}wL^2$.

$$880{,}000 = (\tfrac{1}{8}wL^2)12.$$
$$w = 8 \times 880{,}000 \div (12 \times 15^2)$$
$$= 2610\#/'.$$

Moment of inertia of beam is obtained from the relation $\dfrac{w_1}{I_1} = \dfrac{w_2}{I_2}$.

$$I = \frac{5565}{2610}(280.8) = 600.$$

A 12WF72 section furnishes an adequate modulus of 97.5 and nearly the correct moment of inertia (597.4).

DP42c. *Select an aluminum I-beam to "brick in" with the steel beam of DP42a and to carry a uniform load of 135#/′. The total uniform load including 15#/′ for the weight of the beam will be 150#/′.*

Modulus of elasticity of aluminum alloy $= 10{,}300{,}000\#/□''$.

Hence, $1.0 = \dfrac{5wL^4}{384EI} = \dfrac{5 \times 150 \times 360^4}{384 \times 12 \times 10{,}300{,}000 \times I}$, *or* $I = 265$.

For a stress of 7000#/□″, the reqd. mod. $(\frac{1}{8}wL^2/f) = \frac{1}{8} \times 150 \times 30^2 \times 12 \div 7000 = 28.9$.

The 12″-14.5# I-beam furnishes a modulus of 45.4 which is more than adequate, but its I of 272.6 is required.

(Spec. 14.) In other words a flange width of 6 in. requires a lateral support at least every 20 ft., even if the working stress is low.

It will be noticed that formula (8) has an upper limit set upon the working stress. The reason is that the formula would give a working stress above the maximum allowable fiber stress of 20,000 lb. per sq. in. for values of L/b below 15. The reason for setting up the formula in this form is that many engineers believe there is no weakening influence upon the girder for short lengths of unsupported flange. In agreement with this opinion, no reduction of working stress is specified by formula (8) until the L/b ratio exceeds 15.

As a final word of caution, the writer wishes to emphasize the importance either of providing adequate lateral support or of reducing compression stresses to mitigate against lateral buckling. This is one of the features of design that we cannot afford to neglect.

131. Diagonal Web Buckling. The possibility of web buckling always exists, but it is not expected to be serious in rolled beams. Repeated tests have shown that web buckling, due to *diagonal compression*, will not occur where the value of h/t (distance between flanges divided by the web thickness) is less than 60. Actually the most recent tests (Lehigh University) show that a value of h/t up to 70 is safe. All rolled beam webs come within this range, and almost all are below 50 for h/t.

An analysis of diagonal buckling by simple theoretical considerations alone is unsatisfactory. For example, in Fig. 113, the length of column to be considered is highly questionable. Certainly the length L' would be too great because the column of length L' would have its ends nearly fixed, but a reduction to $L'/2$ may be too liberal. Also, it should be realized that the greatest buckling tendency will not take place in a 45-degree line as indicated on the figure. The maximum diagonal compression is at 45 degrees along the neutral axis, but its direction is changed by the horizontal compression above the neutral axis and also by the tension below the neutral axis. All of this points to the fact that any simplified theory of diagonal buckling is at best merely a semi-rational procedure, the formulated results of which must have constants introduced as justified by tests. For example, if the straight-line column formula, 15,000-50L/r, is to be used and if webs of $50h/t$ are permitted to be stressed to 10,000 lb. per sq. in. in vertical shear, we could rationalize our design procedure by using

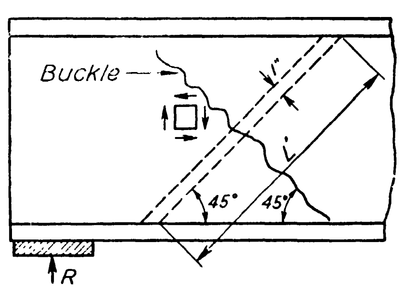


FIG. 113. DIAGONAL COMPRESSION.

the formula

$$15{,}000 - n\frac{h}{t} = 10{,}000 \quad (\text{when } \frac{h}{t} = 50),$$

from which, $50n = 5000$, or $n = 100$. Thus, a semi-rational formula for controlling the allowable shear (or the average diagonal compression which is taken to be numerically the same) on webs where $h/t > 50$ would become

$$s_s = 15{,}000 - 100h/t\,. \tag{9}$$

Such formulas have appeared in some specifications, but the common practice is to allow the same unit shear for all webs for which $h/t < 60$ or 70 and to place *stiffeners* on all thinner webs.

132. Vertical Buckling and Crimpling of Web. Much has been made of the design of unstiffened webs as vertical compression members (at loads and reactions) with every conceivable assumption involved before application of the column formula. For instance, in Fig. 114, which type of failure should we assume? The smallest resistance is provided by

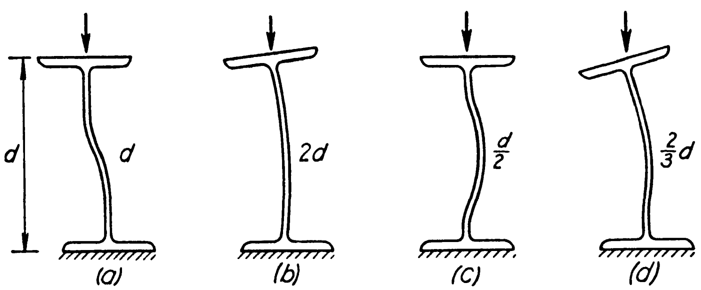


FIG. 114. FAILURE BY BUCKLING FROM VERTICAL LOADING.

(*b*) and the greatest by (*c*); but is it logical to base a design upon the assumption that the upper flange is able to twist and also to move laterally as well? The range of buckling resistance for the cases illustrated by Fig. 114 is indicated by designation of the *free length for buckling* corresponding to each conformation. This length varies from $d/2$ to $2d$, a range of 400 per cent. Exceptional conditions of restraint or lack of restraint at ends and center of beam may make any form of buckling shown in Fig. 114 either possible or probable. Nevertheless, it has been common to base web or stiffener design under concentrated loads or reactions upon a free depth for buckling of $d/2$. If lateral restraint is not provided for the compression flange, a length of d is recommended.

The length of web along the beam that may resist a vertical load or an end reaction has been established as shown in Fig. 115 by drawing 45-degree lines from the ends of the bearing block. Thus the average width for *vertical buckling* of the web is $a + d/4$ at the end reaction and $b + d/2$ at

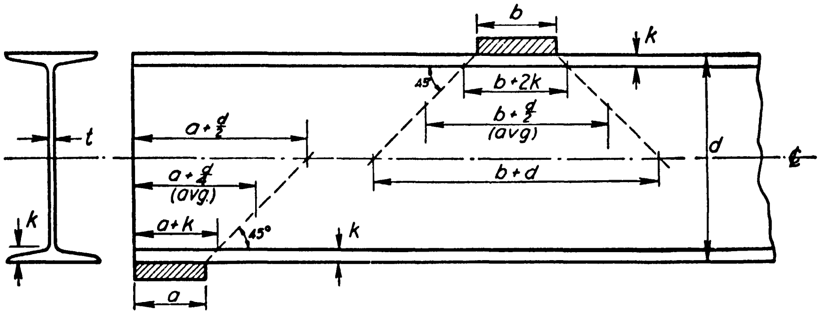


FIG. 115. BEARING AT LOAD AND REACTION.

the applied load concentration. An investigation of *crimpling* at the juncture between the flange fillet and the web should be made for a bearing length of $(a + k)$ at the reaction, or $b + 2k$ under a load. (Spec. 46.) Hence, we may write

(10) $$f_c = \frac{R}{(a + d/4)t} \quad \text{web } \textit{buckling} \text{ at reaction,}$$

(11) $$f_c = \frac{P}{(b + d/2)t} \quad \text{web } \textit{buckling} \text{ at load,}$$

(12) $$f_b = \frac{R}{(a + k)t} \quad \text{web } \textit{crimpling} \text{ at reaction,}$$

(13) $$f_b = \frac{P}{(b + 2k)t} \quad \text{web } \textit{crimpling} \text{ at load.}$$

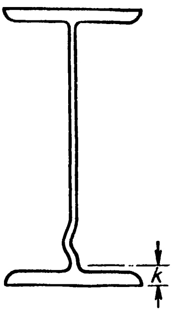


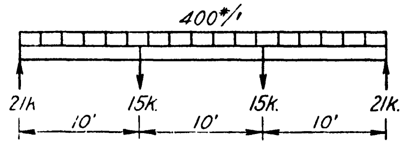
FIG. 116. FAILURE OF WEB BY LOCAL CRIMPLING.

The limitations upon f_c will be obtained from the column formula by use of L as some function of the depth (usually $1/2d$ or $7/10d$) and of r as $0.29t$. The limitation upon f_b may be set in the specifications, or it can be taken as twice the allowable web shear as given by the specifications.

DESIGN FOR BUCKLING RESISTANCE, *DP*43. An ordinary floor girder is designed for flexure and is then checked for all possible conditions of buckling. Flange buckling is not significant here because the value of L/b is less than 15, at which point the *AISC* flange buckling formula starts to control the working stress for the compression flange. For this relatively long beam (30 ft.), the end shear is quite small and the web is not stressed heavily in shear. Accordingly, the diagonal buckling stresses are insignificant. The possibility of vertical buckling over the reaction is found to be serious. It was thought

DP43. *Design a girder of 30′-0″ span into which floor joists frame at the one-third points. The loading is as shown by the sketch. Distributed load including 70# for girder = 400#/′. Use AISC spec.*

Cross-Section:



$M = \frac{1}{8} \times 400 \times 30^2 \times 12$
$+ 15{,}000 \times 120 = 540{,}000$
$+ 1{,}800{,}000 = 2{,}340{,}000''\#.$

$\text{Mod. reqd.} = \dfrac{2{,}340{,}000}{20{,}000} = 117.$

The most economical section is a 21WF59; modulus = 119.3.

Flange Buckling: (*Lateral restraint provided by joists only.*)

Flange width = 8.23″; L/b *ratio* $= \dfrac{10 \times 12}{8.23} = 14.6.$

Allowable compression stress in flange $= \dfrac{22{,}500}{1 + \dfrac{14.6^2}{1800}} = 20{,}100\#/\square''.$

Since this figure is above 20,000, the actual allowable stress is 20,000#/□″, and the 21WF59 section is satisfactory.

Diagonal Buckling of Web:

Web thickness = 0.39″; h/t *ratio* $= \dfrac{20.91 - (2 \times 0.575)}{0.39} = 51.$

Since this value is less than 70, the web is adequate to resist buckling at the full shearing stress of 13,000#/□″. (*Spec. 10 and 45.*)

Actual unit shear $= 21{,}000 \div (20.91 \times 0.39) = 2570\#/\square''.$

Buckling over the Reaction:

Support pad is the width of the flange but only 3″ long.

Length of web resisting buckling $= 3 + d/4 = 8\frac{1}{4}''$ *approx.*

Unit vertical compressive stress $= \dfrac{21{,}000}{8.25 \times 0.39} = 6530\#/\square''.$

L/r *value for web* $= (20.91 - 2 \times 0.575) \div 0.288 \times 0.39 = 176.$

Allowable compression $= \dfrac{18{,}000}{1 + \dfrac{176^2}{18{,}000}} = 6600\#/\square''.$

(NOTE: *The full depth of the web was taken as the buckling length because of the lack of lateral support at the end.*)

Crimpling of Web over the Reaction: (*Spec. 46*)

Depth k from face of flange to toe of fillet = 1.125″.

Bearing on web $= \dfrac{21{,}000}{(3 + 1.12)0.39} = 13{,}100\#/\square''.$ (*24,000 allowed.*)

necessary to consider the clear depth of the web as the possible length for buckling since the beam was simply to be seated on a column bracket without having adequate lateral resistance to flange displacement at the ends. See the controlling possibility of Fig. 114(*a*). The more serious situation represented by Fig. 114(*b*) was considered to be sufficiently improbable to neglect. Web crimpling was not found to be serious. Such crimpling needs investigation for short beams that are heavily loaded in the region where the web shear is high.

133. Grillage under Column. A steel grillage is a common device for distributing a heavy column load over an area of concrete, for example, at the top of a caisson. The bearing on concrete commonly is limited to 600 lb. per sq. in. The proper arrangement is to let the column bear on a thick steel slab resting upon a tier of beams as shown in Fig. 117. These beams in turn bear upon

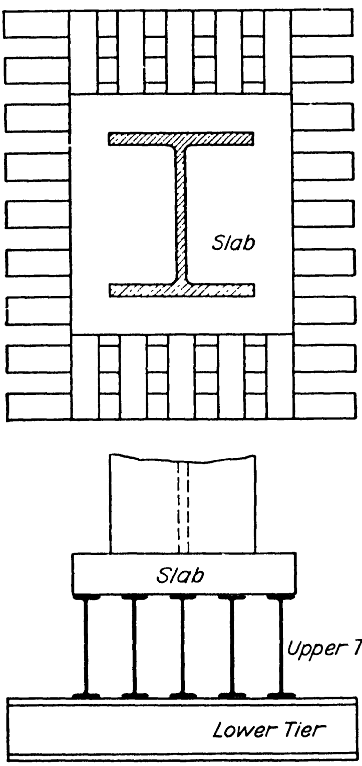


FIG. 117. COLUMN GRILLAGE.

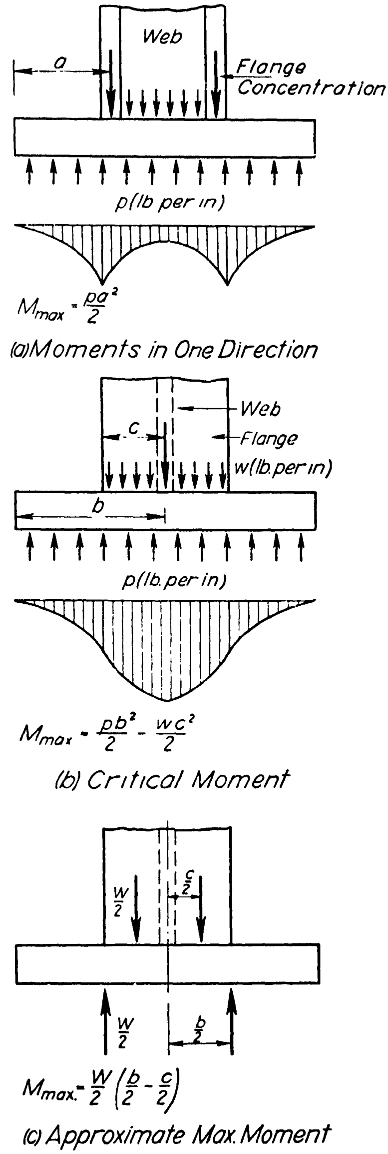


FIG. 118. MOMENTS IN GRILLAGE SLAB.

a second tier at right angles to the first. Hence, the load is finally distributed more or less uniformly over the surface of the caisson or the foundation soil as the case may be.

DESIGN OF A GRILLAGE, *DP*44. The projection of the slab beyond the face of the column section acts as a *cantilever*, but the bending moment often is greater than this would indicate. Based upon the assumption that the pressure of the column section on the grillage slab is uniform, we find that the moment diagram of Fig. 118(*b*) is theoretically correct. In this figure, p represents the bearing pressure in pounds per lineal inch across the grillage slab and w is the pressure per lineal inch produced by the two column flanges. The maximum moment clearly occurs along the center line directly under the web of the column as shown in Fig. 118(*b*). This moment can be reduced by the use of welded stiffeners or distributors as shown on *DS*44 since this increases the value of w and *decreases the web concentration to a negligible value.* The resultant moment is as indicated in Fig. 118(*c*) when distributors are used as in *DS*44.

FLOOR DESIGN

134. Building Floors. The framing of building floors is essentially the same whether the floor is for an office building, a warehouse, or an industrial shed. The usual method is to frame girders between the columns in one direction, and then to connect floor joists perpendicularly between the girders. For low structures where the connections do not

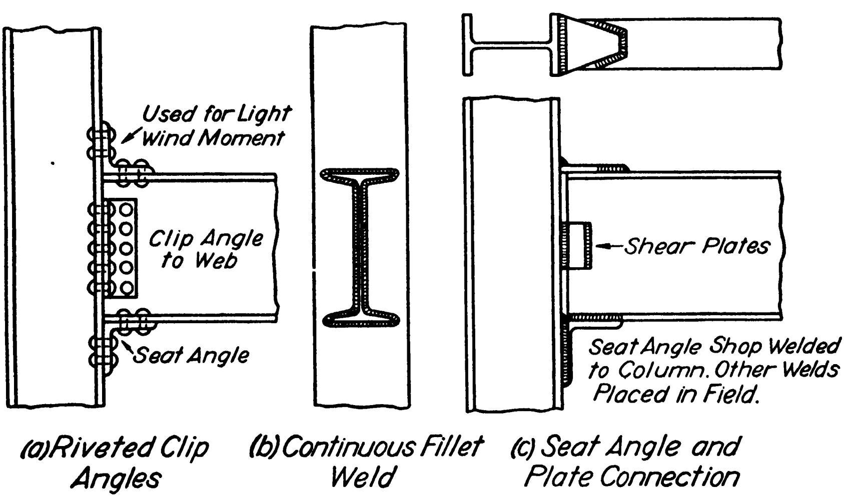


FIG. 119. CONNECTIONS FOR BEAMS AND GIRDERS.

have to resist wind moments, they are usually formed of ordinary clip angles to the web as shown in Fig. 119(*a*). Welded connections may be made by direct welding, or *seat angles* and *plates* may be used (see Fig. 119(*b*) and (*c*)). In order to provide field adjustment for plumbing the columns, a connection of the type (*c*) is necessary at one end of the girder when a direct weld as in (*b*) is used at the other end.

DP44. *Design a steel grillage for the support of a building column which is a 14×16WF426 section stressed to 15,000#/□″. Use AISC spec.*

Load on grillage = 125.25 × 15,000 = 1,880,000#.

Area for bearing on concrete $= \dfrac{1,880,000}{600} = 3140$□″, *i.e., 52″ × 60″.*

Slab Selection:

Try a slab 28″ × 30″.

Unit bearing under slab $= \dfrac{1,880,000}{28 \times 30}$

$= 2240$#/□″.

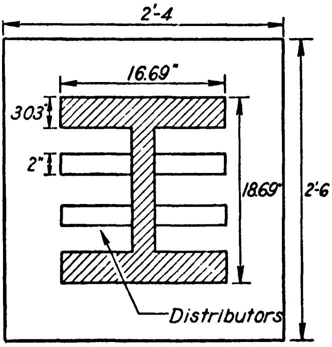
Bending moment acting on long dimension of slab =

$$\frac{1,880,000}{2 \times 30}\left(\frac{28.0}{4} - \frac{16.69}{4}\right)$$

$= 88,500$″#/″.

Required depth $= \sqrt{\dfrac{6 \times 88,500}{20,000}}$

$= 5.15$″; *use* $5\frac{1}{2}$″.



Upper Beam Tier; Length 60″:

For three beams the moment per beam will be

$$\frac{1,880,000}{3 \times 2}\left(\frac{60}{4} - \frac{30}{4}\right) = 2,350,000''\#.$$

$I/c = 2,350,000 \div 20,000 = 117.5.$

Web thickness for bearing $= 1,880,000 \div 3 \times 30 \times 24,000$

$= 0.87$″; *use 20″I @ 100#/′.*

$I/c = 164.8;\ t = 0.87.$

Lower Beam Tier; Length 52″:

For six beams the moment per beam will be

$$\frac{1,882,000}{6 \times 2}\left(\frac{52}{4} - \frac{28}{4}\right) = 941,000''\#.$$

$I/c = 941,000 \div 20,000 = 47.0.$

Web thickness for bearing $= 1,882,000 \div 6 \times 28 \times 24,000$

$= 0.47$″; *use 12″I @ 45#/′.*

$I/c = 47.3;\ t = 0.56.$

Web Buckling: *These beam webs will buckle unless the grillage is poured solid with concrete. The web thickness would need to be greatly increased to be self-supporting.* *See § 132.*

Load Distribution. The load carried by a floor joist usually is taken to be uniform and is composed of (1) its own dead weight, (2) the dead load of the floor for a width equal to the distance between joists, (3) the live load from the same tributary area, and (4) an allowance for partitions. A girder must carry a uniform load caused by its own dead weight and the dead load and live load from the floor directly above its upper flange. It also must carry a set of load concentrations from the reactions of the floor joists. In practice, it is customary to consider all loads except the dead weight of the girder itself to be concentrated at the joist connections. If the joists are quite close together and the span of the girder is reasonably great, so that there are at least four interior joists per panel, the joist reactions to the girder may be considered as a uniformly distributed loading. The error in moment caused by this approximation will be not more than 4 per cent.

Materials. The dead load of the floor must be computed from the approximate weights of the materials to be used. Most industrial building floors are constructed with reinforced concrete slabs. A thickness of 1 in. per ft. of clear span is usually sufficient for a reinforced concrete floor slab, but the minimum overall thickness is about 4 in. A floor surface over the slab may or may not be used. *Precast floor slabs* as thin as 2 in. are available in patented constructions. The use of slag or other light aggregate for slabs is a desirable feature of precast slab construction. The weight is 100–110 lb. per cu. ft.

Another type of floor construction is known as the *battledeck* floor. It consists of flat plates welded together over the beams. For warehouse construction, it is possible to use this floor without any covering, but it is desirable, in order to reduce noise, to cover the plates either with a mastic or a cork composition covering. The battledeck floor combines light weight with low cost but *it is not fireproof.*

Live Loadings. The live load varies considerably, but it is commonly taken at 100 lb. per sq. ft. or less, where no heavy concentrations of machinery or other such loads exist. Store-room floors are designed to carry loads from 100 to 400 lb. per sq. ft. or even more. The live load is also assumed to include any required allowance for impact. For this reason the live load is taken as 100 lb. per sq. ft wherever the gathering of a crowd of people seems probable, although a crowd of people will seldom produce a *static* load of over one half of this amount. Care must be used in load selection. The writer once was asked to explain floor cracks in a meat storage room. The stacked frozen meat was found to weigh 500 lb. per sq. ft., which was five times the design load. At another time he observed bags of Portland cement stacked nine sacks deep on a light wood floor. It was more difficult to explain the lack of a complete collapse than the evident signs of distress.

Load from Crowds. There has been considerable discussion in regard to the load produced by a crowd of people. Tests have shown that a static live load of over 150 lb. per sq. ft. can be produced by selecting individuals and packing them together in a small pen. However, such congestion could only occur in halls, ramps, and other such passages. Although the writer is not able to offer specific data, he does not believe that an unselected crowd of men and women would ever produce a load much in excess of 100 lb. per sq. ft. Incidentally, it seems unlikely that such a dense crowd could produce an appreciable impact on the structure. A more serious live loading may conceivably develop from a less congested group where rhythmic movement is possible. For instance, a football crowd, swaying, cheering and rising in unison might produce reasonably heavy dynamic stresses in the structure. Where seats are provided for a crowd, the static loading will not exceed 50 lb. per sq. ft., and a design load of 100 lb. per sq. ft. allows for impact of 100 per cent. Tests have not indicated dynamic effects of this magnitude.

135. Design of a Floor for an Industrial Building.

PROBLEM. Design the steel framing for supporting the second floor of an industrial building. The floor is supported by the exterior columns and by one line of columns down the center of the building.

Data.

Width of building = 60 ft. center to center of outside columns.

Width of bay or distance between columns = 15 ft.

Live load. The live load may be taken at 75 lb. per sq. ft. since the floor is to be used for a drafting room and office.

Columns. All columns are of 12-in. width in a direction across the building.

Working Stresses.

Bending. 20,000 lb. per sq. in. for fiber stress in beams.

Other working stresses. *AISC* specifications for structural steel for buildings, § 215.

Type of Floor. A " battledeck " floor will be used. This is a floor composed of steel plates welded together over the beams. Plate of $\frac{3}{8}$-in. thickness will be used and the joists will be placed 7 ft.-6 in. apart. A mastic surface and a cork composition floor covering will be selected weighing 20 lb. per sq. ft. Girders frame between columns at 15-ft. spacing and 29-ft. clear span to support joists at 7 ft.-6 in. centers.

Design of a Floor Joist.

Dead load. The $\frac{3}{8}$-in. plate weighs 15 lb. per sq. ft. which added to the weight of the floor covering makes the total weight of the floor 35 lb. per sq. ft. The dead weight of a floor joist will be estimated at 20 lb. per lineal ft.

Uniform load per foot of joist.

$$\begin{aligned} \text{Dead load} &= (7.5 \times 35) + 20 = 282 \text{ lb. per lineal ft.} \\ \text{Live load} &= (7.5 \times 75) \qquad\;\; = \underline{563} \text{ `` `` `` ``} \\ &\qquad\qquad\qquad \text{Total} = 845 \text{ `` `` `` ``} \end{aligned}$$

Bending moment = $\frac{1}{8} \times 845 \times 15^2 \times 12 = 285{,}000$ in-lb.

Required section modulus = $285{,}000 \div 20{,}000 = 14.3$.

Section selected. The lightest standard beam that offers this section modulus is the 8*WF*19 section. However, a depth of not less than $\frac{1}{20}$ of the span is commonly required for rolled beams to prevent excessive deflection. The allowable deflection is $\frac{1}{360} \times 15 \times 12 = 0.5$ in. The deflection of an 8*WF*19 section for a uniform load of 845 lb. per ft. is 0.49 in. This will be acceptable. Hence, although from a consideration of the interaction of beam and plate, which form a T-section, we might have expected to reduce the weight of the joist, actually deflection fixes the minimum section.

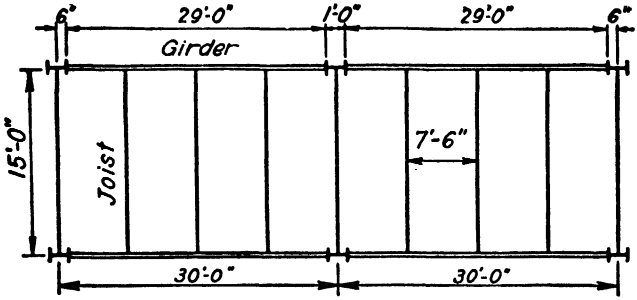


FIG. 120. FLOOR FRAMING.

Connection Between Joist and Girder.

Shear = 845 × 15 ÷ 2 = 6350 lb.

Standard connection. The standard connection for an 8*WF*19 beam, coped to place the beam and girder flanges on the same level, is shown in Fig. 121. The connection is amply strong with ¾-in. rivets.

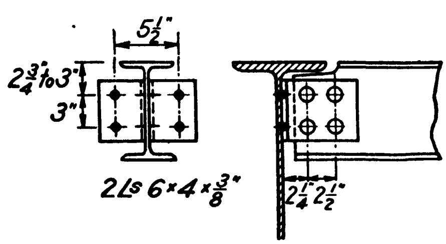


FIG. 121. JOIST CONNECTION TO GIRDER.

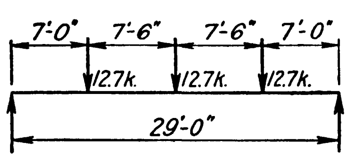


FIG. 122. GIRDER LOADING.

Design of the Girder.

Loads. The girder carries 3 load concentrations of 12,700 lb. at 7 ft.-6 in. centers. (See Fig. 122.) It also supports its own dead weight which is estimated at 60 lb. per ft.

Bending moment. The bending moment is found by taking moments about the center of the beam; $M = 12{,}700 \times 1.5 \times 14.5 - 12{,}700 \times 7.5 + \frac{1}{8} \times 60 \times 29^2 =$ 187,000 ft-lb. (See Fig. 122.)

Section modulus = 187,000 × 12 ÷ 20,000 = 113.

Section selected. A 21*WF*59 beam will furnish a modulus of 119.3, which is satisfactory. This beam also meets the requirements as to depth ratio, i.e., $\frac{1}{20}$ of the span or 17.4" is the minimum allowable depth. It is the most economical section obtainable.

Connection Between Girder and Column.

Shear = 1.5 × 12,700 + 14.5 × 59 = 19,900 lb.

Standard Connection. The standard connection for a 21*WF*59 beam is shown in Fig. 123. The critical condition is that of bearing on the web. This allowable value in bearing is 58,500 lb. with ¾-in. rivets (eccentricity being neglected). The connection, therefore, is amply strong.

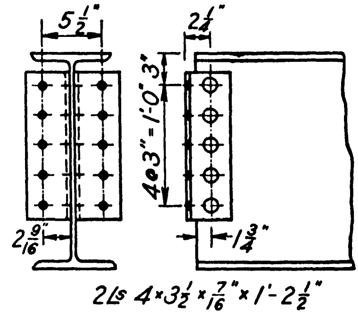


FIG. 123. GIRDER CONNECTION TO COLUMN.

REMARKS. In the design of these beams, it was not necessary to reduce the working stress because of lack of lateral support for the top flange. The floor plates are welded together over the beams and are also welded to the beams. This *stiff plate prevents lateral buckling* of the top flange of the beam. If a concrete floor slab had been used, the top flange of the beams would have been allowed to project ½ in. into the concrete to obtain lateral stiffness. These remarks also apply to the girders.

Spandrel beams and girders are those placed along the side of the building at the wall. A spandrel beam or girder receives but one half of the *dead and live* floor load that an interior beam or girder must carry. However, in most cases the masonry walls are merely enclosures supported by the structural frame of the building. In such construction the spandrel girder must carry a full story of the wall load. This type of construction is always used for tall buildings.

PROBLEMS

163. Redesign the floor system designed in § 135 to carry a concrete floor slab of 4-in. total depth. Place the floor joists at 5 ft.-0 in. centers. Allow 12 lb. per sq. ft. for a wood floor.

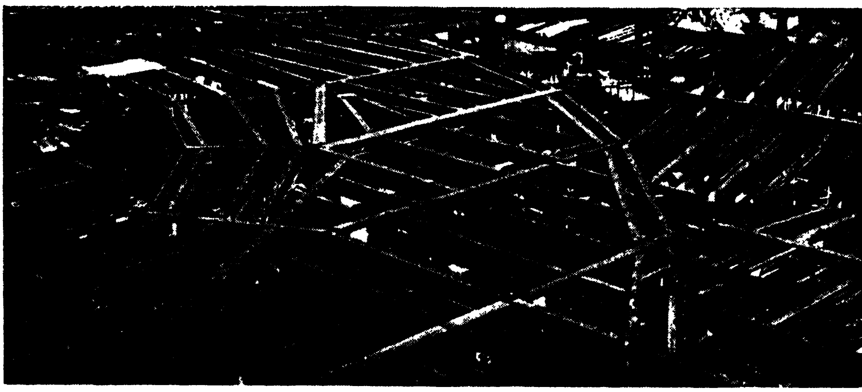
164. Design a floor panel for a warehouse where the columns are at 20-ft. centers in both directions. Place the beams 6 ft.-8 in. apart and use a concrete slab of 6-in. overall depth. The live load is 300 lb. per sq. ft. Columns are 14-in. sections at 87 lb. per ft. Use *AISC* specifications.

165. Design the floor framing for a 15-ft. balcony. The main girders are placed perpendicularly to the wall and are supported at one end by the wall and at the other end by hangers attached to the roof trusses. The roof trusses are spaced on 16-ft. centers. Beams may be spaced from 4 to 6 ft. apart. Take the live load at 100 lb. per sq. ft. and the dead load of the floor at 60 lb. per sq. ft., not including the weight of the beams or girders. Use *AISC* specifications.

166. A one-story industrial building has plan dimensions of 40 ft. × 60 ft. The flat roof of the building supports a condenser box which carries a 10-ft. depth of water. The condenser box is made up of $\frac{5}{16}$-in. plate supported by beams at 45-in. centers or less. The condensers themselves are carried directly to the columns by steel framing *inside* of the box. Design the roof framing of the building to support the load of water. Place columns as needed, but obtain as much clear space as is reasonably possible. Use working stresses as given in the *AREA* specifications.

136. Bridge Floors. The floors of highway bridges are similar in their framing details to building floors. Usually, floor beams are supported by

the trusses at panel points, and stringers are framed between the floor beams just as the joists are framed between the girders in a building floor. The concrete floor slab must be made thick enough to span across the stringers and carry the wheel loads. A depth to the reinforcing steel equal in inches to the span between stringers in feet, but limited to a minimum depth of 5 in., is usually adequate. Highway bridge floors are designed to carry two 15-ton or two 20-ton trucks plus impact of about 30 per cent.



Courtesy Eng. News-Record.

FIG. 124. GRADE SEPARATION — HENRY HUDSON PARKWAY.

Railway bridges may be constructed with open floors or with ballasted floors. An open floor consists of floor beams with two or more stringers that support the ties directly. A floor designed to carry ballast must be covered with steel plates or with a concrete slab, upon which the ballast is placed to support the track. The stringers and floor beams must be designed for the engine loads plus a heavy allowance for impact. The impact allowance for a stringer or floor beam may be nearly 100 per cent. Impact is reduced by the use of ballast.

137. Design of a Floor for a Highway Bridge.

PROBLEM. Design the stringers and floor beams for a through-truss highway bridge of Pratt type. Design the connections between the stringers and the floor beam, and between the floor beam and the vertical post.

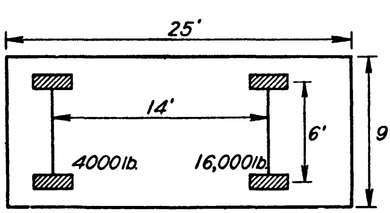


FIG. 125. STANDARD 20-TON TRUCK.

Data.

Panel length = 16 ft.-0 in.
Width of clear roadway = 20 ft.-0 in.
Distance face to face of posts = 21 ft.-9 in.
Width of curb = 9 in.
Loads. Two 20-ton trucks of the type shown in Fig. 125.
Impact = 30% of the live load.

Working Stresses.

Bending on extreme fiber of rolled beams = 18,000 lb. per sq. in.
Shear on field rivets = 11,000 lb. per sq. in.; on shop rivets = 13,500.
Bearing on field rivets = 22,500 lb. per sq. in.; on shop rivets = 27,000.

Specifications. *AASHO* specifications as given in § 216.

Layout of Floor. The outside stringers must be placed as shown in Fig. 126 in order to support the curb properly. The distance between outside stringers is divided equally into 5 spaces which makes the distance between stringers equal to 4 ft.-1 in. The thickness of slab of 6 in. overall is more than adequate to carry the truck wheels when the stringer spacing is 4 ft. A bituminous covering 2 in. thick will be used as a wearing surface.

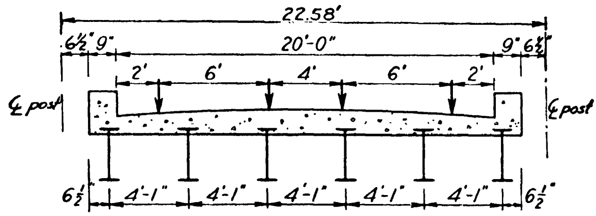


FIG. 126. STRINGER SPACING.

Dead load. The dead load of the slab and covering will be taken at 100 lb. per sq. ft. The weight of a stringer is estimated to be 40 lb. per lineal ft. Total dead load per foot carried by the stringer is 448 lb.

Stringer Selection.

Dead load moment = $\frac{1}{8} \times 448 \times 16^2 \times 12 = 172{,}000$ in-lb.

Placing wheel load for maximum moment. In this case the rear wheel should be placed at the center of the stringer. Following Spec. 66, the proper load is $\frac{4.08}{4.5} \times 16{,}000 = 14{,}500$ lb. This load is increased by $\frac{50}{16 + 125} = 36$ per cent for impact. Whence, $1.36 \times 14{,}500 = 19{,}700$ lb. (Spec. 62.)

$$\text{Live load moment} = \frac{19{,}700 \times 16 \times 12}{4} = 945{,}000 \text{ in-lb.}$$

Section modulus required = $(945{,}000 + 172{,}000) \div 18{,}000 = 62.1$.

Section selected. The $16WF40$ section furnishes a modulus of 64.4. The web thickness of 0.307 in. is barely satisfactory. (Spec. 86.) If the stringers were to be used for a bridge to serve as an overpass above railroad tracks where excessive corrosion should be expected, the $16WF50$ section might be selected. The web thickness is 0.38 in. for this beam.

End shear. The end shear is a maximum when the rear wheel rests at the end of the stringer. No distribution to other stringers is allowed. (Spec. 65.) The rear wheel load of 16,000 lb. is increased 36 per cent for impact and also by a small carry-back shear from the front wheel.

End shear = $(16{,}000 + \frac{2}{16}\,4000)\,1.36 + 448 \times 8 = 26{,}000$ lb.

Allowable end shear for beam web = $16 \times 0.307 \times 11{,}000 = 54{,}000$ lb.

Connection Between Stringer and Floor Beam. The standard connection (*B*-Series) for a 16*WF*40 beam is shown in Fig. 127(*a*). This connection is weakest in bearing on the stringer web. The value of four ¾-in. shop rivets in bearing on a 0.307-in. web is $4 \times 0.75 \times 0.307 \times 27{,}000 = 24{,}800$ lb. which is not sufficient. A 5-rivet connection as shown in Fig. 127(*b*) must be used. Its value in bearing on the web is 31,000 lb.

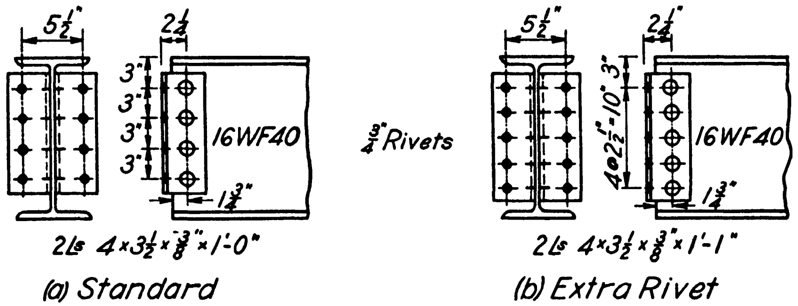


FIG. 127. STRINGER CONNECTIONS.

Design of the Floor Beam.

Dead load. The floor beam must carry a full panel of dead load, and, because this load is concentrated at 6 points by the stringers, it may be distributed uniformly without serious error. The uniform load caused by 16 ft. of slab is 1600 lb. per ft. The uniform load caused by the stringers may be taken as $(16 \times 40) \div 4.08 = 157$ lb. per ft. The weight of the floor beam itself is estimated to be 100 lb. per ft., which makes the total dead load 1857 lb. per lineal ft. This dead load will be taken as extending over the entire span of the floor beam from center to center of trusses, which will compensate for the neglected weight of the curb. (The span of a floor beam is always taken as center to center of trusses. Thus we add to the 20-ft. width of the roadway, 18 in. for curbs, 3 in. for clearance, and 10 in. for the post, to obtain a total of 22.58 ft. as in Fig. 126.)

$$\text{Dead load moment} = \tfrac{1}{8} \times 1857 \times 22.58^2 \times 12 = 1{,}420{,}000 \text{ in-lb.}$$

Placing wheel loads for maximum moment. The rear wheels of the trucks must be placed directly over the floor beams. The reactions will be increased for impact and for the small *carry-back reaction* of the front wheels. This reaction is $(16{,}000 + \tfrac{2}{16}\, 4000)\, 1.36 = 22{,}500$ lb. Laterally, the truck concentrations will be placed as shown in Fig. 126. Each truck is placed symmetrically in its own traffic lane. The loads could be moved laterally to produce *exact maximum moment*, but this procedure seems unjustified since the possibility of obtaining the condition of two heavily loaded trucks passing each other with all four rear wheels over a floor beam in the exact position for maximum moment and at the correct speed to produce maximum impact is so remote as properly to be neglected.

Live load moment at center of 22.58-ft. span $= (45{,}000 \times 11.29 - 45{,}000 \times 5)12 = 3{,}400{,}000$ in-lb.

Total bending moment $= 3{,}400{,}000 + 1{,}420{,}000 = 4{,}820{,}000$ in-lb.

Required section modulus $= 4{,}820{,}000 \div 18{,}000 = 268$.

Section selected. The lightest weight beam that will furnish this section modulus is the 27*WF*106 which has a modulus of 277.2. Shallower and heavier sections would save head room. Possible second choices are 24*WF*110, or 21*WF*122.

End shear of floor beam. The maximum end shear is obtained for the same placing

of loads as for maximum moment, Fig. 126. (Again the loads are not crowded to one side of the bridge because of *improbability.*)

Shear = 45,000 + 1857 × 11.29 = 66,000 lb.

The unit shear in the web is 66,000 ÷ (27.1 × 0.535) = 4600 lb. per sq. in.

Standard End Connection. The standard connection (*B*-Series) for a 27-in. beam is shown in Fig. 128. There are seven ¾-in. rivets in bearing through the web and 14 rivets in single shear between connection angles and post. The web thickness is 0.535 in. The 14 rivets in single shear at 11,000 lb. per sq. in. for field rivets (Spec. 70) have a value of 14 × 4840 = 67,700 lb. The 7 shop rivets in bearing on the web have a value of 7 × 0.535 × 0.75 × 27,000 = 76,000 lb. Shear controls the design and the connection is adequate. (See § 33 for an exact computation of rivet stress in a standard connection.)

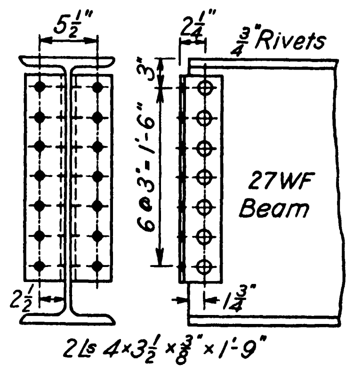


Fig. 128.
Floor-Beam Connection.

Remarks. In the design of both the stringer and the floor beam above, the estimate of the dead weight of the beam itself was sufficiently close to the true weight so that a revision of calculations was unnecessary. If the estimate of weight is not in error by more than 25 per cent, the error may usually be neglected without affecting the design. The net change in the modulus of the floor beam in the above example would have been only 1.0 if its weight had been estimated 25 per cent too low or too high.

The exterior stringers carry a smaller amount of load than the interior stringers. However, they are subject to a heavy impact shock from the possible effect of a truck hitting the curb. It is common practice to use the same size of stringer at the edge as under the interior of the slab.

PROBLEMS

167. Repeat the design of the bridge floor designed in § 137 by making use of two 15-ton trucks as the live load. A 15-ton truck is assumed to have the same dimensions as a 20-ton truck, but the wheel loads are ¾ as great.

168. Determine the sizes of stringers and floor beams for a low-truss highway bridge where the panel length is 14 ft.-2 in. and the width of roadway between curbs is 18 ft. Assume the distance center to center of trusses to be 20 ft.-10 in. Use a 6-in. total depth of slab without wearing surface and place the stringers as near as possible to 3 ft.-6 in. apart. Use two 20-ton trucks for live load. Follow the *AASHO* specifications.

169. Determine the required size of an interior floor beam for a low-truss highway bridge of Warren type where the panel length is 8 ft.-0 in. The slab is 10 in. deep overall and is not covered. No stringers are used; instead, the slab spans between floor beams. The width of roadway is 22 ft. and the distance center to center of trusses is 23 ft.-10 in. Use two 20-ton trucks for the live load. Determine the required weight for an end floor beam. Use the same depth as for an interior beam and allow for 60 per cent impact. Follow the *AREA* specifications.

170. Design the stringers and floor beams for a single-track railway bridge with through girders. The ties rest directly upon the stringers. Allow 500 lb. per ft. for the dead weight of the track (10 × 10 ties, 10 ft. long, at 14-in. centers; 6 × 8 guard rails;

two 100-lb. rails). The stringers are placed 7 ft.-0 in. apart and the girders are 16 ft.-0 in. on centers. The distance between floor beams is 13 ft.-6 in. Make your design conform to the *AREA* specifications as given in § 218. Use Cooper's *E*-60 live loading and allow for the proper amount of impact.

Strengthening Old Structures

138. Design of a Girder Made of a Strengthened Section. Occasionally it is necessary to strengthen a rolled beam by use of cover plates to increase its resistance to moment; or, by use of side plates or stiffener angles to increase its resistance to shear or buckling. In designing a new riveted structure, we would connect strengthening plates to the beam by rivets, while in a welded structure or in an old riveted structure which needed strengthening, the connection would be made by welding.

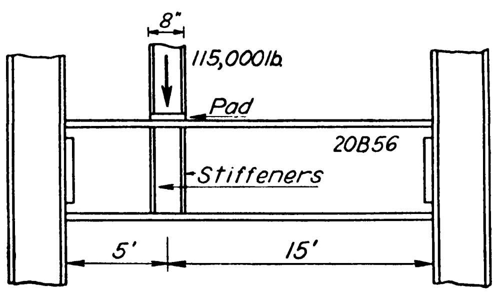


Fig. 129. Strengthened Girder.

Problem. Design a welded building beam to carry a concentrated column load at its quarter point. A used 20-in., 56-lb. Bethlehem beam is available. It must be strengthened and re-used. The upper flange is unsupported laterally except at the ends and at the concentrated load. Section modulus of the 20*B*56 section is 109.7.

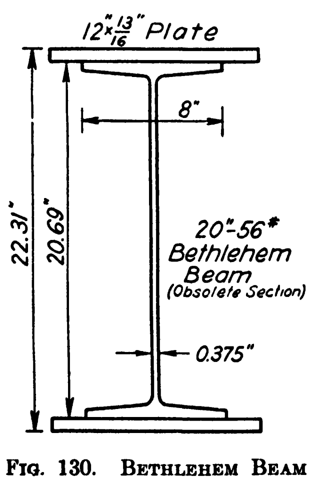


Fig. 130. Bethlehem Beam Section.

Data.

Section. Old style 20-in., 56-lb. Bethlehem beam (see Fig. 130).

Span of beam = 20 ft.-0 in.

Concentrated load at quarter point = 115,000 lb.

Working stresses (reduced for second-hand material).

Flexure of rolled shapes = 18,000 lb. per sq. in.

Shear on webs of beams = 12,000 lb. per sq. in.

$$\text{Compression in beam flange} = \frac{20{,}000}{1 + \dfrac{L^2}{2000b^2}}$$

Compression in web = 15,000 lb. per sq. in.

Shear on welds = 11,300 lb. per sq. in. on throat of fillet.

Specifications. *AISC* and *AWS* specifications except for working stresses.

Design of Cover Plates.

Width. The cover plate will be made 4 in. wider than the flange of the girder, or 12 in.

Allowable flange stress. The stress must be reduced to allow for the buckling tendency of the unsupported compression flange for a length of 15 ft. (Spec. 10.)

$$\text{Allowable stress} = \frac{20{,}000}{1 + \dfrac{180^2}{2000 \times 12^2}} = 17{,}900 \text{ lb. per sq. in.}$$

Bending moment $= (115{,}000 \times {}^{15}\!/_{20} \times 5) + 4500 = 436{,}000$ ft-lb.

The 4500 ft-lb. is the moment at the quarter point caused by the estimated dead load of 120 lb. per ft.

$$\text{Section modulus required} = \frac{436{,}000 \times 12}{17{,}900} = 293.$$

Modulus furnished by 20*B*56 section = 109.7.

Modulus required in plates = 293.0 − 109.7 = 183.3 approximately.

Approximate area required in plates = 183.3 ÷ 10 = 18.3 sq. in. (Try two 12 × ${}^{13}\!/_{16}$-in. covers furnishing 19.5 sq. in.)

Required thickness of cover plates. The moment of inertia of two 12 × ${}^{13}\!/_{16}$-in. cover plates spaced 20.69 in. apart is 2260. This value is added to the moment of inertia of the rolled section. $I = 2260 + 1135 = 3395$. The section modulus is 3395 ÷ 11.15 = 305. This is slightly greater than the modulus required. The actual weight of the section is 124 lb. per lineal ft.

Maximum end shear = ¾ × 115,000 + 1240 = 87,400 lb. The figure 1240 is the end shear caused by the weight of the beam at 124 lb. per ft.

Welding on the Cover Plates. The cover plate will be welded to the edge of the flange by means of a ${}^{5}\!/_{16}$-in. intermittent fillet. The value of this fillet in shear is 2500 lb. per lineal in., or, for two fillets, 5000 lb. per lineal in. The actual horizontal shear per lineal inch is

$$\frac{VA'\bar{y}}{I} = \frac{87{,}400}{3395}(9.75 \times 10.75) = 2700 \text{ lb.}$$

Use 3-in. lengths of intermittent welds spaced 2½ in. in the clear or 5½ in. on centers.

Web Shear.

$$s_s = \frac{87{,}400}{0.375 \times 20.69} = 11{,}300 \text{ lb. per sq. in.}$$

This shear is below the maximum of 12,000 allowed for a web not more than $70t$ in depth. 70 × 0.375 = 26.2 in. Accordingly, side plates will not be needed to increase the thickness of the web.

Stiffeners at Load. Minimum effective length of web to resist vertical compression = $0.5d + 8 = 18.35$ in. (See Fig. 129.)

Effective bearing area = 18.35 × 0.375 = 6.9 sq. in.

Required area = 115,000 ÷ 15,000 = 7.7 sq. in.

Use 4 stiffener plates 4 × ⅜ in. under flanges of column.

Welds on stiffeners may be ${}^{5}\!/_{16}$–2–4 (2-in. welds on 6-in. centers). Value = 130k.

Riveted Construction. The design of the girder might have been carried out for riveted construction. The main difference would be that the rivets through the cover plates and flanges would reduce the net section of the tension flange and probably make it

control the design. Also, the stiffeners would be changed to angles riveted to the web with ¾-in. rivets through a 2½-in. or preferably a 3-in. leg. In all cases it is necessary to *mill* the stiffeners to *bear* on the loaded flanges or else to *weld* the two together. Only the outstanding legs of stiffener angles bear on the flanges since the other legs must be cut short to clear the fillet between the flange and the web of the beam. The stiffener angles must be checked for bearing on the outstanding legs.

PROBLEMS

171. Redesign the strengthened beam of § 138 using riveted cover plates and riveted stiffener angles. Allow 30,000 lb. per sq. in. for bearing on outstanding legs of stiffener angles. Deduct all rivet holes when calculating the tensile fiber stress.

172. Redesign the strengthened beam from § 138 changing the load to 150,000 lb. Use 2 cover plates on each flange. Make the second cover plate only 10 in. wide and weld it to the first 12-in. plate. It will be necessary to use welded side plates to take care of the excess end shear. Are these side plates needed for the full length of the beam?

173. A 30*WF*180 beam section has been specified for use on a certain job. By mistake a 30*WF*124 section was shipped to the field. The web thickness is satisfactory, but the section modulus of the larger beam is required. Design cover plates to be welded to the flanges to reproduce the modulus of the 180-lb. beam. Use working stresses recommended by the *AISC* and *AWS* specifications.

174. Rework Problem (173) using shop riveted cover plates. Make use of the *AASHO* specifications and working stresses. The cover plates should be made only long enough to meet the requirements of a uniform loading.

139. Tables for Beam Design. Every structural handbook contains certain tables for aid in beam design. If used properly, such tables are valuable, but they are not helpful as devices to avoid the logical thinking that has been developed here. The results are usually unfortunate when such tables reach the hands of unqualified designers. Standard tables that give *section properties* for designing and detailing are the basic ones. The data presented are: *weight* per foot, *area* of section, *depth* of section, *flange width* and *web thickness,* along with *moments of inertia, section moduli,* and *radii of gyration* about the two principal axes. All of the *dimensions* that may be necessary for detailing are also given. The second table in point of usefulness is the economy table in which all available beam sections are arranged in the order of increasing weight for providing given section moduli. The use of this table aids us in the selection of the proper section for economy, or, if we choose a shallower section of heavier weight, the excess cost is readily evaluated. The third table considered is one that gives all data on standard beam connections. This table is useful, but the mistake must not be made of accepting the listed connection values unless the specifications to be used are exactly those for which the table was arranged.

Safe Load Tables. The final table that needs consideration is the table of allowable loads given in most steel handbooks. The tables in the handbook of *Steel Construction* (*AISC*) are typical. They give the *total allow-*

able load in kips (uniformly distributed) for different spans and for simple reactions. Additional data are: *deflection* in inches, allowable *total shear* on the web, allowable *end reaction,* and *length of bearing* to develop the allowable web shear. The use of such tables is an aid to the designer who has need for them daily but they may be of little value to others. They must be used with these points in mind.

1. The allowable load is uniformly distributed. The beam is simply supported.
2. The allowable load is based upon a specified working stress in flexure which may not be the one we wish to use.
3. The deflections listed are for uniform loads and simple supports.
4. The allowable web shear is based upon a fixed allowable unit shearing stress. Diagonal buckling is not considered since $h/t < 70$.
5. The allowable end reaction is obtained for a 3½-in. length of end bearing and for resistance to crimpling based upon a specified bearing stress. There is no consideration of possible vertical buckling.
6. The values of standard end connections neglect the moment of eccentricity and are based upon one set of allowable stresses in shear and bearing.

In other words, a set of safe load tables is made up to meet the requirements of one set of specifications. Such tables must be corrected before they can be used with other specifications.

CHAPTER 9

COMBINED DIRECT STRESS AND FLEXURE

140. Members that Resist Direct Stress and Flexure. All truss members which lie in a horizontal or inclined plane must resist a bending moment caused by their own weight. All such members, therefore, resist direct stress and flexure. Flexure is of less importance in the design of tension members than compression members because the direct pull of a tension member tends to reduce lateral deflection while the thrust of a compression member tends to increase lateral deflection. The result for the compression member is that the applied centric force becomes slightly eccentric. This is one of the reasons why specifications limit the slenderness ratio of compression members and struts below the limit for tension members. Whenever possible, a compression member should be designed sufficiently stiff so that the lateral deflection caused by its own weight will be negligible. Tension members should be designed with a large enough moment of inertia about the horizontal axis so that the flexural stress caused by dead load bending moment may be neglected. The specifications regarding slenderness ratio are usually such that the dead load flexure need not be considered.

Some members of trusses, particularly of roof trusses, must actually be designed to carry transverse loads. For instance, purlins are frequently placed away from the panel points of a roof truss with the result that the top chord must act as a beam between panel points. Occasionally it is necessary to hang light fixtures or shafting from the lower chord of a roof truss away from a panel point. In such instances the member must be designed so that the fiber stress for combined direct stress and flexure will not exceed a proper allowable stress in tension or compression. The *AISC* Code limits the working stress to an intermediate value when the allowable stresses in direct stress and flexure are unequal. (Spec. 4.)

Columns may undergo flexure from the effect of an eccentric load or from the influence of wind shear. Building columns frequently support beams attached by clip angles to their flanges. The beam reaction has an eccentricity of at least one half of the width of the column. The portals of bridges and the transverse bents and portals of mill buildings resist heavy bending moments caused by wind shear. These wind moments must be considered in the design of the columns.

141. Flexure of a Diagonal Member of a Bridge Truss Caused by Its Own Weight.

PROBLEM. Compute the bending stress in a diagonal member of a low truss Warren highway bridge caused by its own dead weight.

Data.

Panel length = 15 ft.-0 in.
Length of diagonal = 18 ft.-0 in.
Section. Two 6 × 4 × ⅜-in. angles with short legs turned in and laced together. (See Fig. 131.)

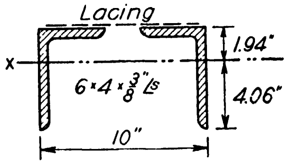


FIG. 131. CROSS-SECTION.

Weight of Member.

Weight of angles = 2 × 12.3 × 18 = 442 lb.
Weight of lacing bars and tie plates (assumed) = 70 lb.
Total = 512 lb.

Bending moment = ⅛WL = ⅛ × 512 × 15 × 12 = 11,500 in-lb.

Properties of the Section.

Moment of inertia = 13.5 × 2 = 27.0 in.^{4}
Radius of gyration = 1.93 in. (about x-x axis).
Slenderness ratio = (18 × 12) ÷ 1.93 = 112.

Flexural Stress.

$$S = \frac{Mc}{I} = \frac{11{,}500 \times 4.06}{27.0} = 1730 \text{ lb. per sq. in.}$$

REMARKS. The bending moment computed above is high for two reasons. The gusset plate cuts down the length of the member for flexure and it also partially fixes the end of the member so that the bending moment is reduced below $WL/8$. The stress of 1730 lb. per sq. in. is not excessive and may be neglected unless the member also carries a high secondary flexural stress. A provision for a flexural stress of 30 per cent of the primary stress is made in setting the working stress.

THEORY OF COMBINED ACTION

142. Tension or Compression with Flexure. The simplest possible approach to this problem is to add the two fiber stresses as defined by the direct stress formula and the flexure formula. Thus we obtain

$$f = \frac{P}{A} + \frac{Mc}{I}. \tag{1}$$

This equation expresses the combined stress correctly, but there may be some question, especially for columns, as to what limitation should be placed upon the combined fiber stress. Some specifications (*AREA* for example) state quite plainly that the combined fiber stress shall not be greater than the allowable stress as controlled by the column formula.

For use with such specifications, this formula may be rearranged as follows:

$$f = \frac{P}{A} + \frac{Mc}{Ar^2}, \tag{2}$$

$$A = \frac{P}{f} + \frac{Mc}{fr^2}, \tag{3}$$

$$A = \frac{P + Mc/r^2}{f}. \tag{4}$$

The use of the formula (4) is limited to the case where the same working stress is permitted for direct stress and for flexure. It also neglects the influence of lateral deflection upon flexure stresses. For determining the allowable load P of eccentricity e, we may solve equation (4) by substituting Pe for M. Thus we obtain

$$P = \frac{fA}{1 + ec/r^2}, \text{ and also } A = \frac{P}{f}\left(1 + \frac{ec}{r^2}\right). \tag{5}$$

143. Influence of Deflection. It will be possible to show that the influence of deflection upon the design of members for the combination of direct stress and flexure is negligible. Of course, deflection may be neglected for tension members since its small influence is toward the reduction of stress, but, even for columns, *the actual influence of lateral deflection will be found to be small.*

The moment including the influence of deflection may be written as

$$M_t = M \pm P\Delta. \tag{6}$$

Here, M is the bending moment in the member due in part to eccentricity of the direct load P and including the moment caused by lateral forces. The value of Δ for a uniform lateral load of w is

$$\Delta = \frac{5}{384}\frac{wL^4}{EI} = \frac{5}{48}\frac{ML^2}{EI} = \frac{ML^2}{10EI} \text{ (nearly)}. \tag{7}$$

When the deflection moment caused by the centric load is taken into consideration, we should write

$$\Delta = \frac{M_tL^2}{10EI}.$$

We may substitute this value of Δ in equation (6) to obtain

$$M_t = M \pm \frac{M_tPL^2}{10EI}, \tag{8}$$

or

$$M_t = \frac{M}{1 \pm \dfrac{PL^2}{10EI}}. \tag{9}$$

From this result we may obtain an expression for the area of cross-section by introducing the value of M_t for M in equation (4). The resulting equation is

$$(10) \qquad A = \frac{P + (Mc/r^2) \div \left(1 \pm \dfrac{PL^2}{10EI}\right)}{f} \quad \text{(plus sign for tension member).}$$

Again, this expression is dependent upon the use of a single working stress for direct load and for flexure. Since the factor $PL^2/10EI$ occurs in only one of the two terms, and since it is to be added or subtracted from unity while its value is seldom greater than 0.03, we conclude that it is unlikely to influence the area by more than one or two per cent at most.

144. Different Working Stresses. The working stress for direct stress in a column is controlled by a *column formula* while the working stress for beam compression is either a fixed value or it is controlled by the *beam-flange compression formula.* It is possible in some instances, therefore, to effect a slight economy by the use of two separate working stresses in equation (4). The revised formula becomes,

$$(11) \qquad A = \frac{P}{f_c} + \frac{Mc}{f_b r^2}.$$

Naturally, f_c is controlled by the column formula and f_b is controlled by the beam compression formula.

This procedure is not entirely logical, however. For example, the value of f_b may be controlled by a tendency to buckle in one direction, while f_c may have to be chosen for buckling about the other axis because of the direction of the lateral forces. Also, we know that most column failures start locally, while both f_c and f_b are based upon the action of the column as a whole. Nevertheless, equation (11) is usually considered to be a rational attempt at improved design.

145. Design Procedures. There are two satisfactory design procedures. The experienced designer is most likely to guess at a trial section and then to compute its actual stress by equation (1). He will pass the design if the actual stress approximates the allowable stress defined by the column formula. Otherwise, he will revise the trial section. This procedure is conservative. The second procedure is to obtain the values of A, c, and r for the trial section and then to compute the required area by equation (11). A simple comparison of the trial area with the area needed will indicate the direction for revision. The second design procedure will reach the same result as the first when the allowable stresses are the same for *column action* and *beam action.* When different working stresses seem justified, the use of equation (11) is the logical procedure for design.

Design Problems

146. Design of Truss Members for Direct Stress and Bending. Examples *DP*45, *DP*46. These two problems illustrate the two techniques of design mentioned above. In *DP*45 there is a single working stress because this is a tension member for which the allowable tensile stress is the same as the stress permitted for flexure. Accordingly, the procedure is to choose a section and then to compute the maximum combined fiber stress. The author purposely set the allowable working stress at 22,500 lb. per sq. in. which is higher than the standard working stress in use when this book was written. The designer should not become so accustomed to one set of allowable stresses that change is difficult. In two decades, standard allowable stresses for buildings have risen from 16,000 to 20,000 lb. per sq. in. and they will probably go higher as materials are improved.

The design problem *DP*46 illustrates the design procedure when the allowable working stress for direct load is not the same as for flexure. One area is found for compression and a second area to resist flexure — the total area required being the sum of the two. It must be noted, however, that the calculated area for flexure is dependent upon the properties of the section (r and c) so that *a change of section would vary the required area.* Attention is also called to the fact that the smaller extreme fiber distance must be used since this c-value controls the compression fiber stress. Evidently, the tension fiber stress due to flexure cannot be significant because it is opposed to the average compressive stress produced by the direct load. The other point of interest is that the tendency toward buckling here is controlled by the *larger radius of gyration* because the purlin loading makes the direction of buckling self-evident.

PROBLEMS

175. Calculate the stress in an upper chord member of a bridge truss caused by its own weight. Panel length, 20 ft.-6 in. The section is composed of two 9-in., 13.4-lb. channels and a 16 × 3/8-in. cover plate. Channels are placed 10.75 in. back to back. Assume that the weight of lacing and tie plates adds 150 lb. to the total weight.

176. Calculate the stress in the bottom chord of a bridge truss caused by its own weight. Panel length, 16 ft. The section is composed of two 5 × 3½ × 5/8-in. angles with 5-in. legs turned down. Add 50 lb. for the weight of the tie plates.

177. Revise Problem *DP*45 for a maximum allowable stress of 16,000 lb. per sq. in.

178. Design a lower chord member of a roof truss to carry a direct tension of 30,000 lb. and a vertical concentrated load of 500 lb. located 6 ft. from a panel point. The panel length is 15 ft. Two 5/8-in. holes must be drilled through the outstanding legs of the chord angles at the point of application of the concentrated load to provide a connection for a shaft bearing. The member is horizontal. *AISC* specifications.

179. Revise Problem *DP*46 by permitting working stresses to be increased 33 per cent for wind. Wind pressure produces one-half of the total stress.

180. Design a top chord member of a roof truss to carry a direct stress of 40,000 lb. and a normal purlin reaction of 3600 lb. located 2 ft.-3 in. from a panel point. Use the simple-span bending moment. The panel length is 6 ft.-8 in. Over 33⅓ per cent of the direct stress and of the purlin reaction are caused by wind. *AISC* specifications.

181. A horizontal top chord of a roof truss carries a direct stress of 60,000 lb. caused by dead load and snow load. A cinder concrete slab spans from roof truss to roof truss placing a uniform load of 1000 lb. per ft. on the top chord. The panel length is 6 ft.-0 in. The wind stresses are negligible. Design a top chord section using a moment of $wL^2 \div 10$. *AREA* specifications.

DP45. *The lower tension chord of a Fink roof truss carries a shaft load at its center in addition to its direct stress. Design the member for riveted construction. Let the legs of the chord angles be turned down.*

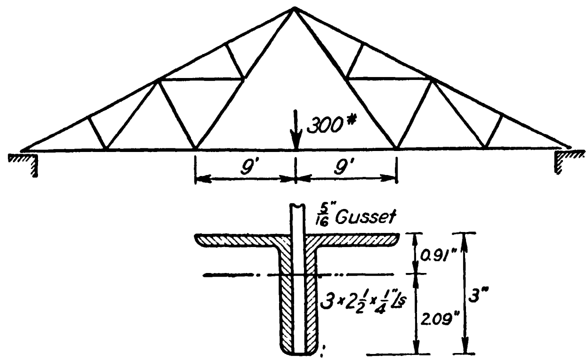
Data:

Direct stress = 11,000#.

Panel length = 18'-0".

Transverse load = 300# at center of member.

Working stress = 22,500#/□".



Minimum Section: *Try two $2\frac{1}{2} \times 2\frac{1}{2} \times \frac{1}{4}''$ ∠s.*

Gross area = $2 \times 1.19 = 2.38$□".

Direct unit stress = $11{,}000 \div 2.38 = 4600$#/□".

Gross section modulus = $2 \times 0.39 = 0.78$ (angle legs turned down).

$$\text{Bending moment} = \left(\frac{300 \times 18}{4} + \tfrac{1}{8} \times 8.2 \times 18^2\right)12 = 20{,}200''\#.$$

Flexural stress = $20{,}200 \div 0.78 = 25{,}900$#/□".

Total stress = $4600 + 25{,}900 = 30{,}500$#/□".

Revised Section: *Try two $3 \times 2\frac{1}{2} \times \frac{1}{4}''$ ∠s (3" legs turned down).*

Gross area = $2 \times 1.31 = 2.62$.

Gross section modulus = $2 \times 0.56 = 1.12$.

$$\text{Bending moment} = \left(\frac{300 \times 18}{4} + \tfrac{1}{8} \times 9.0 \times 18^2\right)12 = 20{,}600''\#.$$

$$\text{Total stress} = \frac{11{,}000}{2.62} + \frac{20{,}600}{1.12} = 22{,}600\#/\square''.$$

Remarks: *The use of gross area and gross section modulus is correct because no holes are punched in the angles near the center at the section of maximum moment. The shaft connection can be made by a clamp. The bending moment is greatly reduced at the end where net section would have to be considered.*

DP46a. *Design a top chord member for a roof truss to carry a direct compressive stress and a purlin reaction at 2 ft. from a panel point. Use AISC specifications. (Note Spec. 4.)*

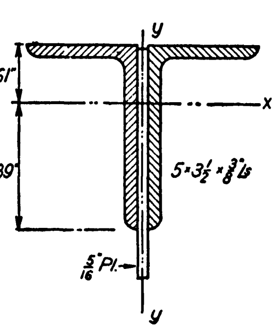
Data:

Design stress = 65,000#.

Purlin reaction perpendicular to member = 5000#.

Panel length = 5'-0".

Trial Section: *Two angles 5 × 3½ × ⅜" placed back to back on opposite sides of a 5/16" gusset with 3½" legs turned out.*



Gross area = 2 × 3.05 = 6.1□".

$r_{y-y} = 1.44$; $r_{x-x} = 1.60$.

Section modulus about x-x axis = 4.6.

Simple beam moment reduced 20% for continuity

$$= \left(\frac{5000 \times 3 \times 2 \times 12}{5}\right)0.8 = 57{,}600''\#.$$

$$\text{Allowable column stress} = 17{,}000 - 0.485\left(\frac{60}{1.60}\right)^2 = 16{,}300\#/\square''.$$

Allowable beam stress = 20,000 since $L/b < 15$.

$$\text{Required area} = \frac{65{,}000}{16{,}300} + \frac{57{,}600 \times 1.61}{20{,}000 \times 1.60^2} = 5.8\square''. \quad \text{[Equation 11]}$$

Area furnished = 6.1□".

DP46b. *Repeat the design problem DP46a by use of AREA working stresses. The column formula is $15{,}000 - \frac{1}{4}(L/r)^2$, and the beam-flange formula is $18{,}000 - 5(L/b)^2$ for values of L/b not greater than 40.*

$$\text{Allowable column stress} = 15{,}000 - 0.25\left(\frac{60}{1.60}\right)^2 = 14{,}600\#/\square''.$$

$$\text{Allowable beam stress} = 18{,}000 - 5\left(\frac{60}{7}\right)^2 = 17{,}600\#/\square''.$$

$$\text{Required area} = \frac{65{,}000}{14{,}600} + \frac{57{,}600 \times 1.61}{17{,}600 \times 1.60^2} = 6.5\square''. \quad \text{[Equation 11]}$$

Area furnished by 5 × 3½ × 7/16" ∠s is 7.06□".

Remarks: *Notice that the r-value in the direction of the purlin loading is used in the column formula since buckling is forced to occur in this direction rather than in the direction of the minimum r-value.*

147. Design of a Steel Building Column. EXAMPLE *DP*47. The end conditions of the columns of a mill bent are partly unknown. The *assumption* here is that the buckling tendency of the column acting in the plane of the bent will reach its maximum at the mid-height from the base to the connection of the lower chord of the roof truss. Thus, the moment of the horizontal reaction about this point is taken as the bending moment used to increase the area required to resist buckling. Then, of course, the maximum moment at the connection with the lower chord of the truss must be used to check the stress existing there. The stress at that point should be less than the allowable compressive stress for a *beam* since the buckling tendency as a *column* is small there.

The buckling tendency of the column in the direction perpendicular to the plane of the roof truss (the weak direction of the column section) is controlled by the *lateral resistance produced by the girts.* Since only alternate girts are usually attached to the diagonal side bracing, it is proper to consider the distance between alternate girts (or 10 ft.) as the free length for lateral buckling due either to column action or to beam action. This happens to be the controlling condition for buckling.

The increase of working stresses for wind allowance is always stated in such a manner that the increased working stress is not permitted to reduce the area below that required for dead load and live 'oad alone. Hence, we use the working stress for column compression without revision (because wind does not increase the direct stress by more than 33 per cent), but we increase the allowable beam stress 33 per cent since the bending moment is entirely caused by wind.

148. Eccentric and Lateral Loading on Column. EXAMPLE *DP*48. This example is a repetition of the same general procedure used in the design of the mill building column *DP*47. Again, use is made of different working stresses for direct compression and for flexure even though in this case the free length for buckling of the beam flange is exactly the same as its free length for buckling as a column. It may be well to reconsider the reason for this distinction. If the entire section is stressed in compression, both flanges and web tend to buckle together. On the other hand, beam flexure produces tension in one flange, which helps to maintain its straightness, while only the compression flange tends to buckle. The aid given by the tension flange in resisting the buckling tendency of the compression flange is one excuse for allowing greater compression stresses in beam flanges than in columns with centric loads. When we consider the fact that the force needed to prevent buckling is relatively small, we are inclined to justify this distinction.

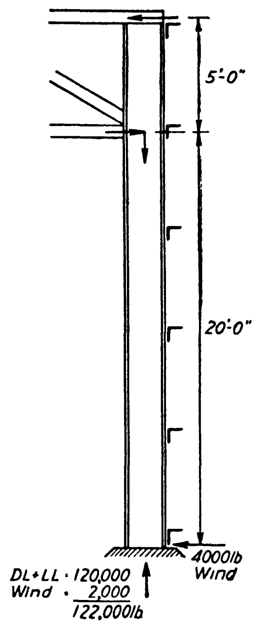
149. Practical Considerations. If we carry the matter of design for direct stress and flexure to its logical theoretical conclusion, we will find it necessary to design all tension and compression members for direct stress and flexure. Truss members all resist secondary flexural stresses. All except vertical truss members must resist bending from the influence of their own weights. Columns are never loaded axially, or, at least, *the occurrence of a column without bending is exceedingly rare*—so rare that it cannot even be produced in a laboratory without the greatest difficulty. But in most of the instances mentioned, the influence of flexure is not very serious and it may be neglected without danger. The working stresses for truss design are expected to allow for flexural (secondary) stresses of about 30 per cent of the primary stresses. There is no reason, therefore, why the specified working stresses for building columns should not be considered to cover an equal allowance for slight eccentricities that de-

DP47. *Design a mill building column to resist a vertical compression of 122,000 lb. caused by D.L., L.L., wind, and also for a wind shear of 4000 lb. The column is not fixed at its base, but is so restrained that its tendency toward buckling reaches a maximum halfway between the base and the lower chord connection. AISC working stresses. Counting from the top, the 1st, 3rd and 5th girts act as bracing struts.*

Column Moment:

Maximum wind moment at critical section

$= 4000 \times 10 \times 12 = 480{,}000''\#.$



Trial Section: *10WF45.*

$L/r = 20 \times 12 \div 4.33 = 55.5.$

L/r laterally between alternate girts $= 10 \times 12 \div 2.0 = 60.$

Working stress for column buckling

$= f_c = 17{,}000 - 0.485 \times 60.0^2 = 15{,}250\#/\square''.$

Working stress for beam buckling when L is taken for safety as twice the distance between girts, i.e.,

$L/b = 10 \times 12 \div 8.02 = 15.$

$$f_b = \frac{22{,}500}{1 + \frac{15^2}{1800}} = 20{,}000\#/\square''.$$

Increment for Wind:

Since the vertical load is not appreciably increased by wind, the working stress f_c is 15,250 and the wind load is neglected. But for moment, the working stress f_b is to be increased by 33% to 26,700#/□" since all moment is produced by wind.

Required Area:

$$A = \frac{P}{f_c} + \frac{Mc}{f_b r^2} \quad \textit{(Spec. 4 and Equation 11)}$$

$$= \frac{120{,}000}{15{,}250} + \frac{480{,}000 \times 5.06}{26{,}700 \times 4.33^2} = 7.87 + 4.85 = 12.72\square''.$$

Area furnished $= 13.24\square''.$

Fiber stress just below chord connection $= \dfrac{122{,}000}{13.24} + \dfrac{960{,}000}{49.1} = 28{,}800\#/\square''.$

Since this is over 26,700, the 10WF49 section will be required.

Remark: *The change to the heavier section should not affect r, f_c or f_b appreciably.*

DP48. *Design a column that carries a central load, an eccentric load, and a lateral load. The column is considered to be pin connected at top and bottom. AISC spec.*

Data:

Total vertical load = 120,000#.

Maximum moment = 225,000 + 120,000 = 345,000″#.

First guess at column area = 120,000 ÷ 10,000 = 12□″.

First Trial Section:

Try an 8×8WF40 section.

Area = 11.76□″; S = 35.5.

$$\text{Maximum fiber stress} = \frac{120{,}000}{11.76} + \frac{345{,}000}{35.5} = 20{,}000\#/\square''.$$

Second Trial Section:

Since this is too high, we will try a 10×10WF49 section.

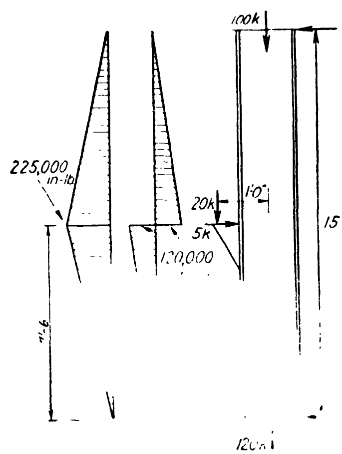
Area = 14.40; S = 54.6; $r_{xx} = 4.35$; $r_{yy} = 2.54$.

Allowable Stresses:

L/r for buckling = 15 × 12 ÷ 2.54 = 71.0.

L/r for buckling [illegible] × 12 ÷ 10.0 = 18.[illegible]

[illegible]



Area of Secti[illegible]

$$\text{Required area} = \frac{120{,}000}{14{,}500} + \frac{345{,}000 \times 5.0}{19{,}100 \times 4.35^2} \quad \text{[Equation 11]}$$

$$= 8.3 + 4.8 = 13.1\square''.$$

Area furnished = 14.4□″, but no weight reduction is possible.

Remarks: *This column will buckle in its weak direction as controlled by the minimum r of 2.54. As a beam its 10″ flange is unsupported laterally for the full length of 15′. However, in computing the required area we must use the larger value of r since* Ar^2 *is the moment of inertia for flexure. It must be remembered that the formula for required area was derived by using the basic beam-flexure formula,* $f = Mc/I$.

velop from unpredictable loadings. For the same reason, we seldom consider flexure caused by the dead weight of horizontal struts and tension members that are not parts of a truss. Long horizontal tension members sometimes do need intermediate support.

The fact that deflection changes the eccentricity of the load for members resisting direct stress and flexure, continues to be given more consideration than it deserves. For typical instances the influence is found to be in the nature of one or two per cent and it may accordingly be neglected. Structural design in its very nature is subject to greater approximations than this.

CHAPTER 10

STRESS AND STABILITY

150. Theory of Elasticity. We could not progress very far in the study of stress analysis by the mathematical theory of elasticity without recourse to an understanding of advanced mathematics that is not presupposed here. This book is devoted to applications of theory instead of development of theory. For these reasons certain problems in design will be solved by the use of formulas taken from other studies without explanation of the basic theory. These problems will be the determination of principal stresses, buckling stresses for plates, stresses in rollers, stress concentrations at holes, stresses in flat loaded slabs or plates, and torsion stresses in structural beams. Evidently, to cover this amount of material in one short chapter will necessitate extreme abbreviation. Hence, we will merely present the formulas, define their terms, show applications, and give a few of the many references to their development and their special uses. This will serve the purpose of familiarizing the reader with the kinds of problems that have been studied and solved. He will need to read further if his work demands other than standardized applications.

MAXIMUM STRESSES

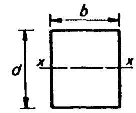
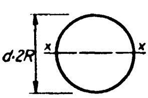
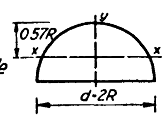
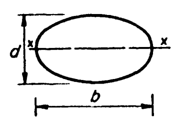
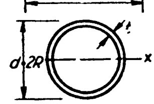
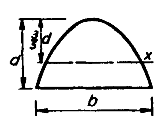
151. Critical Stresses. We have studied the design of tension members, columns, and beams and have considered in each instance only a single controlling stress. However, the designer will inevitably be confronted from time to time with structural design problems where two kinds of stress combine at a given point. The problem is to determine *the direction of the maximum stress and its magnitude.* For example, tensions or compressions may occur at right angles to each other and produce a resultant stress greater than either. The calculations of such resultants are common engineering procedure. On the other hand, we are prone to forget that a direct stress and a shear acting on the plane A may produce a greater direct stress or a greater shear on the plane B.

Failures of Materials. Whether failure is caused by direct stress, by shear, or by strain has never been determined for the general case. It seems probable that each may control failure for special materials and special conditions of loading. For the purpose of design, however, we will

accept the maximum stress-shear theory of failure; that is, we will so design the structure that the maximum direct stress will be within its specified allowable value and the maximum shearing stress will be below its specified limitation. This necessitates a knowledge of how to compute the maximum unit direct stress at a point and the corresponding maximum unit shearing stress.

TABLE 24

PROPERTIES OF SIMPLE SECTIONS

		Area	I_{xx}	r_{xx}	S_{xx}
Rectangle		bd	$\frac{bd^3}{12}$	$0.29d$	$\frac{bd^2}{6}$
Circle		πR^2	$\frac{\pi R^4}{4}$	$0.25d$	$\frac{\pi R^3}{4}$
Semi-circle		$\frac{\pi R^2}{2}$	$I_{xx}\ 0.11R^4$ $I_{yy}\ \frac{\pi R^4}{8}$	$0.26R$ $0.25d$	$0.19R^3$ $\frac{\pi R^3}{8}$
Ellipse		$\frac{\pi bd}{4}$	$\frac{\pi bd^3}{64}$	$0.25d$	$\frac{\pi bd^2}{32}$
Thin ring		$2\pi Rt$	πtR^3	$0.35d$	πtR^2
Parabola		$\frac{2bd}{3}$	$\frac{8bd^3}{175}$	$0.26d$	$\frac{8bd^2}{105}$

152. Formulas for Determining Principal Stresses and Shears. These results will be presented as a group of special cases that cover many practical problems.

CASE 1. *Axial Tension or Compression.* The application of a direct unit stress s is accompanied by the development of a shear on any plane inclined to the cross-section normal to the load. If we define s_n as the *unit* stress normal to the plane A-A in Fig. 132 and s_t as the *unit* stress along the plane (shear) while s is the *unit* axial stress, we may state

(1) $$s_n = s\cos^2\theta = \tfrac{1}{2}s\,(1 + \cos 2\theta);^*$$

(2) $$s_t = s\sin\theta\cos\theta = \tfrac{1}{2}s\sin 2\theta.$$

* Note in all cases that s, s_1, s_2, s_n, and s_t are *unit stresses* and only represent *forces* when the area on which the stress acts is unity.

When s_n and s_t reach their maximum values, they are called the *maximum stress* and the *maximum shear* respectively. The minimum direct stress, which is the second principal stress, occurs at 90° to the maximum direct stress. Thus we obtain

(3) $$\text{Max. } s_n = s \quad \text{when } \theta = 0°;$$

(4) $$\text{Min. } s_n = 0 \quad \text{when } \theta = 90°\cdot$$

(5) $$\text{Max. } s_t = \frac{s}{2} \quad \text{when } \theta = 45° \text{ (or } 135°).$$

The maximum shear is always one half the difference of the principal stresses as will be shown in equation (9).

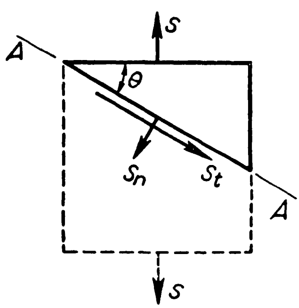


FIG. 132. AXIAL STRESS.

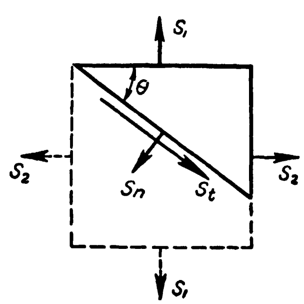


FIG. 133. BIAXIAL STRESS.

CASE 2. *Biaxial Stresses.* The next step is the case of direct stresses applied at 90° to each other in a single plane as illustrated by Fig. 133. We will take s_1 and s_2 as principal stresses, that is, *direct stresses unaccompanied by shears.* We may write

(6) $$\begin{aligned} s_n &= s_1 \cos^2 \theta + s_2 \sin^2 \theta \\ &= \tfrac{1}{2}(s_1 + s_2) + \tfrac{1}{2}(s_1 - s_2) \cos 2\theta; \end{aligned}$$

(7) $$s_t = \tfrac{1}{2}(s_1 - s_2) \sin 2\theta.$$

Naturally, the maximum normal stress s_n is either s_1 or s_2. In general terms we may say

(8) $$\text{Max. } s_n = s_1 \text{ or } s_2 \text{ when } \theta = 0° \text{ (or } 90°);$$

(9) $$\text{Max. } s_t = \tfrac{1}{2}(s_1 - s_2) \text{ when } \theta = 45° \text{ (or } 135°).$$

Note that s_t will reach a maximum value when s_1 and s_2 are of opposite sense so that $s_t = \tfrac{1}{2}(s_1 + s_2)$. If s_1 and s_2 are both tension or both compression, the controlling shear is $\frac{1}{2}s_1$ or $\frac{1}{2}s_2$, and it occurs in a plane inclined at 45° to the plane of the paper in Fig. 133.

Equal and Opposite Principal Stresses — Pure Shear. If the biaxial stresses are equal in value but *opposite in sense*, equations (6) and (7) may be rewritten as follows:

$$s_n = s(\cos^2\theta - \sin^2\theta) = s\cos 2\theta; \tag{10}$$

$$s_t = \tfrac{1}{2}(2s)\sin 2\theta = s\sin 2\theta. \tag{11}$$

From these equations we see that there will be no normal stresses on planes at 45° with the principal stresses since *cos* 90° = 0. The value of the shear s_t reaches its maximum value on the same plane and becomes numerically equal to s. Hence, on these planes we have the state of *pure shear.* This case is not a particularly important one since shear in beams and other structures is usually combined with direct stress, as will be discussed below.

CASE 3. *Direct Stresses Arising from Shear.* This is essentially a reversed condition from the one just discussed. Knowing the value of an applied shear we are to find the normal stress and the shear on an inclined plane. From Fig. 134, we write

$$s_n = s_s\sin 2\theta; \tag{12}$$

$$s_t = s_s\cos 2\theta. \tag{13}$$

The maximum values of s_n occur for $\theta = 45°$ and 135°. These values are

$$\text{Max. } s_n = \pm s_s; \tag{14}$$

$$\text{Max. } s_t = s_s \text{ when } \theta = 0° \text{ (or 90°)}. \tag{15}$$

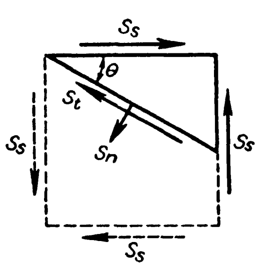


FIG. 134. PURE SHEAR.

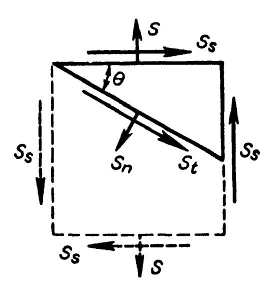


FIG. 135. AXIAL STRESS PLUS SHEAR.

CASE 4. *One Axial Stress plus Shear.* This is a common and an important case because the principal stresses become significant for design. In Fig. 135 we combine Cases 1 and 3 to obtain

$$s_n = \tfrac{1}{2}s(1 + \cos 2\theta) + s_s\sin 2\theta; \tag{16}$$

$$s_t = \tfrac{1}{2}s\sin 2\theta + s_s\cos 2\theta. \tag{17}$$

The principal stresses are

(18) $$\text{Max. } s_n = \frac{s}{2} \pm \sqrt{\frac{s^2}{4} + s_s^2}.$$

These stresses occur when $\tan 2\theta = -\frac{2s_s}{s}$. Also

(19) $$\text{Max. } s_t = \sqrt{\frac{s^2}{4} + s_s^2}.$$

SIGNS

In equation (17) the plus sign would become minus for the forces of Fig. 135. Signs for all equations must be adjusted to fit the directions of the forces.

Tan 2θ is negative when 2θ is in the 4th quadrant.

This maximum shear occurs on planes at 45° to the planes of the principal stresses.

CASE 5. *Biaxial Stress Plus Shear.* This is the most general case of stresses in a plane. With reference to Fig. 136, we may combine Cases 2 and 3 to obtain

(20) $$s_n = \tfrac{1}{2}(s_1 + s_2) + \tfrac{1}{2}(s_1 - s_2) \cos 2\theta + s_s \sin 2\theta;$$

(21) $$s_t = \tfrac{1}{2}(s_1 - s_2) \sin 2\theta + s_s \cos 2\theta.$$

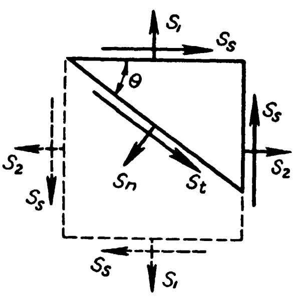


FIG. 136. BIAXIAL STRESS PLUS SHEAR.

The principal stresses become

(22) $$\text{Max. } s_n = \tfrac{1}{2}(s_1 + s_2) \pm \sqrt{\frac{(s_1 - s_2)^2}{4} + s_s^2}.$$

These maximum stresses occur when $\tan 2\theta = -\frac{2s_s}{s_1 - s_2}$.

The principal shears are

(23) $$\text{Max. } s_t = \sqrt{\frac{(s_1 - s_2)^2}{4} + s_s^2}.$$

These shears occur on planes at 45° to the planes of the principal stresses. *The maximum shear is one half the difference between the principal stresses.*

OBSERVATION. Preceding equations may be obtained as special cases of equations (22) and (23) when s_1, s_2, or s_s is zero.

153. Illustrations of Combined Stress Calculations. There are many instances of combined stresses where the ordinary design procedure does not take the real principal stresses into consideration. For example, we know that hot driven rivets have initial tension stresses. Such rivets are designed for shear or bearing with no reference to combined stresses. The justification is that all test joints that have been used to set allowable working stresses in shear and bearing also have had rivets with initial tension. Therefore, combined stresses are considered indirectly in riveted joint design.

BOLT SELECTION, *DP*49. The design problem *DP*49 shows how the maximum tension and the maximum shear can be determined in a bolt or rivet resisting tension and shear. This combined stress is quite significant for a bolt since working stresses for bolts are not intended to allow for stress combinations. When large anchor bolts are being designed, the safety of the structure may be involved to as great an extent as in the

DP49. *An anchor bolt carries an initial tension of 12,000# and a shear of 8000#. Select a diameter such that the combined tension will not exceed 24,000 and the combined shear will not exceed 15,000#/□".*
Trial diameter = 1.0"; area = 0.785□".
Unit direct stress, s = 12,000 ÷ 0.785 = 15,300#/□".
Unit cross shear (average), s_s = 8000 ÷ 0.785 = 10,200#/□".

Combined Stress:

$$Max.\ s_n = \frac{s}{2} + \sqrt{\frac{s^2}{4} + s_s^2} = \frac{15{,}300}{2} + \sqrt{\frac{15{,}300^2}{4} + 10{,}200^2}$$

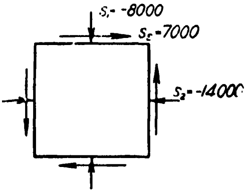
$$= 7650 + 12{,}750 = 20{,}400\#/\square''.$$

$$Max.\ s_t = \sqrt{\frac{s^2}{4} + s_s^2} = \sqrt{\frac{15{,}300^2}{4} + 10{,}200^2} = 12{,}750\#/\square''.$$

The 1" diameter is larger than needed, but a ⅞" bolt would not serve.
At the root of thread, the net tension is 12,000 ÷ 0.55 = 21,800#/□".
If the bolt undergoes flexure, s_s might more properly be taken as the maximum beam shear on a circular section, which is ⅓ larger than the average unit shear.

DP50. *A beam web near the flange is stressed to 14,000#/□" in horizontal compression and to 7000#/□" in vertical and horizontal shear. Can a vertical load be placed on the beam that will stress the web in compression in a vertical direction to 8000#/□" without exceeding a permissible combined compression of 20,000#/□"?*

Combined Stress: *Equations (18) and (22).*



$$Max.\ s_n = \tfrac{1}{2}(s_1 + s_2) \pm \sqrt{\frac{(s_1 - s_2)^2}{4} + s_s^2}$$

$$= -11{,}000 - 7600 = -18{,}600\#/\square''$$

(when $s_1 = -14{,}000$, and $s_2 = -8000$).
Without the vertical load, the combined compression is

$$Max.\ s_n = \frac{s}{2} \pm \sqrt{\frac{s^2}{4} + s_s^2} \qquad (s \text{ being } -14{,}000)$$

$$= -\frac{14{,}000}{2} - \sqrt{\frac{(-14{,}000)^2}{4} + 7000^2} = -7000 - 9900$$

$$= -16{,}900\#/\square''.$$

The maximum shear in either case is the value of the radical, i.e., 7600#/□" with the vertical load and 9900#/□" without it.

Remarks: *The design is satisfactory and the vertical load may be carried safely.*

design of a main member. A determination of the maximum tension and of the maximum shear is the only safe procedure.

BEAM STUDY, *DP*50. The design problem *DP*50 shows how the combination of stresses may properly influence the selection of a beam for special loadings. The addition of a vertical load on the top flange is shown to increase the diagonal compression in the web from 16,900 to 18,600 lb. per sq. in. It is interesting that this same load application happens to reduce the web shear at the point under consideration from 9900 to 7600 lb. per sq. in. Of course, *either* combined shear is greater than the applied shear of 7000 lb. per sq. in. It is to be noted that beam design does not ordinarily make use of combined stress calculations. Nevertheless, if unusual loads are to be considered, a study of the combined stresses will be the designer's best approach to conservative design.

BEARING STRESSES

154. Stress Concentrations at Loads. The application of a perfectly concentrated load to a structure is a physical impossibility since there is always some small *finite area of contact.* The theory of elasticity assumes that point concentrations can exist. The results obtained are essentially correct at a very small distance away from the load where plastic yielding does not occur. The cases to be studied will be the usual ones of concentrated and distributed loads on semi-infinite flat areas and the case of bearing pressure between a ball or roller and a flat plate. The formulas given are from *Theory of Elasticity*, S. Timoshenko, 1934.

CASE 1. *Line Loads on a Flat Surface.* The radial compressive stress at A at any distance R from the load line is expressed as

$$s_c = \frac{2p \cos \theta}{\pi R}. \tag{24}$$

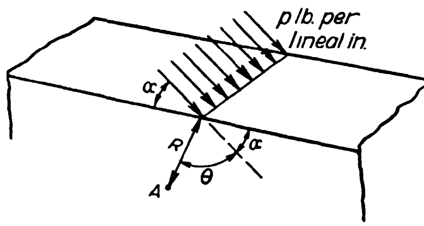


FIG. 137. LINE LOADING.

This equation holds for any inclination of the line loading, even when it is tangential to the surface. In each case the angle θ, however, is measured from the direction of the applied loading.

CASE 2. *Uniform Load of Plate Width.* The direct stress immediately underneath the load, of course, is the same as the loading or w lb. per sq. in. At any point A below the load, the maximum compressive stress directed along the bisector of the angle θ is

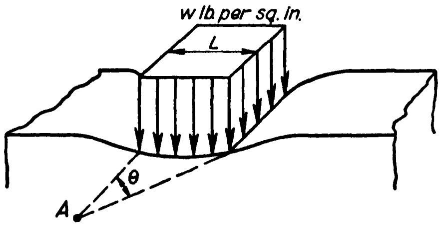


FIG. 138. UNIFORM LOADING.

$$s_c = \frac{w}{\pi}(\theta + \sin \theta). \tag{25}$$

The maximum unit shear at A becomes

$$s_s = \frac{w \sin \theta}{\pi}. \tag{26}$$

CASE 3. *Load Applied through Rigid Block.* The deflection is taken to be constant at all points under the load. The important stress at the surface of contact may be expressed as

$$s_c = \frac{p}{\pi\sqrt{a^2 - x^2}}. \tag{27}$$

This stress becomes infinite along the outer edges of the rigid block (when $x = a$). Along these lines plastic flow will occur.

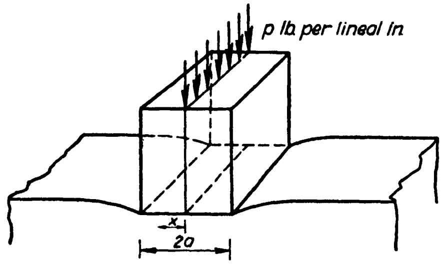


FIG. 139. RIGID BLOCK LOADING.

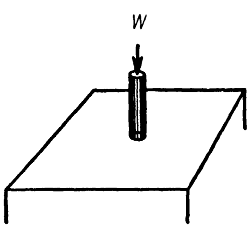


FIG. 140. PUNCH LOAD.

CASE 4. *Circular Area of Contact on Face of Large Thick Plate.* A uniform pressure of w lb. per sq. in. over a circle of radius R gives rise to a maximum shear of $0.33w$ at $0.64R$ below the center of the loaded area. A total load W applied through a circular punch (rivet punch) of radius R gives rise to a contact stress of

$$s_c = \frac{W}{2\pi R\sqrt{R^2 - x^2}}. \tag{28}$$

In this equation, x is the radius to the point considered, and $x < R$. The minimum bearing pressure at the center of the die is one half of the average unit pressure.

CASE 5. *Bearing Roller on Flat Plate.* The value of Poisson's ratio will be taken as 0.3 for steel. For Fig. 141 the maximum contact pressure then becomes

$$s_c = 0.418\sqrt{\frac{pE}{R}}. \tag{29}$$

The value of R is the radius of the roller. The maximum shearing unit stress is approximately $0.3s_c$.

CASE 6. *Bearing Between Rollers or Rockers.* Again, the value of Poisson's ratio is taken as 0.3. The following formula expresses the maximum value of the bearing pressure in Fig. 142:

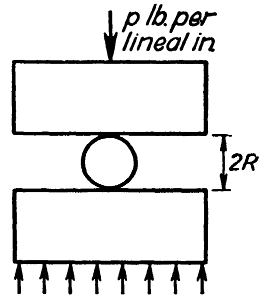


FIG. 141. ROLLER BEARING.

$$s_c = 0.418\sqrt{pE\,\frac{R_1 + R_2}{R_1R_2}}. \tag{30}$$

If the case is that of a roller bearing in a circular groove or channel, the value of R_1 (radius of groove) must be preceded by the minus sign. Then, cancellation gives

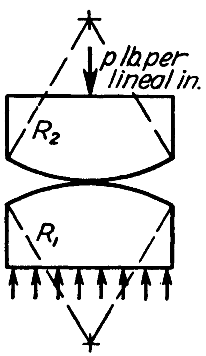


FIG. 142. DOUBLE ROCKERS.

$$s_c = 0.418\sqrt{pE\,\frac{R_1 - R_2}{R_1R_2}}. \tag{31}$$

CASE 7. *Bearing Pressure of Ball on Flat Plate.* For a value of Poisson's ratio of 0.3 and a total load of W, the maximum unit bearing pressure becomes

$$s_c = 0.388\sqrt[3]{\frac{WE^2}{R^2}}. \tag{32}$$

The value of R is the radius of the ball.

155. Bearing Design. EXAMPLES $DP51$, $DP52$, $DP53$. These illustrative examples show how to apply the bearing formulas given in § 154 to actual design problems. The knife-edge problem, $DP51$, is used to illustrate the high stresses involved when it is necessary to carry a heavy load on a narrow steel bearing. The edge is assumed to be ground to a radius of $\frac{1}{10}$ in. and even then the bearing must be made of high carbon steel to carry the small load of 50 lb. per lineal in. Again, in example $DP52$ we find that a 6-in. roller will only carry a load of 230 lb. per lineal in. when the bearing pressure is limited to 20,000 lb. per sq. in.

It will be interesting to compare the design of 72-in. rockers obtained in $DP53$ with the requirements of the *AREA* specifications for metal having an elastic limit of 27,000 lb. per sq. in. (Spec. 104.) We must introduce a factor 2 to allow for the fact that both bearing surfaces are curved. Compare equations (29) and (30). Thus we obtain

$$p = 5000 = \left(\frac{27{,}000 - 13{,}000}{20{,}000 \times 2}\right) 600d,$$

from which

$$d = \frac{5000}{210} = 24 \text{ in.} \quad (72 \text{ in. in } DP53)$$

This example points out clearly the extreme conservatism involved in the use of bearing formulas derived by the mathematical theory of elasticity. This theory always gives the maximum *point* or *line stress* which may cover such a minute area that it is not a very serious factor in design. A slight flow will relieve this stress in most instances. It becomes

DP51. *A knife-edge bearing of a testing machine carries a load of 50 lb. per lineal in. If the knife edge is in reality rounded to a radius of 1/10", what elastic limit must be provided for a safety factor of 2.0?*

$$\text{Equation (29)} \quad s_c = 0.418\sqrt{\frac{pE}{R}}$$

$$= 0.418\sqrt{\frac{50 \times 30{,}000{,}000}{0.1}} = 51{,}000\#/\square''.$$

For a factor of safety of 2.0 the minimum elastic limit should be 100,000#/□', which may be obtained with high carbon steel.

DP52. *Find the allowable load on a 6" roller (12" long) bearing on a thick steel plate when s_c is limited to 20,000#/□".*

$$\text{Equation (29)} \quad s_c = 0.418\sqrt{\frac{pE}{R}}$$

$$20{,}000 = 0.418\sqrt{\frac{p \times 30{,}000{,}000}{3.0}}.$$

$$p = \left(\frac{20{,}000}{0.418 \times 3160}\right)^2 = 230\#/''.$$

$$\text{Total } W = 12 \times 230 = 2760\#.$$

DP53. *Two cast steel rockers 12" long must carry a total bearing load of 60,000#. Find the radius to which the bearing surfaces of the rockers must be turned if the maximum bearing pressure is limited to 27,000#/□".*

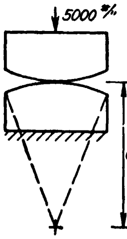
$$p = 60{,}000 \div 12 = 5000\#/''.$$

$$\text{Equation (30)} \quad s_c = 0.418\sqrt{pE\,\frac{R_1 + R_2}{R_1R_2}}.$$

$$\text{When } R_1 = R_2,\ s_c = 0.418\sqrt{\frac{2pE}{R}}.$$

$$\text{Hence,} \quad 27{,}000 = 0.418\sqrt{\frac{2 \times 5000 \times 30{,}000{,}000}{R}}.$$

$$R = 300{,}000{,}000{,}000\left(\frac{0.418}{27{,}000}\right)^2 = 72''.$$



Remarks: *The required radius by AREA spec. is 24"; § 155.*

significant, however, when repetition can produce fatigue cracks that actually start in just such limited areas. Only rapidly moving bearings would need to be designed to meet the conservative procedure of *DP*53.

Plate Design

156. Theory of Plate Stresses. The plate or slab is a structure that resists bending in more than one direction. Like beams, plates may have simply supported or fixed edges. We will give the stress formulas to cover both types of support for circular and rectangular plates uniformly loaded. These data come from the writings of S. Timoshenko; additional cases may be studied in his *Theory of Elasticity* or in *Applied Elasticity* by Timoshenko and Lessels. Also see *Stress and Strain*, R. J. Roark.

Case 1. *Circular Plate — Uniform Load — Simply Supported Edges.* The maximum stress occurs at the center. If W is the total load, t is the plate thickness and μ is Poisson's ratio, we may write

$$\text{(33)} \qquad \text{Max. } s = \frac{3W}{8\pi t^2}(3 + \mu).$$

Case 2. *Circular Plate — Uniform Load — Clamped or Fixed Edges.* Although fixed edges are seldom exactly achieved, practical conditions may approach this ideal. The maximum stress occurs at the edge in a radial direction. This stress is

$$\text{(34)} \qquad \text{Max. } s = \frac{3W}{4\pi t^2}.$$

The tangential stress at the edge is

$$\text{(35)} \qquad s = \frac{3W\mu}{4\pi t^2}.$$

Case 3. *Rectangular Plate — Uniform Load — Simply Supported Edges.* In his book *Formulas for Stress and Strain*, R. J. Roark has expressed the maximum stress in convenient form. The load per unit of area is w, the shorter span is b, the thickness of the slab is t, and the ratio n is b/c, the *shorter* span divided by the *longer*. Poisson's ratio is taken as 0.3 to obtain

$$\text{(36)} \qquad \text{Max. } s = \frac{3wb^2}{4t^2(1 + 1.61n^3)}.$$

For a square plate we may write

$$\text{(37)} \qquad \text{Max. } s = \frac{0.287wb^2}{t^2}.$$

Case 4. *Rectangular Plate — Uniform Load — Clamped or Fixed Edges.* The maximum stress occurs at the center of the long edge. It is expressed

by the equation

$$\text{(38)} \qquad \text{Max. } s = \frac{wb^2}{2t^2(1 + 0.623n^6)}.$$

For a square plate we obtain

$$\text{(39)} \qquad \text{Max. } s = \frac{0.308wb^2}{t^2}.$$

This stress is greater than the stress in the same plate when simply supported. At the center of the short edge the stress is approximately

$$\text{(40)} \qquad s = \frac{wb^2}{4t^2}.$$

At the center of the plate the controlling stress is

$$\text{(41)} \qquad s = \frac{3wb^2}{4t^2(3 + 4n^4)}.$$

157. Plate Design. Examples *DP*54, *DP*55, *DP*56. Circular plates occur as ends to pressure drums, cylinders, and tanks, and also as manhole covers. When flat ends are designed less conservatively than in *DP*54, they will still carry the load but there may be noticeable bulging. The bulging produces a domed head, thereby developing direct stresses that aid in reducing the flexural moment. Ordinary design properly does not depend upon such *inelastic* deformation.

Cast iron floor plates and manhole covers must be designed for flexural resistance at relatively low working stresses. The example *DP*55 shows the proper procedure. Ribbed plates are common and they may be designed to provide the same moment resistance (that is, moment of inertia divided by extreme fiber distance) as the solid plate. The ribs should be placed in compression since the larger area (for cast iron) is needed at the tension fiber.

The battledeck floor designed in *DP*56 is a satisfactory floor, particularly for warehouse use. It carries heavy loads well, but it is slippery unless covered. Note also that it is not fireproof. It becomes evident in the design that the plate is stressed nearly to the same amount whether the edges are simply supported or clamped.

Buckling of Plates

158. Buckling of Flange and Web Plates. The possible buckling of the flange plates and web plates of girders or of the cover plates and diaphragms of compression members is a very important consideration. These matters are studied in detail by S. Timoshenko in his *Theory of Elastic Stability*. A few of the simpler results obtained there will be given below. Reference should be made to the original source for other edge conditions and for special studies of stiffened plates.

General Formula Defining Critical Buckling Stress. We will give in general terms the expression for the critical unit stress that starts buckling and then we will need tables of constants k that apply for special

DP54. *The flat plate in the end of a pressure drum must resist an internal pressure of 100#/□″. The drum is 10″ in diameter. Consider the plate to have fixed edges and obtain its thickness for a maximum stress of 22,000#/□″.*

Equation (34) $s = \frac{3W}{4\pi t^2}$.

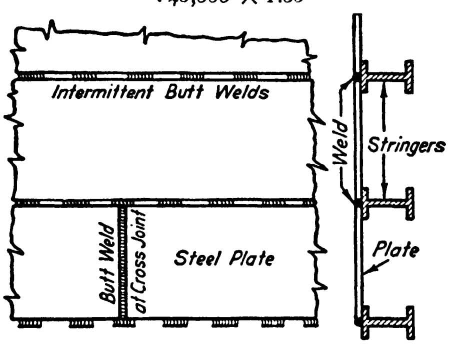
Hence, $t = \sqrt{\frac{3W}{4\pi s}} = \sqrt{\frac{3(100 \times \pi \times 25)}{4\pi \times 22{,}000}} = 0.29''$.

DP55. *A cast iron floor plate covers an opening 30″ × 50″. It carries a load of 145#/□′ or 1#/□″ nearly. What should be the thickness to hold the tensile stress down to 10,000#/□″?*

Equation (36) $s = \frac{3wb^2}{4t^2(1 + 1.61n^3)}$,

$$10{,}000 = \frac{3 \times 1 \times 30^2}{4t^2(1 + 1.61 \times 0.6^3)}.$$

From which, $t = \sqrt{\frac{2700}{40{,}000 \times 1.35}} = 0.23''$.



(a) Plan (b) Section

DP56. *A battledeck floor plate spans 5′ between stringers and 10′ between floor beams. If the design load is 200#/□′ and the working stress is 20,000#/□″, find the plate thickness. Since the edges are continuous but neither free nor fixed, we will use an average thickness between those given by equations (36) and (38).*

Equation (36) $t = \sqrt{\frac{3wb^2}{80{,}000(1 + 1.61n^3)}} = \sqrt{\frac{3 \times 1.39 \times 60^2}{80{,}000(1 + 0.20)}}$,

$= 0.40''$.

Equation (38) $t = \sqrt{\frac{wb^2}{40{,}000(1 + 0.623n^6)}} = \sqrt{\frac{1.39 \times 60^2}{40{,}000(1.01)}}$,

$= 0.35''$.

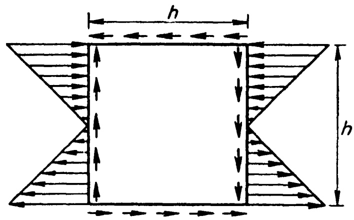
Use a ⅜″ plate.

DP57. *Determine the factor of safety against web wrinkling that exists in a plate girder where $h/t = 170$. The stiffeners form square panels.*

Actual shear = 6000#/□″.

Allowable fiber stress = 20,000#/□″.

For shear alone, the critical stress is determined for a value of k of 9.4 as shown by Case 5 of Table 25.



$$\text{Critical } s_s = \frac{9.4\pi^2 E}{12(1-\mu^2)}\left(\frac{t}{h}\right)^2$$

$$= \frac{9.4 \times 3.14^2 \times 30{,}000{,}000}{12(1-0.09)170^2}$$

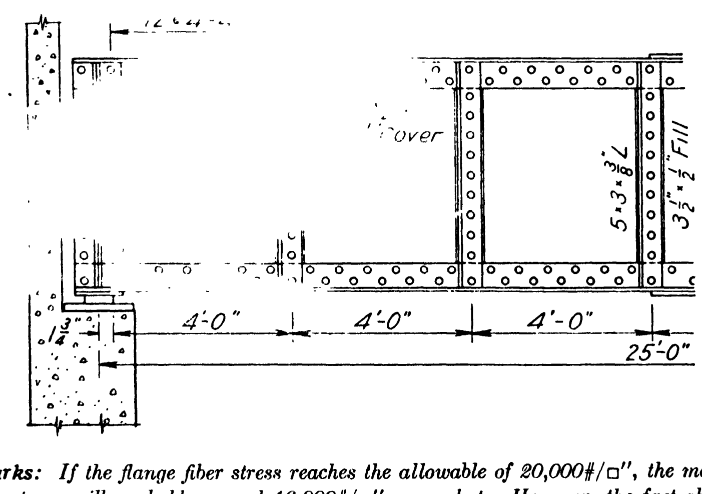
$$= 8830\#/\square''.$$

$$\frac{\text{Actual shear}}{\text{Critical shear}} = \frac{6000}{8830} = 0.68.$$

Then, for flexure, when s_s/s_s cr. = 0.7, we obtain a value of k of 17. (Case 6, Table 25.)

Hence, the critical flexural stress is

$$s = \frac{17 \times 3.14^2 \times 30{,}000{,}000}{12(1-0.09)170^2} = 16{,}000\#/\square''.$$



Remarks: *If the flange fiber stress reaches the allowable of 20,000#/□″, the maximum web stress will probably exceed 16,000#/□″ somewhat. However, the fact should be considered that the flanges tend to fix the web plate along two edges. This restraint increases the critical stress considerably. This may explain the fact that the critical shear found here (8830) is so much lower than the allowable of 13,000 permitted by AISC specifications.*

stresses, particular loadings, and different edge conditions. The general formula for the critical stress is

(42) $$s_{\text{critical}} = \frac{k\pi^2 E}{12(1-\mu^2)}\left(\frac{t}{b}\right)^2.$$

In this formula μ is Poisson's ratio, which will usually be taken as 0.3, t is the thickness of plate, and b is the length of the loaded edge. The critical stress may be an axial compressive stress, a flexural stress, or a shearing stress. The same formula controls all three types of critical stresses. The values of k vary with the edge condition and also with the plate shape, or with a/b (see Table 25). In the special case of flexure combined with shear, the critical shearing unit stress is first determined (Case 5) and the ratio of the *actual unit shearing stress* to this *critical unit shearing stress* is evaluated. From this ratio we can select the new value of k (Case 6 from Table 25) which defines the critical flexural stress. In each case, of course, we must finally introduce a factor of safety of about 2.0 to determine the allowable stress.

Stress Concentration

159. Influence of Stress Raisers. It has been shown by the theory of elasticity that changes of cross-section, holes, and all other discontinuities increase stresses greatly. On the other hand, a slight plastic flow will usually reduce or even eliminate these stress concentrations. However, it is well for the structural engineer to be acquainted with theoretical upper limits of stress for he will need to consider this influence when repeated stresses are involved.

Stress Increase at a Hole. A simple punched or drilled hole is a stress raiser of serious proportions.

Case 1. *Simple tension or compression member.*

(43) $$s_{\text{max.}} = 3s = 3\frac{P}{A}.$$

The stress distribution on the net section is illustrated by Fig. 143.

Case 2. *Equal biaxial stresses — both tension or both compression.*

(44) $$s_{\text{max.}} = 2s.$$

Case 3. *Equal biaxial stresses — one tension and one compression.*

(45) $$s_{\text{max.}} = 4s.$$

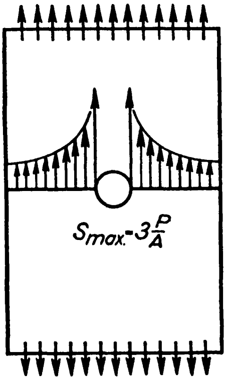
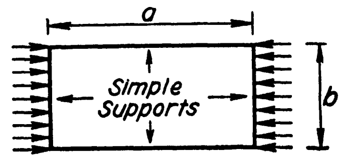


Fig. 143. Stress on Net Section of Plate.

Corner Fillet as a Stress Reducer. For the special case of an angle where the ratio of leg length to thick-

TABLE 25

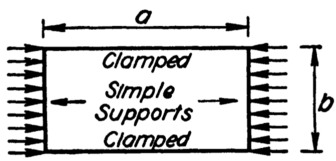
Coefficients for Critical Buckling Stress*



Case 1

where compressive stress is critical

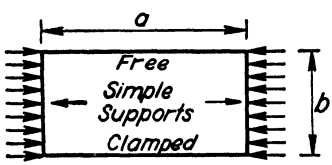
a/b	0.2	0.3	0.4	0.5	0.6	0.7	
k	27.0	13.2	8.41	6.25	5.14	4.53	
a/b	0.8	0.9	1.0	1.1	1.2	1.3	1.4
k	4.2	4.04	4.0	4.04	4.13	4.28	4.47



Case 2

where compressive stress is critical

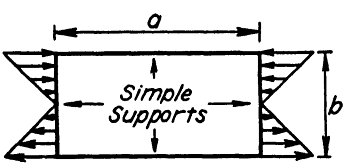
a/b	0.4	0.5	0.6	0.7	0.8	0.9	1.0
k	9.44	7 69	7.05	7.00	7.29	7.83	7.69



Case 3

where compressive stress is critical

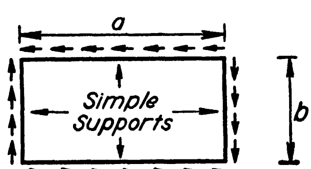
a/b	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7
k	1.70	1.56	1.47	1.41	1.36	1.34	1.33	1.33
a/b	1.8	1.9	2.0	2.2	2.4			
k	1.34	1.36	1.38	1.45	1.47			



Case 4

where bending stress is critical

a/b	0.4	0.5	0.6	0.67	0.75	0.8	0.9	1.0
k	29.1	25.6	24.1	23.9	24.1	24.4	25.6	25.6
a/b	1.5							
k	24.1							

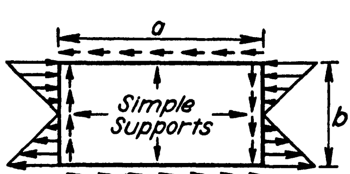


Case 5

where shear is critical

a/b	1.0	1.2	1.4	1.5	1.6	1.8	2.0
k	9.4	8.0	7.3	7.1	7.0	6.8	6.6
a/b	2.5	3.0					
k	6.3	6.1					

Case 6



where bending stress is critical for values of $a/b > 0.5 < 1.0$

s_s/s_s cr.	0.0	0.2	0.3	0.4	0.5	0.6	0.7
k	26.0	25.0	24.0	23.0	22.0	20.0	17.0
s_s/s_s cr.	0.8	0.9	1.0				
k	14.0	10.0	0.0				

Values of k are approximate but satisfactory for comparison. The critical value of the unit shear (s_s cr.) is obtained from Case 5.

* See *Theory of Elastic Stability*, S. Timoshenko, 1936 ed.

In some cases $\mu = 0.25$ has been used in computing k and in other cases $\mu = 0.3$. However, the influence of μ is not great enough to make this discrepancy serious.

ness was 5.5, Roark, Hartenberg, and Williams* report the following factors of flexural stress concentration determined experimentally for varying fillet radii (see Fig. 144).

$\frac{R}{t} =$	0.125	0.15	0.20	0.25	0.30	0.40	0.50	0.70	1.00
$k =$	2.50	2.30	2.03	1.88	1.70	1.53	1.40	1.26	1.20

In other words, the corner stress varies from 1.2 to 2.5 times the stress calculated by the flexure formula and decreases as the radius of fillet is increased.

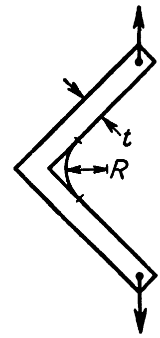


Fig. 144. Fillet as a Stress Reducer.

Torsion of Beam Sections

160. Beam Stresses Produced by Torsion. Torsion of rolled beams produces shear directly and flexure secondarily. Since the beam will also be carrying loads that produce bending moment, it will already be resisting web shear and flange stresses. The combined flange or web stresses control the design.

Flange Flexure. The illustration Fig. 145 shows how lateral flange reactions are developed by torsion. These reactions occur at the ends of the span. If h is the depth between the centroids of the flanges, we may write

$$R = \frac{T}{2h} \quad (T = Pe, \text{ Fig. } 145). \tag{46}$$

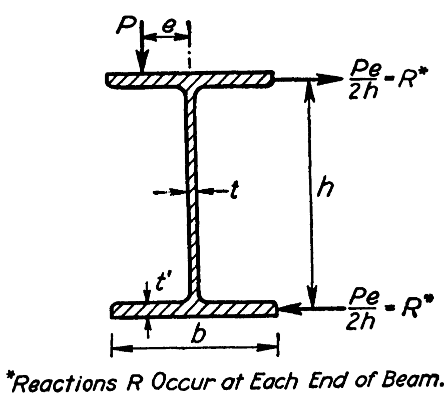


Fig. 145. Lateral Forces Caused by Torsion.

Evidently, the reaction R occurs at *each* end of the span. The corresponding distributed lateral flange loading (taken as uniform) will be found by dividing the end reaction by one half of the span. Thus,

$$w = \frac{T}{Lh}. \tag{47}$$

In this expression, w is the lateral load per foot acting on either flange. We may designate the flange thickness by t' and its width by b. Then its section modulus is $1/6t'b^2$ and its fiber stress produced by torsion will be

* Engineering Experiment Station Bulletin, No. 84, University of Wisconsin, 1938. Also, see *Stress and Strain*, R. J. Roark, McGraw-Hill, 1938.

approximately

$$f_{\text{torsion}} = \frac{\frac{1}{8}wL^2}{S} = \frac{3TL}{4ht'b^2}. \tag{48}$$

This fiber stress is to be added directly to the flexural fiber stress f caused by bending.*

Web and Flange Shear. An approximate method of analysis devised by W. B. Campbell may be used for I-beams and wide flange beams. The general formulas are

$$s_s = \frac{Tt}{J} \text{ for the web shear,} \tag{49}$$

and

$$s_s = \frac{Tt_1}{J} \text{ for the flange shear.} \tag{50}$$

In these formulas the value of J may be found from the expression

$$J = 0.4dt^3 + 0.8t_1^3(b - t). \tag{51}$$

As elsewhere, t is the web thickness, t_1 the average flange thickness, b the flange width, and d the depth of the beam. When the unit web shear is found by the aid of formula (49), it is to be added to the web shear at the juncture with the flange as found from the VQ/It (or V/ht) formula. The flange shear from torsion (equation 50) is not increased appreciably by the shear that accompanies flexure, which is always small for the flange.

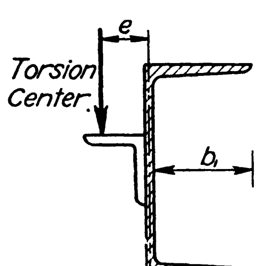


FIG. 146. ELIMINATION OF TORSION.

161. Torsional Center for a Channel. A channel loaded along its upper flange actually *twists* as it deflects. In order to avoid such torsion or to evaluate its effect, we must locate the torsional center through which a load can be applied without twisting the section. According to Seely, this eccentric distance is approximately

$$e = \frac{\frac{1}{2}b_1}{1 + \frac{A_w}{6A_f}}. \tag{52}$$

In this formula, b_1 is the extension of the flange beyond the web, A_w is the web area, and A_f is the area of one flange. See Fig. 146.

* Equation (48) applies only to beams with end connections that will resist torque. The wind connections of building frames provide such resistance. The flange stress will be smaller for beams with light end connections, but the distortions and the web stresses will be increased. However, formulas (47) to (51) are conservative in most instances. See Transactions, *ASCE*, 1936, paper by Lyse and Johnson.

162. Design for Torsion. EXAMPLES *DP*58, *DP*59. The example *DP*58 illustrates the method of checking a wide flange beam section for its resistance to both torsion and direct flexure. It is important to note that under torsion the maximum shearing stress may be in the flange instead of the web. The problem *DP*59 shows rather clearly the serious eccentricity of load that will occur when a load is applied centrally to the flange of a standard channel. We would find a high flange stress if we applied equation (48) to such a case.

PROBLEMS

182. A 21*WF*59 section is stressed to 20,000 lb. per sq. in. at the extreme fiber and has an average web shear of 10,000 lb. per sq. in. Find the maximum direct stress and maximum shear at the juncture of web and flange which is 1⅛ in. from the extreme fiber.
Ans. 22,100 and 13,300 lb. per sq. in.

183. Obtain the information of Problem 182 for four points between the neutral axis and the flange and plot curves for maximum direct stress and for maximum shear.

184. Design an anchor rod at *AISC* working stresses to resist a tension of 40,000 lb. and a cross shear of 19,000 lb. There are no threads. *Ans.* 1¾-in. diam.

185. A knife-edge bearing carries a load of 200 lb. per lineal in. The safe bearing value is 200,000 lb. per sq. in. Find the radius to which the edge must be rounded.
Ans. 0.026 in.

186. Select bridge rollers by *AREA* specifications and working stresses to carry a total end reaction of 300,000 lb. Then check your design by the theory of elasticity.

187. A cast iron cover plate for a circular floor opening is 48 in. in diameter and carries a total uniform load of 175 lb. per sq. ft. Determine its thickness for a tension working stress of 12,000 lb. per sq. in. Edges are simply supported. Repeat for clamped edges. *Ans.* 0.27 in., 0.21 in.

188. Select a battledeck floor plate to span 6 ft. between floor joists and 18 ft. between floor girders by use of *AISC* working stresses. Edges may be considered clamped since they are welded to the beams. The load is 100 lb. per sq. ft.
Ans. 0.3 in., use $\frac{5}{16}$-in. plate.

189. The cover plate for a column is supposed to have a thickness of $\frac{1}{40}$ times the distance between rivets if it is to be stressed to 17,000 lb. per sq. in. Determine the critical buckling stress if the panel shape factor a/b is 1.0. Case 2, Table 25 is applicable.
Ans. 130,000 lb. per sq. in.

190. Repeat *DP*57 for $h/t = 160$, actual shear of 5000 lb. per sq. in., allowable fiber stress of 18,000 lb. per sq. in., and $\mu = 0.25$.

191. Find the radius of weld fillet at a right angle corner that will reduce the flexural stress concentration to about 25 per cent. The plates to be joined by welding are of 2-in. thickness. *Ans.* 1.4 in.

192. Locate the torsional center for the 18-in., 42.7-lb. channel.
Ans. ⅞ in. behind web.

193. Load an 18-in., 45.8-lb. channel as a beam on a span of 18 ft. so that its flexural stress from ordinary bending will be 12,000 lb. per sq. in. The load is uniformly distributed. Determine the combined fiber stress if the load position is in line with the center of the web. Equation (48) is applicable. Use an average flange thickness for t'.

163. Applicability of Theoretical Formulas. The use of theoretical formulas given in this chapter must be tempered by a knowledge of test results. When stresses combine into a maximum stress or a maximum shear, we normally expect to let this theoretical maximum condition con-

DP58. *A 12WF36 beam section with heavy end connections undergoes a flexural moment of 450,000″# and a torsional moment of 20,000″#. The vertical shear is 10,000#. Check this section by AISC specifications and select another if it is unsatisfactory. The beam spans 18′-0″.*

Beam Stresses:

$$\text{Flexural stress} = \frac{450{,}000}{45.9} = 9800\#/\square''.$$

$$\text{Average unit web shear} = \frac{10{,}000}{0.305 \times 12.24} = 2680\#/\square''.$$

Torsion Stresses:

$$\begin{aligned} \text{Equation (51)}\quad J &= 0.4dt^3 + 0.8(t_1)^3(b - t) \\ &= 0.4 \times 12.24 \times 0.305^3 + 0.8 \times 0.54^3(6\ 26) \\ &= 0.14 + 0.79 = 0.93. \end{aligned}$$

Web shear from torsion, Equation (49)

$$s_s = \frac{Tt}{J} = \frac{20{,}000 \times 0.305}{0.93} = 6560\#/\square''.$$

$$\text{Total web shear} = 2680 + 6560 = 9240\#/\square''.$$

$$\text{Flange shear from torsion} = \frac{Tt_1}{J} = \frac{20{,}000 \times 0.54}{0.93} = 11{,}600\#/\square''.$$

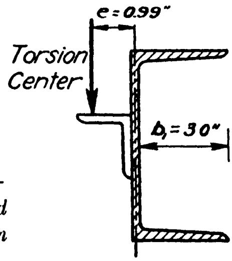
$$\text{Equation (48)}\ f_{torsion} = \frac{3TL}{4ht_1b^2} = \frac{3 \times 20{,}000 \times 18 \times 12}{4 \times 11.7 \times 0.54 \times 6.56^2}$$

$$= 12{,}000\#/\square''.$$

$$\text{Total fiber stress} = 9800 + 12{,}000 = 21{,}800\#/\square''.$$

Since this stress is more than 20,000, the 12WF40 section will be used. Its wider flange will effectively reduce torsion stresses.

DP59. *Locate the torsional center for a 15″-33.9# channel.*



Equation (52)

$$e = \frac{\frac{1}{2}b_1}{1 + \frac{A_w}{6A_f}} = \frac{1.5}{1 + \frac{6.0}{6 \times 1.95}} = 0.99''.$$

Distance from back of flange to load point = 0.99 − 0.2 = 0.79″. (Web thickness = 0.40″.) A load applied at the center of the flange would have a torsional lever arm of 0.79 + 1.70 = 2.49″.

Remarks: *If loaded centrally and not provided with lateral restraint, the channel would be seriously stressed by torsion.*

trol the design. The only escape from this necessity is in a few instances where the existence of such combined stresses is allowed for by a reduced working stress. Thus the allowable rivet shear reflects the knowledge that hot driven rivets resist initial tension and that the combined shear is greater than the average unit shear.

Bearing stresses found by theoretical analysis are point or line stresses that may not be very significant when the load is not repeated rapidly. In machinery design, such highly localized stresses may result in ultimate failure. This observation also applies to stress concentrations at holes and at changes of cross-section as well. The neglect of high localized stresses presupposes a slight plastic flow which is only permissible if the load is static or is repeated infrequently.

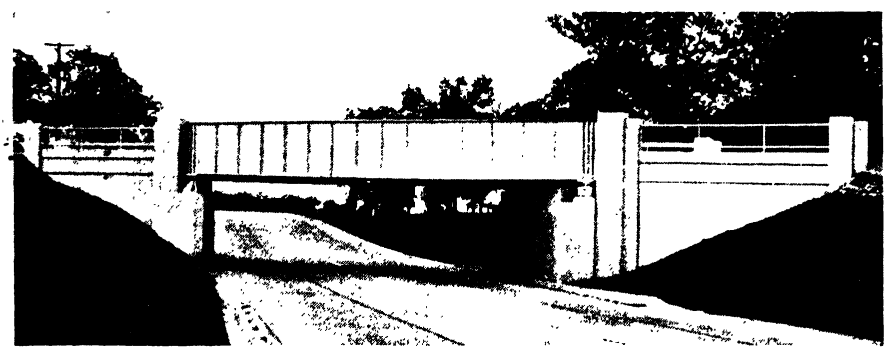
Theoretical formulas for plate stresses are a good guide to design. Of course, the maximum stress occurs only at a point or along a line, but this stress is not reduced greatly at a short distance away. It is therefore a significant stress. The buckling formulas for plates are as logical as the Euler formula for columns. They apply properly to thin plates but not to thick ones. The web of a plate girder may have a depth-thickness ratio of 170 which is equivalent to an L/r value of 590, for comparison with limitations on column formulas.

Torsion of structural sections has long been neglected in design. The theoretical formulas presented here are approximate because the actual coefficients of rolled sections are quite complex, but nevertheless, the results are useful. The illustrative problems show that torsion may be a serious factor in producing beam failures. Design including the effect of torsion need not be particularly difficult or tedious.

CHAPTER 11

DESIGN OF PLATE GIRDERS

164. Plate Girders for Use in Buildings. Where heavy loads must be carried for spans above 40 ft. and for nearly all spans over 60 ft., it will be found that the largest rolled girder sections are usually too small. Hence, a built-up girder or a truss must be used. The designer has the choice of welded or riveted construction. The welded girder is quite common for use in buildings. Its use in bridge design has been delayed by a lack of information about the impact resistance of weld metal. Tests have demonstrated that the proper kinds of welds do possess excellent resistance to impact. We can expect, therefore, that welded bridges will become as common as riveted ones.



Courtesy C. M. St. P. & P. R.R. Co.

FIG. 147. ATTRACTIVE PLATE-GIRDER BRIDGE.

The weight of a welded girder is considerably less than the weight of a riveted girder (about 15 per cent less). The reduction in weight comes from the following changes:

1. The full gross area of the tension flange is available, while rivet holes must be deducted from the tension flange of a riveted girder.*

2. One sixth of the web area may be considered as a part of each flange. This allowance is but one eighth for a riveted girder because of rivet holes in the web.

* Recent specifications for building girders (*AISC*, given in § 215) do not require the deduction of rivet holes. It remains to be seen whether this specification is accepted by all authorities.

3. The effective depth of a welded girder is greater than the effective depth for a riveted girder because the center of gravity of the flange is not lowered by down-turned angle legs. The flange may consist of plates only.

4. Stiffener plates are used in place of stiffener angles so that the weight of the angle legs in contact with the web is saved.

5. There is no need for filler plates under the stiffener angles, as used in riveted girders.

Plate-Girder Theory. Since the basic assumptions and the theory of plate-girder design have been discussed in detail in Chapter 6 of Vol. 1, *Theory of Modern Steel Structures*, the reader is merely referred to that source for such background information. The theory of the plate girder is also presented in other standard textbooks.

WELDED GIRDER

165. Design of a Welded Building Girder.

PROBLEM. A welded girder is to be designed to carry a concentrated load plus a uniform load. These are static loads obtained from heavy equipment placed in an industrial building. The ends of the girder are supported on concrete pilasters. The compression flange is supported laterally against buckling.

Data.

Span = 50 ft.-0 in.
Depth. Not limited.
Concentrated center load = 67,000 lb.
Uniform load = 2750 lb. per ft. not including the weight of the girder.
Impact. No allowance for impact.

Working Stresses.

Flexure. 20,000 lb. per sq. in. where flanges are supported laterally.
Fillet welds. 13,600 lb. per sq. in. shear on throat (special welding).

Specifications.

AISC specifications for Buildings, § 215, p. 396.
AWS Code for Fusion Welding in Buildings, § 217, p. 420.

165a. Design of the Web and Stiffeners.

Depth of girder. Taken at $\frac{1}{10}$ of the span or 5 ft.-0 in.
Web depth = 58 in. (Economical depths range from $\frac{1}{8}$ to $\frac{1}{12}$ of the span.)
The weight of the girder is estimated at 200 lb. per ft.
End shear $= (67{,}000 \div 2) + (25 \times 2950) = 107{,}300$ lb.
Web thickness $= 107{,}300 \div (13{,}000 \times 58) = 0.143$. (Spec. 10.)
Minimum web thickness $= 58 \div 170 = 0.34$. (Spec. 42.) Use $\frac{3}{8}$-in. web.
Unit web shear $= 107{,}300 \div (58 \times 0.375) = 4930$ lb. per sq. in.
Need for Stiffeners. If h/t is equal to or greater than 70, *intermediate stiffeners* are required wherever h/t exceeds $8000/\sqrt{s_s}$.

Near the reaction point, $8000/\sqrt{s_s} = 8000 \div \sqrt{4930} = 114$. (Spec. 45.)
Value of $h/t = 58 \div 0.375 = 155$. Hence, stiffeners are needed.

$$\text{Stiffener spacing} = d = \frac{270{,}000t}{s_s}\sqrt[3]{\frac{s_s t}{h}} \quad \text{(Spec. 45)}$$

$$= \frac{270{,}000 \times 0.375}{4930}\sqrt[3]{\frac{4930}{155}} = 65 \text{ in.}$$

Stiffeners will be placed at 58-in. spacing which is the clear depth of the web. They will be used throughout the length of the span.

Stiffener plates for girders of ordinary depth should be made about as wide in inches as the depth of the girder in feet. Accordingly, 5-in. stiffener plates will be used. The thickness will be made $\frac{1}{12}$ of the width to meet Spec. 17 conservatively, or, $t = \frac{5}{12} = 0.415$ in. Use $5 \times \frac{7}{16}$-in. plates placed in pairs on opposite sides of the web. See Fig. 148 for details.

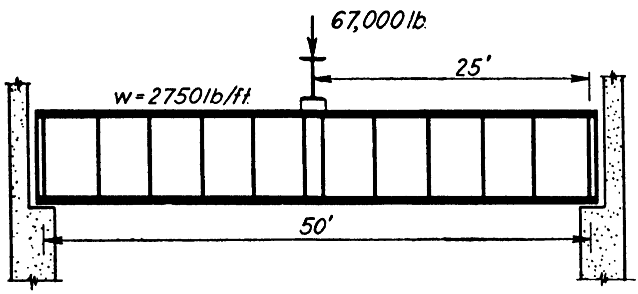


FIG. 148. GIRDER DIMENSIONS AND LOADS.

Strength of Stiffener Welds. At 13,600 lb. per sq. in. on the throat, the value of a fillet weld in shear or tension is 1200 lb. per lineal in. per $\frac{1}{8}$ in. of fillet. This means that a $\frac{5}{16}$-in. weld has a value of 3000 lb. per lineal in., a $\frac{3}{8}$-in. weld has a value of 3600 lb., etc. Intermittent welds are used where the full length of fillet is not required. The minimum intermittent length of weld is 2 in. and these 2-in. welds should not be spaced more than 4 in. in the clear.

Welding stiffeners to web. It has been common practice to rivet stiffener angles to the web with $\frac{3}{4}$-in. rivets at about 5-in. centers. A $\frac{3}{4}$-in. rivet in double bearing on a $\frac{3}{8}$-in. web has a value of 11,200 lb. Four $\frac{1}{4}$-in. fillet welds will be used in connecting each pair of stiffener plates to the web. These welds have a value of $4 \times 2400 = 9600$ lb. per lineal in. The equivalent of the rivet strength requirement is 1.4 in. of weld in 6 in., but the minimum intermittent weld of 2 in. of fillet in each 6-in. length will be used.

End Stiffeners. The reaction to be transferred into the web is 107,300 lb. The stiffeners act as a column whose height is usually taken at one half of the depth of the web. This is a stiff short column for which the allowable stress is nearly the maximum of 17,000 lb. per sq. in.; $107{,}300 \div 17{,}000 = 6.32$ sq. in. of area. It is wise to furnish about $\frac{2}{3}$ of this area or 4.2 sq. in. in one pair of end stiffeners because it is not unlikely that one pair will receive more than one half of the total reaction. (An even distribution of end reaction could only be obtained by use of a pin connected end bearing.) Use two pairs of intermediate stiffeners as end stiffeners. The area furnished by each pair is 4.38 sq. in. Place the end stiffener plates not more than 6 in. apart.

Welding end stiffeners to web. A reaction equal to $\frac{2}{3} \times 107{,}300$ lb. or 71,600 lb. must be transferred into the web by one pair of stiffeners. The length of $\frac{1}{4}$-in. weld required is $71{,}600 \div 2400 = 30$ in. The length furnished by 4 intermittent welds of

minimum size ($\frac{1}{4}$–2–6) in the 58-in. depth of the web is 77 in. Accordingly, the weld used on the intermediate stiffener is also ample for the end stiffeners.

Stiffeners under Concentrated Load. The stiffeners to be placed under the concentrated load will also be two pairs of intermediate stiffeners. Since the intermediate load is less than the end reaction, the arrangement of stiffeners used at the end reaction will be amply strong. These stiffeners should be spaced far enough apart to support the load properly. (See Fig. 149.)

165b. Design of the Flange (*assuming lateral support*).

$$\text{Maximum moment} = \left(\frac{67{,}000 \times 50}{4} + \frac{2950 \times 50^2}{8}\right) 12 = 21{,}100{,}000 \text{ in-lb.}$$

Approximate effective depth. Assumed as depth of web + 1 in. = **59 in.**
Flange area required = 21,100,000 ÷ (59 × 20,000) = 17.9 sq. in.
Effective area of web = $\frac{1}{6} \times \frac{3}{8} \times 58$ = 3.6 sq. in.
Area in flange plates = 17.9 − 3.6 = 14.3 sq. in.

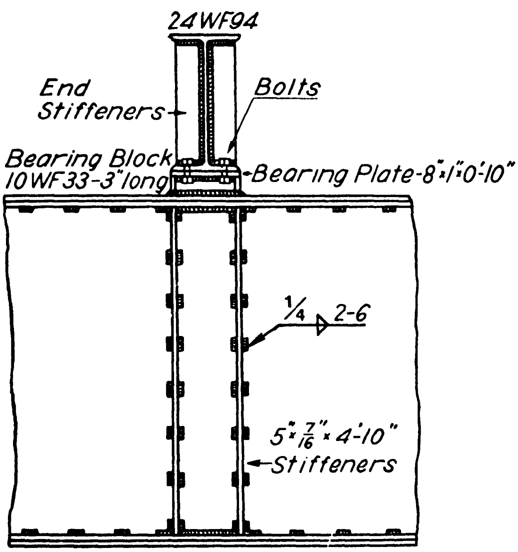


Fig. 149. Load Seat.

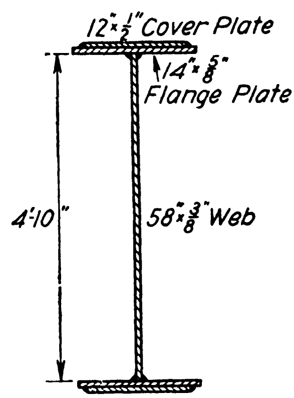


Fig. 150. Cross-Section.

Flange Section. Use one 14 × $\frac{5}{8}$-in. flange plate and one 12 × $\frac{1}{2}$-in. cover plate. The gross area furnished is 14.75 sq. in.

Gross moment of inertia.

Web = $\frac{1}{12} \times 58^3 \times 0.375$ = 6,100 in.^{4}
14 × $\frac{5}{8}$-in. plates = $2 \times 8.75 \times 29.31^2$ = 7520 × 2 = 15,040
12 × $\frac{1}{2}$-in. plates = $2 \times 6.0 \times 29.87^2$ = 5360 × 2 = 10,720

Total I = 31,860 in.^{4}

$$\text{True fiber stress} = \frac{21{,}100{,}000 \times 30.12}{31{,}860} = 19{,}900 \text{ lb. per sq. in.}$$

The flange as selected above will be used. (See Fig. 150.)

Cut-off Point of Cover Plate. The net moment of inertia of the web and of the 14-in. plates is 21,140 in.^{4} The allowable moment is

$$\frac{20{,}000 \times 21{,}140}{29.62} = 14{,}300{,}000 \text{ in-lb.}$$

The bending moment at the quarter point is 13,400,000 in-lb. The 12-in. cover plate could be cut off at that point but in Fig. 151 it is shown extending to the support.

Flange Welds. The purpose of the flange welds is to resist the horizontal shear between web and flange or between component parts of the flange. The shear per lineal inch may be found by the exact formula, $\frac{VQ}{I}$, or by the approximate formula, V/h where

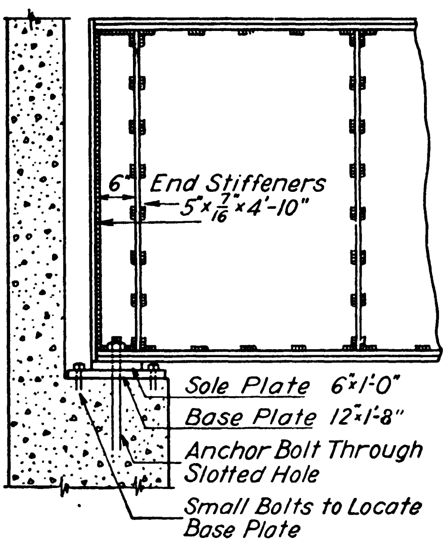


FIG. 151. END SEAT

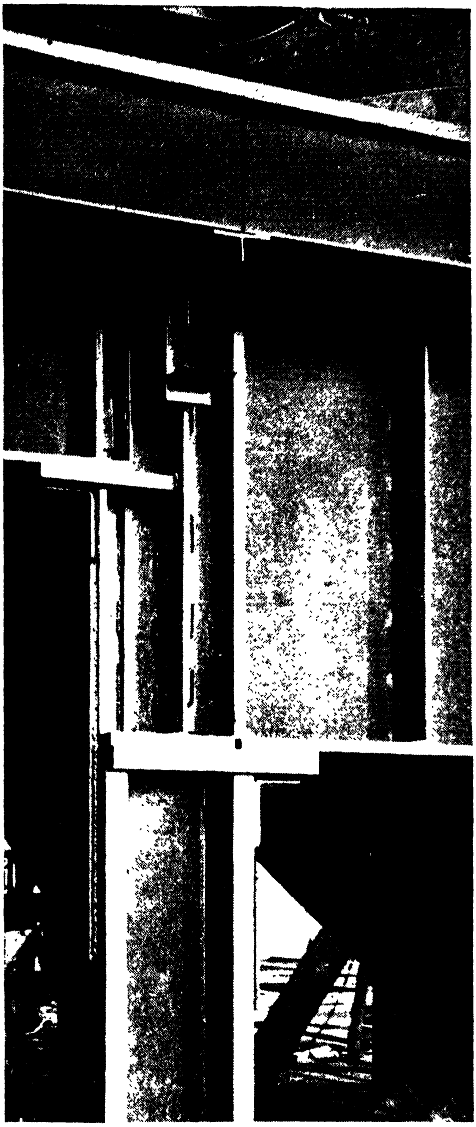
V = total shear at the section,
I = gross moment of inertia of the cross-section,
Q = statical moment about the neutral axis of the area of flange outside of the section on which shear is being calculated,
h = depth between top and bottom welds.

Weld between web and flange. Shear $= \frac{VQ}{I} = \frac{107{,}300}{31{,}860}(14.75 \times 29.5) =$ 1460 lb. per lineal in. A $\frac{5}{16}$-in. intermittent weld 2 in. long, spaced 4 in. in the clear on each side of the web, produces a shear value of 2000 lb. per lineal in. This is ample resistance. Note that these calculations are changed if the cover plate is cut off at the quarter point.

Weld between two flange plates. Shear $= \frac{VQ}{I} = \frac{107{,}300(6 \times 29.87)}{31{,}860} = 604$ lb. per lineal in. The minimum $\frac{1}{4}$-in. intermittent weld of 2 in. length, spaced 4 in. in the clear, offers strength far in excess of the requirement.

Approximate formula. Note that the shear between flange and web as given by the approximate formula is $\frac{107{,}300}{58} = 1850$ lb. per lineal in. This value is greater than the exact shear of 1460 lb. and, accordingly, it would produce a safe design. The shear between plates could be approximated as $\frac{6}{14.75} \times 1850 = 750$ lb. per lineal in., also a safe value.

165c. Bearing for Concentrated Load. A short length of 10*WF*33 beam section makes an excellent bearing block for the concentrated load. This section is welded to the top flange and its web bears directly over the web of the girder while its flanges bear directly over the stiffeners. The detail is shown in Fig. 149.



Courtesy Eng. News-Record

FIG. 152. WELDED COLUMN CAP AND GIRDER CONNECTIONS.

This rather complex welded connection formed a part of the supporting structure of the coal bunker for the Mercury Steam-Electric Station of the General Electric Company. A plate girder with welded stiffeners extends to the right.

Bearing for End Reaction. A sole plate 12 in. wide and 6 in. long will localize the bearing pressure and transmit it properly into the end stiffeners. Since the girder rests upon concrete, a bearing plate 15 in. $\times$ 12 in. is required to keep the bearing pressure below 600 lb. per sq. in. Plates 1 in. thick are ample because the cantilever extension of the base plate beyond the sole plate is only 3 in. The detail is shown in Fig. 151.

Riveted Girder

166. Design of a Riveted Girder.

Problem. Redesign the girder designed in § 165 by changing to riveted construction. Follow *AISC* specifications but reduce effective section for rivet holes.

Data.

Span = 50 ft.-0 in.
Concentrated center load = 67,000 lb.
Uniform load = 2750 lb. per ft. applied by a floor slab to the upper flange.
Impact. No allowance for impact.

Working Stresses.

Follow *AISC* specifications, § 215, p. 396.

166a. Design of the Web.

Depth. Taken at $1/10$ of the span or 5 ft.-0 in. Angles placed 60.5 in. back to back.

End shear = 107,300 lb.

Web thickness. Minimum thickness where 6-in. angle legs are used is $(60 - 11.5) \div 170 = 0.285$. Use $5/16$-in. web.

Web shear = $107{,}300 \div (60 \times 0.312) = 5750$ lb. per sq. in.

166b. Selection of Flange Section (*assuming lateral support*).

Effective depth. Taken at depth of web or 60 in.

Maximum moment = 21,100,000 in-lb.

Required net area = $21{,}100{,}000 \div (20{,}000 \times 60) = 17.6$ sq. in.

Area furnished by web = $1/8 \times 60 \times 5/16 = 2.34$ sq. in.

Net area of flange = $17.6 - 2.34 = 15.26$ sq. in.

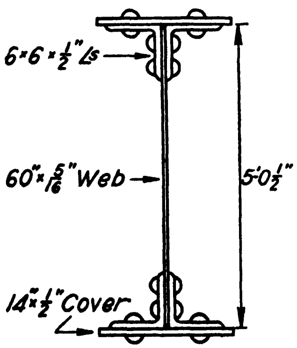


Fig. 153. Riveted Cross-Section.

Angles. Use two $6 \times 6 \times 1/2$-in. angles. Deduct two $7/8$-in. holes for $3/4$-in. rivets from each angle. Net area = 9.75 sq. in.

Cover plate. The net area to be furnished by the cover plate is $15.26 - 9.75 = 5.51$ sq. in. A $14 \times 1/2$-in. cover with two $7/8$-in. holes deducted furnishes a net area 6.12 sq. in. but a $7/16$-in. cover is inadequate.

Cut-off Point for Cover Plate. With the cover cut off, the section is capable of resisting a moment of $(17.6 - 5.51) \times 60 \times 20{,}000 = 14{,}500{,}000$ in-lb. The moment at the quarter point is 13,400,000 in-lb. and the cover plate will be cut off there. The more usual procedure is to plot the moment diagram and determine the location of the point where the bending moment is

14,500,000 in-lb. Then the cover is cut off at 1 ft. from this point toward the end of the beam.

Checking Stress in Flange. The fiber stress in the flange will be checked by the computation of the net moment of inertia of the section. Rivet holes will be deducted from both the tension and compression flanges and the neutral axis will be retained at the mid-height.

Moment of inertia of gross section.

Web = $\frac{1}{12} \times \frac{5}{16} \times 60^3$	=	5,620 in.^{4}
Angles about gravity axis	=	80
Angles transferred to N.A. = $5.75 \times 4 \times 28.6^2$	=	18,800
Cover plate about N.A. = $7 \times 2 \times 30.5^2$	=	13,040
Total gross I	=	37,540 in.^{4}

Moment of inertia of rivet holes.

Web = $2 \times (\frac{7}{8} \times \frac{5}{16})(2.5^2 + 7.5^2 + 12.5^2 + 17.5^2 + 22.5^2 + 27.75^2)$	=	980 in.^{4}
Down-turned legs of angles = $4(\frac{7}{8} \times \frac{1}{2}) \times 27.75^2$	=	1350
Out-turned legs and cover plate = $4(\frac{7}{8} \times 1) \times 30.25^2$	=	3210
		5540 in.^{4}

Net moment of inertia = 37,540 − 5540 = 32,000 in.^{4}

$$\text{Fiber stress. } f = \frac{Mc}{I} = \frac{21,100,000 \times 30.75}{32,000} = 20,300 \text{ lb. per sq. in.}$$

Remarks. The reduction of gross moment of inertia by the moment of inertia of holes in *both tension and compression sides of the beam* may seem overly conservative. However, if holes are deducted from the tension flange only, the upward movement of the neutral axis must be considered, and the extreme fiber distance must be increased (c is increased about $2\frac{1}{4}$-in.). If the fiber stress be computed in this way, it will again be found to be approximately 20,000 lb. per sq. in. The *AISC* specifications permit the entire neglect of rivet holes. (Spec. 41.) The corresponding maximum fiber stress based upon gross moment of inertia is only 17,300 lb. per sq. in.

166c. Stiffeners. Standard stiffener angles for use with 6-in. flange angles are $5 \times 3 \times \frac{3}{8}$ in. They are used in pairs on opposite sides of the web and are riveted to the web with $\frac{3}{4}$-in. rivets at 5-in. centers. Filler plates $3\frac{1}{2} \times \frac{1}{2}$ in. must be used between flange angles under the stiffeners or else the stiffener angles must be *crimped* around the flange angles. The practice of crimping stiffeners is considered objectionable.

Maximum spacing of interior stiffeners.

$$d = \frac{270,000t}{s_s} \sqrt[3]{s_s t/h} = \frac{270,000 \times 0.312}{5750} \sqrt[3]{\frac{5750 \times 0.312}{48.5}} = 48.7 \text{ in.}$$

End Stiffeners. The end stiffeners act as a short column; the required area is 107,300 ÷ 17,000 = 6.32 sq. in. Two-thirds of this area should be furnished by one pair of stiffener angles since the bearing on the two pairs may not be equal. $6.32 \times \frac{2}{3} = 4.22$ sq. in. Use $5 \times 3 \times \frac{3}{8}$-in. angles which furnish 5.72 sq. in. per pair.

Bearing on outstanding angle leg. Only the outstanding leg can be in bearing on the flange angles. (Because of the curved fillet on the inside of the flange angles, the leg of the stiffener angle in contact with the web must be cut short.) Bearing stress = $\frac{2}{3} \times 107,300 \div (9.25 \times 0.375)$ = 20,700 lb. per sq. in., which is quite low.

Rivets to web. The rivets connecting the end stiffener angles to the web must transfer the end reaction into the web through bearing. The value of a ¾-in. rivet in double bearing on a $\frac{5}{16}$-in. web is 9370 lb. Each pair of stiffener angles must contain (⅔ × 107,300) ÷ 9370 = 8 rivets. Rivets will be used at 5-in. spacing as for intermediate stiffeners. The use of 11 rivets justifies acceptance of loose fills. (Spec. 27.)

Stiffeners at Concentrated Load. Two pairs of 5 × 3 × ⅜-in. stiffener angles will be used here. In Fig. 154, the stiffener angles at the center should be turned back to back so that their outstanding legs will be under the load.

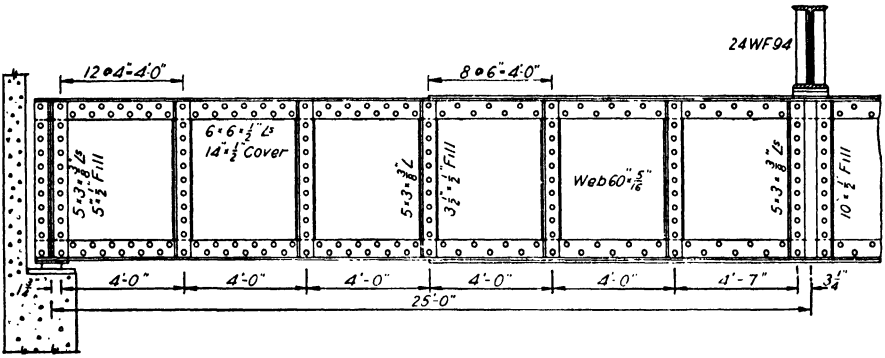


FIG. 154. RIVETED GIRDER.

166d. Rivets Between Flange Angles and Web.

Rivet value. The value of ¾-in. rivets in double bearing on a $\frac{5}{16}$-in. web is 9370 lb.

Rivet spacing. The approximate formula for rivet spacing to be used here is

$$Vs = Rh.$$

V = total vertical shear (107,300 lb.).
s = rivet spacing.
R = rivet value (9370 lb.).
h = depth between lines of rivets (60.5 − 2 × 3.5 = 53.5).

Hence, we may write

$$s = \frac{Rh}{V} = \frac{9370 \times 53.5}{107,300} = 4.7.$$ Use a 4-in. rivet spacing.

Resultant rivet shear. The vertical shear per rivet for a floor load of 2750 lb. per lineal foot resting on the upper flange is 2750 × 4.0 ÷ 12 = 920 lb. The horizontal shear per rivet is $\frac{Vs}{h} = \frac{107,300 \times 4.0}{53.5} = 8100$ lb. The resultant shear is $\sqrt{930^2 + 8100^2}$ = 8150 lb. per rivet. The effect of the vertical shear is negligibly small.

Rivet details. Use the rivet spacing of 4 in. between the end and the quarter point. A 6-in. spacing is sufficient between the quarter points and the center of the girder.

Rivets between Angles and Cover Plate. Two rows of ¾-in. rivets will be used at their maximum spacing of 6 in. A check on the horizontal shear will show that this spacing, which is the greatest permitted by some specifications, provides excess resistance to shear.

166e. End Bearing and Bearing under Concentrated Load. These details might be the same as those for the welded girder of § 165. The only special requirement would be

the use of a few countersunk rivets through the flange at the end and possibly at the concentrated load.

167. Weight Estimates of Riveted and Welded Girders.

Riveted Girder.

Web. 60 × 5/16 in., one piece, 51 ft.-0 in. long	=	3,260 lb.
Angles. 6 × 6 × 1/2 in., 4 pieces, each 51 ft.-0 in. long	=	4,000
Covers. 14 × 1/2 in., 2 pieces, each 25 ft.-0 in. long	=	1,190
Stiffener angles. 5 × 3 × 3/8 in., 32 pieces, each 4 ft.-11 1/2 in. long	=	1,570
Filler plates. 3 1/2 × 1/2 in., 32 pieces, each 4 ft.-1/2 in. long	=	780
Total	=	10,800 lb.

Unit weight = 212 lb. per ft.

Welded Girder.

Web. 58 × 3/8 in., one piece, 51 ft.-0 in. long	=	3780 lb.
Flange plates. 14 × 5/8 in., 2 pieces, each 51 ft.-0 in. long	=	3040
Cover plates. 12 × 1/2 in., 2 pieces, each 25 ft.-0 in. long	=	1020
Stiffener plates. 5 × 7/16 in., 28 pieces, each 4 ft.-10 in. long	=	1010
Total	=	8850 lb.

Unit weight = 174 lb. per ft.

Comparison of Weights. The welded girder weighs 18 per cent less than the riveted girder. Its cost, however, probably would not be as much as 18 per cent less than the cost of the riveted girder because welding has usually cost more than punching holes and driving rivets.

PROBLEMS

194. Redesign the riveted girder of § 166 using *AISC* specifications and apply Spec. 41.

195. Redesign the welded girder designed in § 165, but change the span to 40 ft.-6 in. Compute the weight.

196. Redesign the riveted girder designed in § 166, but change the span to 40 ft.-6 in. Compare the weight of this girder with the weight of the similar welded girder from Problem 195.

197. Design a welded girder of 60 ft.-9 in. span to carry a uniform live load of 4000 lb. per ft. of span. Select a channel as a flange section to be strengthened by the use of a welded cover if necessary. The flanges have adequate lateral support. Turn the channel flanges in. Design by use of the *AASHO* specifications. Allow for impact. Compute the weight.

198. Design a riveted girder to replace the welded girder of Problem 197. Compare its weight with the weight of the welded girder.

199. Design the web at the section of maximum shear and the flanges at the section of maximum moment for a railway plate girder of riveted construction where the span is 80 ft.-0 in. The structure is of the deck type. Assume the dead load including the weight of two girders to be 1600 lb. per ft. of bridge. The equivalent uniform live load for maximum moment is 7500 lb. and for maximum shear is 9300 lb. per ft. of track. Use the *AREA* specifications. Let the flanges be supported laterally at panel points 16 ft. apart. This is a single track structure.

200. Redesign the girder of Problem 199 for welded fabrication. Use *AREA* and *AWS* specifications.

201. Design a building girder to span 48 ft.-6 in. and to carry a total load including its own weight of 3000 lb. per ft. The depth is limited to 42 in. Design for both welded and riveted construction and compare the weights. Assume that the top flange is supported laterally and follow the *AISC* and *AWS* specifications.

202. Design a riveted girder to support a flat roof for a span of 80 ft. Use a depth of 8 ft. back to back of angles. The load on the girder is 1500 lb. per ft. Follow the *AASHO* specifications. The weight of this girder may be compared with the weight of the riveted truss from Problem 216, for the data are identical.

203. Redesign the girder of Problem 202 for fabrication by arc welding. Since the loading is light, a depth of less than 8 ft. may be preferred.

168. Plate-Girder Design. The plate girder is one of the most common fabricated steel structures. Its design should therefore be simplified as much as possible so that designers will continue to take advantage of its wide usefulness. The welded girder is designed rapidly because its gross section is effective. Also, its simple cross-section makes it possible for the designer to guess at the effective depth without the possibility of serious error. Some specifications permit the gross cross-section of the riveted girder to be considered effective. The neglect of rivet holes in the tension flange is difficult to justify theoretically. Design procedures neglecting the effect of rivet holes should be considered as experimental until adequate tests are reported to justify the entire neglect of rivet holes in beams and girders.

Until a few years ago the plate girder was considered to be uneconomical for spans above 100 ft. A recent girder span, the approach to Cleveland's Main Avenue bridge, has a length of 271 ft. The Connecticut highway department has designed a 300-ft. girder span for the new Hartford bridge over the Connecticut River. Even though a plate-girder bridge proves to be heavier than a truss bridge of equal strength, it may be chosen because of several advantages: its fabrication and erection costs are less per pound of weight, it can be erected more rapidly than a truss, and its appearance is in harmony with modern architectural design.

CHAPTER 12

ROOFS FOR INDUSTRIAL BUILDINGS

169. Design of Roofs for Industrial Buildings. The roof covering for an industrial building such as a shop, foundry, car barn, warehouse, or for a gymnasium is usually of light weight self-supporting material such as corrugated steel or corrugated asbestos. These materials will span from 2 to 5 ft. between purlins. The purlins support the roof loads between trusses which in turn are supported upon columns or masonry walls.

170. Roof Loads. Industrial building roofs must be designed to carry their own dead load plus the snow load and wind load.

DEAD LOAD. The weight of the roof includes the weights of covering, purlins, trusses, and bracing.

Weight of Corrugated Steel. For a load of about 40 lb. per sq. ft. of roof surface, the following thicknesses of corrugated steel are recommended where the depth of corrugation is 2½ in.

No. 22 gage (U. S. standard), wt. 1.5 lb. per sq. ft., span 3 ft.-6 in.
No. 20 gage (U. S. standard), wt. 1.8 lb. per sq. ft., span 3 ft.-10 in.
No. 18 gage (U. S. standard), wt. 2.3 lb. per sq. ft., span 4 ft.-6 in.
For 30 lb. per sq. ft., decrease to the next even gage number.
For 50 lb. per sq. ft., increase to the next even gage number.

Weight of Corrugated Asbestos. For a load of from 30 to 40 lb. per sq. ft. of roof surface, the following thicknesses of corrugated asbestos are recommended where the depth of corrugation is at least 2½ in.

¼-in. thickness, wt. 3 lb. per sq. ft., span 4 ft.-0 in.
⅛-in. " " 4 lb. per sq. ft., span 5 ft.-0 in.
⅜-in. " " 4½ lb. per sq. ft., span 6 ft.-0 in.

Weight of Steel Purlins. The unit weight varies from 2 to 4.5 lb. per sq. ft. of *roof surface.* The low value applies for light loads and short spans between roof trusses, while the larger value applies for heavy wind or snow loads and long spans between trusses.

Weights of Roof Trusses. For roof trusses of from ⅓ to ¼ pitch, the weight usually varies from 2 to 3.5 lb. per sq. ft. of *roof surface* where the span is 40 ft. For longer spans, add a pound for each 10 ft. up to 80 ft. For flat roofs, add from ½ lb. to 1 lb. to the above figures, and for very steep roofs, reduce these values from ½ lb. to 1 lb.

Courtesy Eng. News-Record

FIG. 155. ERECTION OF AN INDUSTRIAL BUILDING.

Weights of Struts and Diagonal Bracing. The bracing may be assumed to add from 1 to 2 lb. per sq. ft. of *roof surface* to the dead load. The lower value will apply for trusses of short span for which there is little or no bracing in the plane of the lower chords.

SNOW LOAD. The snow load varies from 0 to 50 lb. per sq. ft. of *roof surface.* For roofs from ⅓ to ¼ pitch, a snow load of 15 to 20 lb. is common in the central states and from 5 to 10 lb. in the southern states. These values may be halved for very steep roofs and doubled for flat roofs. They are all given in pounds per square foot of roof surface.

WIND LOAD. Maximum wind loads vary for different parts of the country and are usually specified in the building code.* A load of 20 lb. per sq. ft. on a vertical surface is a common value for low structures, while 30 lb. per sq. ft. is usually recommended for higher structures and for structures that are unprotected by near-by buildings.

The wind pressure P in pounds per square foot on a vertical surface may be changed into normal pressure P_n in pounds per square foot of inclined surface by the straight line formula.

$$P_n = \frac{P\theta}{45} \text{ (where } \theta \text{ is the angle of inclination in degrees).}$$

RIVETED TRUSS DESIGN

171. Design of a Roof for a Gymnasium. The procedure developed for the design of this roof will apply to the design of any roof supported by simple trusses.

PROBLEM. Design a Fink type roof truss with purlins and bracing to serve as a roof for a small gymnasium. The truss is to be supported upon brick bearing walls. Make a detail drawing of the structure. The design is controlled by a City Building Code which specifies special working stresses but otherwise accepts the *AISC* specifications.

Data.

Dimensions of building to outside of walls. 37 × 50 ft.
Clear height desired = 20 ft.
Wind pressure = 20 lb. per sq. ft. on a vertical surface.
Snow load = 30 lb. per sq. ft. on a horizontal surface.
Type of construction. Riveted steel structure.

Allowable Stresses. (City Building Code)

Tension = 18,000 lb. per sq. in.

$$\text{Compression} = \frac{18{,}000}{1 + \dfrac{L^2}{18{,}000r^2}}, \text{ not to exceed 15,000 lb. per sq. in.}$$

Rivet shear = 13,500 lb. per sq. in.
Rivet bearing = 24,000 with single shear and 30,000 with double shear.

* A most complete set of recommendations on Wind Stresses was made by Subcommittee 31 of the *ASCE*. See Proceedings, *ASCE*, June 1939, pp. 969–1000.

171a. General Design.

Thickness of walls. 12 in. bearing walls are required by nearly all building codes and will be used here.

Span of roof truss = 37 ft.-0 in. less 1 ft.-0 in. = 36 ft.-0 in.

Spacing of roof trusses. The economic spacing of short span roof trusses usually is between 15 and 20 ft. Two end trusses and two interior trusses will be used. The spacing will be 16 ft.-3 in.

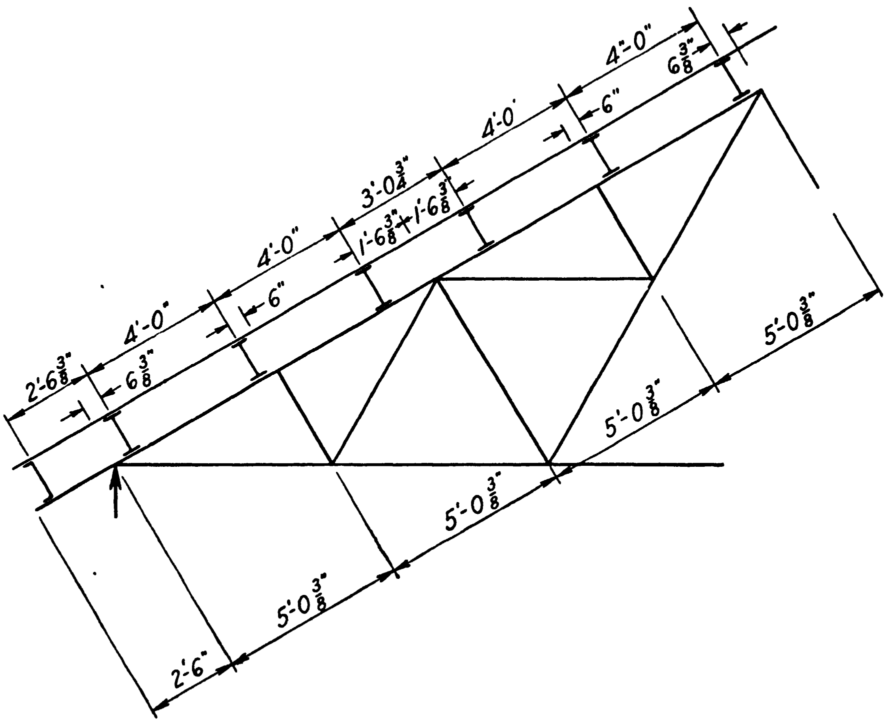


FIG. 156. PURLIN SPACING.

Roof covering. ¼-in. corrugated asbestos roofing will be used to cover the roof and the gable ends of the building. This material is capable of spanning 4 ft.-0 in. between purlins and as much as 6 ft.-0 in. between girts.

Pitch of roof. The height of the roof will be made ¼ of the span or 9 ft.-0 in. This is a ¼ pitch.

Bracing. Diagonal bracing will be used in the plane of the upper chords. Two longitudinal struts will be used as near as possible to the one-third points of the span in the plane of the lower chords.

Type of roof truss. A Fink roof truss will be used with the upper chord divided into four equal panels. The panel length is 5 ft.-⅜ in. An eave overhang of 2 ft.-6 in. is allowed. (See Fig. 156.)

Arrangement for Transportation. In order to ship the trusses to the site by truck, it is necessary to fabricate them in three parts and then to connect the parts in the field. The truss is divided up as shown in Fig. 157, which makes necessary the arrangement of field connections at L_2, L_3 and U_4.

171b. Weight Estimate and Loadings. *Dead Load.*

Roof covering. ¼-in. corrugated asbestos weighing 3 lb. per sq. ft. of roof surface will be satisfactory.

Insulation board. ½-in. insulation board weighing 1½ lb. per sq. ft. is attached to the under side of the purlins. Purpose, to improve the inside appearance of the roof and to reduce heat loss.

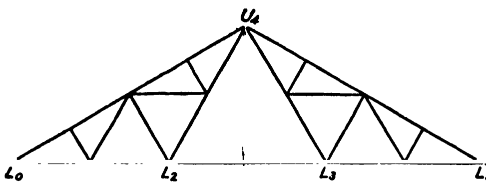


FIG. 157. FIELD CONNECTIONS.

Purlins. Steel purlins at 4-ft. centers to carry medium loads for a medium span length of 16 ft. are estimated to weigh 3.0 lb. per sq. ft. of roof surface.

Roof trusses. Estimated at 2.0 lb. per sq. ft. of roof surface.

Struts and bracing. Adequate bracing will weigh about 1½ lb. per sq. ft. of roof surface.

Total dead weight = 3 + 1½ + 3 + 2 + 1½ = 11.0 lb. per sq. ft.

Dead weight per panel = 11.0 × 5.03 × 16.25 = 900 lb. This load is to be considered as acting at the upper chord panel points.

Snow Load.

Location. The building is assumed to be in the central part of the United States.

Snow load on a horizontal surface = 30 lb. per sq. ft.

Snow load on a roof of ¼ pitch = 20 lb. per sq. ft. of roof surface.

Panel concentration = 20 × 16.25 × 5.03 = 1640 lb.

Wind Load.

Location. Unexposed location protected by surrounding buildings.

Pressure on a vertical surface = 20 lb. per sq. ft.

Pressure on a roof of ¼ pitch. ¼ pitch makes an angle of 26°-40′ with the horizontal plane.

$$P_n = \frac{P\theta}{45} = \frac{20 \times 26.6}{45} = 11.8 \text{ lb. per sq. ft. of roof surface.}$$

Panel concentration = 11.8 × 16.25 × 5.03 = 960 lb.

171c. Design of Purlins. Purlins on a sloping roof must resist bending in two planes, perpendicular and parallel to the plane of the roof. Reference has been made in this book to the bending of unsymmetrical sections, but space does not permit offering a study of other than the simplest purlin section here. Accordingly, the purlins will be selected from among the smaller beam sections, which are the only rolled sections that are symmetrical about both major axes. For this section (and for channels) the true fiber stress can be found merely by computing the fiber stresses caused by bending in the two principal planes, and by adding these stresses algebraically. Loads are assumed to act through the center of gravity of the section in order to avoid reference to torsion. See Fig. 158.

Loadings to be Considered. Two combinations of load must be considered: (1) dead load plus full snow load, and (2) dead load plus one half snow load and full wind load. The purlin spacing for computing these loads will be taken as 4 ft.

Dead load on a purlin = 7.5 × 4.0 × 16.25 = 490 lb. (7.5 lb. per sq. ft. is the weight of purlins, roofing, and insulation board.)

Snow load on a purlin = 20.0 × 4.0 × 16.25 = 1300 lb.

Wind load on a purlin = 11.8 × 4.0 × 16.25 = 770 lb.

Components of Dead Load and Snow Load.

Dead load perpendicular to roof	= 490 × 0.89 =	440 lb.
Snow load perpendicular to roof	= 1300 × 0.89 =	1160 lb.
Dead load parallel to roof	= 490 × 0.45 =	220 lb.
Snow load parallel to roof	= 1300 × 0.45 =	580 lb.

Maximum Loading. The total dead load plus snow load per purlin taken perpendicular to the roof is 1600 lb. while the component of load taken parallel to the roof is but 800 lb. This latter load is so great that one sag rod must be used per bay. The loading to be used will consist of *dead load plus full snow load,* because, when wind is considered, the snow load is reduced 50 per cent and all working stresses are increased 33⅓ per cent at the same time. The point of maximum stress in the purlin will occur either at the center of the span due to bending in a plane normal to the slope of the roof or else near the quarter point of the span caused by flexure in both planes. The wind load is not large enough to influence the section of the purlin. Sag rods provide reactions parallel to the roof midway between trusses. Hence, a sag rod is not stressed by wind.

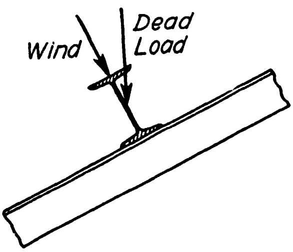


FIG. 158. PURLIN LOADS.

Bending Moments — Neglecting Continuity.

$$\text{Moment at center of purlin} = \frac{1600 \times 16.25 \times 12}{8} = 39{,}000 \text{ in-lb.}$$

$$\text{Moment at quarter point about major axis of purlin} = 0.75 \times 39{,}000 = 29{,}300 \text{ in-lb.}$$

$$\text{Moment at quarter point about minor axis of purlin} = \frac{400 \times 8.12 \times 12}{8}$$

$$\text{(neglecting continuity)} = 4870 \text{ in-lb.}$$

Trial Section. Try a 5-in., 10-lb. standard I-beam for which the values of the section moduli are 4.84 and 0.82 respectively. This is the shallowest section that will provide the minimum required depth of $\frac{1}{40}$ of the span. (Some codes require $\frac{1}{30}$ of span.)

Maximum Stress.

At center of span = 39,000 ÷ 4.84 = 8000 lb. per sq. in.

At quarter point = (29,300 ÷ 4.84) + (4870 ÷ 0.82) = 12,000 lb. per sq. in.

Deflection Perpendicular to Roof.

$$\frac{5}{384} \times \frac{1600 \times 16.25^3 \times 1728}{30{,}000{,}000 \times 12.1} = 0.43 \text{ in.}$$

This deflection is not excessive, and, since the 5-in. purlin meets the requirement that the depth shall not be less than $\frac{1}{40}$ of the span, it will be used.

Check on Assumed Weight.

7 purlins at 10 lb. per ft. = 70 lb. per ft. of roof.

Weight per sq. ft. = 70 ÷ 22.6 = 3.1 lb.

The estimated weight was 3 lb. per sq. ft.

Connection to Roof Truss. Purlins may be bolted to the roof trusses with at least two ½-in. bolts at each connection. Field rivets are preferred.

171d. Design of Sag Rods.

Tension in sag rod. The combined purlin reaction on one side parallel to the roof produces the design stress in the sag rod. $(7.5 + 20)0.45 \times 8.12 \times 22.6 = 2270$ lb. (7.5 lb. is the weight per sq. ft. of purlins, roofing, and insulation; 20 lb. is the snow load.)

Net area required = 2270 ÷ 18,000 = 0.126 sq. in.

Size used. The diameter required to furnish this area is 7/16 in. Adding 1/16 in. to allow for stress concentration at the root of the thread makes the *required minimum diameter* ½ in. A ⅝-in. non-upset rod furnishes a diameter of 0.507 in. at the root of the thread. This is the smallest sag rod commonly used. A detail of the sag rod connection at the ridge is shown in Fig. 159.

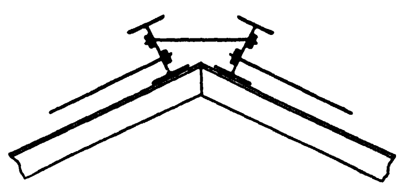


FIG. 159. SAG-ROD CONNECTION.

171e. Analysis of Stresses. The stress analysis for a Fink truss is often performed graphically because of the numerous sloping members. Stress diagrams for dead load and for wind load are shown in Fig. 160. The stresses for snow load are obtained by multiplying the dead load stresses by the ratio 1640/900. In the analysis of wind stresses, the right-hand reaction is assumed to resist all of the horizontal component of the wind. Accordingly, two wind stress diagrams were drawn: (1) for wind from left, and (2) for wind from right. It happens in this structure that the maximum stresses for the left half and right half of the truss are identical.

171f. Maximum Stresses. The dead-load, snow-load, and wind-load stresses are combined for the maximum stresses in Table 26. Three combinations are to be considered: (1) dead load plus full snow load, (2) dead load plus full wind load plus one half snow load, and (3) wind load minus dead load for a member that reverses. No member in this truss has a reversal stress. The final column of Table 26 gives an alternate design stress caused by a uniform vertical load of 31 lb. per sq. ft. of roof surface (D.L. plus Snow Load). The stresses as given by this alternate loading are sufficiently close to the maximum stresses to be satisfactory for design purposes for many roof trusses. For the particular truss which is being designed here, these alternate stresses actually control the design, for in no case is the exact maximum stress 33⅓ per cent larger than the alternate stress (stress caused by dead load plus snow load), and by Spec. 6 and Spec. 7, one is allowed to increase the working stress when wind is considered in the analysis. Accordingly, the alternate stresses caused by dead load plus full snow load (31 lb. per sq. ft. of roof surface) will be used with normal working stresses for the design of members and connections.

171g. Design of Truss Members. Design details for members and connections are summarized in Table 27. Most members are composed of one or two minimum angles, 2 × 2 × ¼ in. The minimum thickness of ¼ in. is controlled by Spec. 15; the 2-in. leg being required to hold a ⅝-in. rivet. All rivets and bolts will be made ⅝ in. Gusset plates 5/16 in. thick will be used *to increase the values of the rivets* in bearing for double angle members even though ¼-in. gussets would be stiff enough for this light truss.

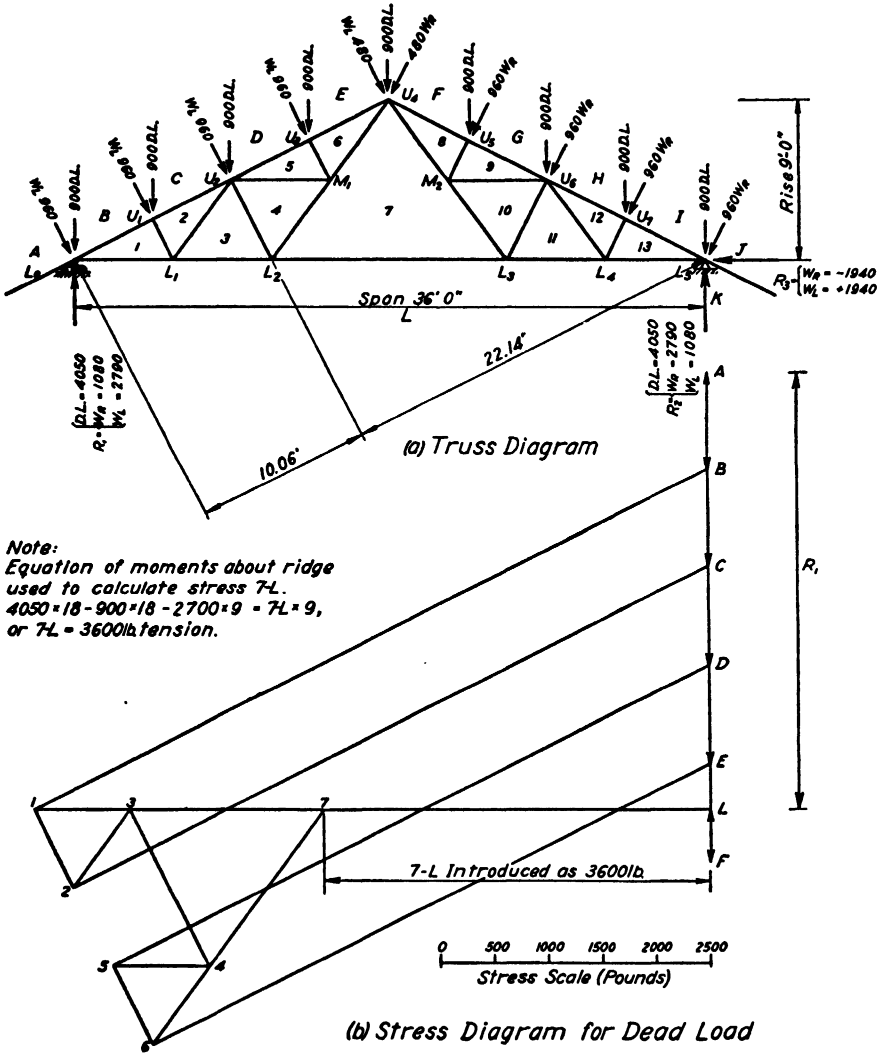
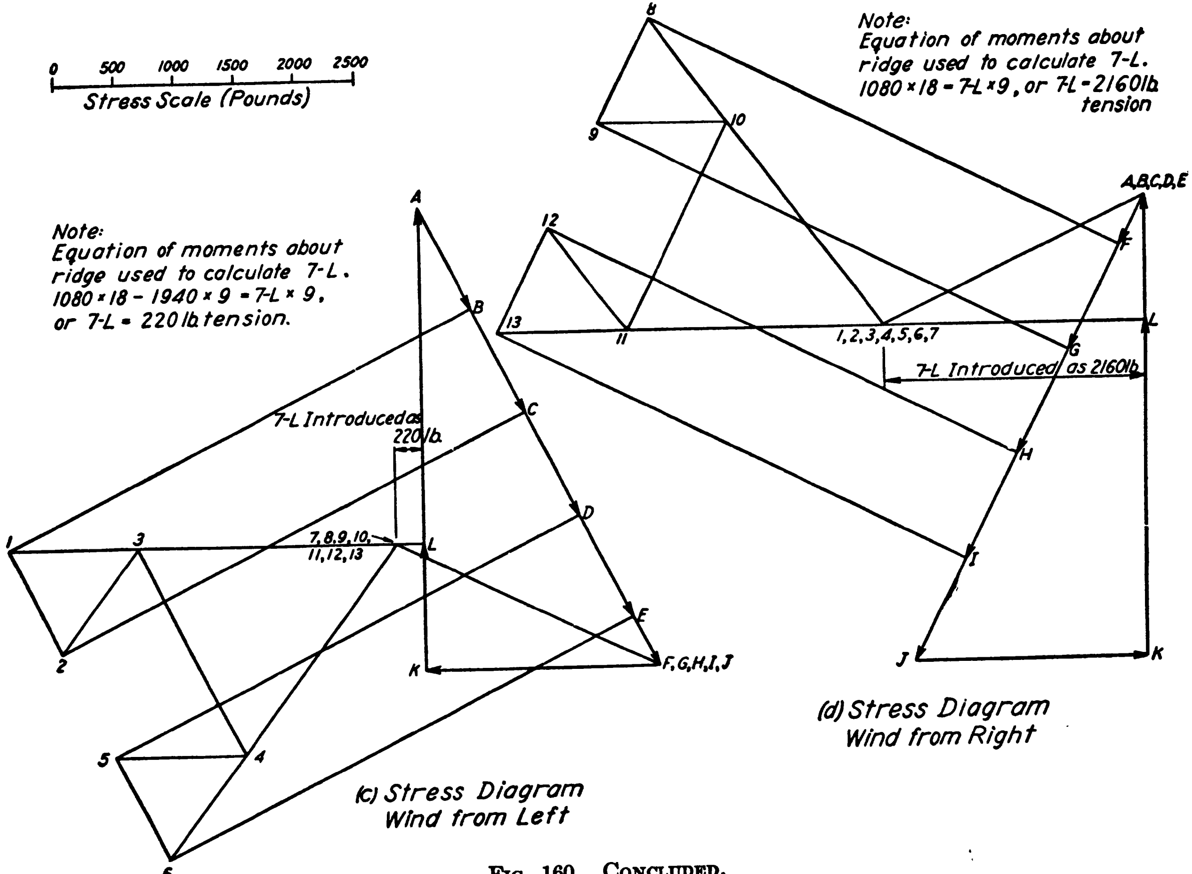


FIG. 160. STRESS DIAGRAMS FOR A FINK ROOF TRUSS.



(c) Stress Diagram
Wind from Left

(d) Stress Diagram
Wind from Right

FIG. 160. CONCLUDED.

TABLE 26 — STRESSES IN MEMBERS OF THE FINK ROOF TRUSS OF FIG. 160

Stress		Dead Load (1)	Full Snow (2)	One Half Snow (3)	Wind from Left (4)	Wind from Right (5)	Columns to be Combined for Max. (6)	Maximum Stress (7)	Stress Caused by an Alternate Vertical Load of 31 lb. per sq. ft. (8)	Member
Upper Chord	1B	−7050	−12,840	−6420	−4320	−2440	1 & 2	−19,890	−19,890	U_1L_0
	2C	−6650	−12,100	−6050	−4320	−2440	1 & 2	−18,750	−18,750	U_1U_2
	5D	−6250	−11,400	−5700	−4320	−2440	1 & 2	−17,650	−17,650	U_2U_3
	6E	−5850	−10,640	−5320	−4320	−2440	1 & 2	−16,490	−16,490	U_3U_4
	13I	−7050	−12,840	−6420	−2440	−4320	1 & 2	−19,890	−19,890	U_7L_5
	12H	−6650	−12,100	−6050	−2440	−4320	1 & 2	−18,750	−18,750	U_7U_6
	9G	−6250	−11,400	−5700	−2440	−4320	1 & 2	−17,650	−17,650	U_6U_5
	8F	−5850	−10,640	−5320	−2440	−4320	1 & 2	−16,490	−16,490	U_5U_4
Lower Chord	1L	+6300	+11,460	+5730	+3430	+2160	1 & 2	+17,760	+17,760	L_0L_1
	3L	+5400	+ 9,840	+4920	+2380	+2160	1 & 2	+15,240	+15,240	L_1L_2
	7L	+3600	+ 6,560	+3280	+ 220	+2160	1 & 2	+10,160	+10,160	L_2L_3
	11L	+5400	+ 9,840	+4920	+ 220	+4300	1 & 2	+15,240	+15,240	L_3L_4
	13L	+6300	+11,460	+5730	+ 220	+5400	1 & 2	+17,760	+17,760	L_4L_5
Left Web	1–2	− 810	− 1,470	− 740	− 960	0	1,3,4	− 2,510	− 2,280	U_1L_1
	2–3	+ 900	+ 1,640	+ 820	+1070	0	1,3,4	+ 2,790	+ 2,540	U_2L_1
	3–4	−1620	− 2,950	−1480	−1920	0	1,3,4	− 5,020	− 4,570	U_2L_2
	4–5	+ 900	+ 1,640	+ 820	+1070	0	1,3,4	+ 2,790	+ 2,540	U_2M_1
	5–6	− 810	− 1,470	− 740	− 960	0	1,3,4	− 2,510	− 2,280	U_3M_1
	6–7	+2700	+ 4,920	+2460	+3200	0	1,3,4	+ 8,360	+ 7,620	U_4M_1
	4–7	+1800	+ 3,280	+1640	+2130	0	1,3,4	+ 5,570	+ 5,080	M_1L_2
Right Web	13–12	− 810	− 1,470	− 740	0	− 960	1,3,5	− 2,510	− 2,280	U_7L_4
	12–11	+ 900	+ 1,640	+ 820	0	+1070	1,3,5	+ 2,790	+ 2,540	U_6L_4
	11–10	−1620	− 2,950	−1480	0	−1920	1,3,5	− 5,020	− 4,570	U_6L_3
	10–9	+ 900	+ 1,640	+ 820	0	+1070	1,3,5	+ 2,790	+ 2,540	U_6M_2
	9–8	− 810	− 1,470	− 740	0	− 960	1,3,5	− 2,510	− 2,280	U_5M_2
	8–7	+2700	+ 4,920	+2460	0	+3200	1,3,5	+ 8,360	+ 7,620	U_4M_2
	10–7	+1800	+ 3,280	+1640	0	+2130	1,3,5	+ 5,570	+ 5,080	M_2L_3

TABLE 27 — Summary of Design of Gymnasium Roof

Member			Design Stress	Section Selected	How Placed	Holes Out	Area in sq. in.: Gross Area	Area in sq. in.: Net Effective Area	Slenderness Ratio, L/r: Actual	Slenderness Ratio, L/r: Allowed	Value of Member, lb.	Connection: 5/8″ Shop Rivets	Connection: 5/8″ Field Bolts
Upper Chord	U_1L_0	1B	−19,890	2 angles 3½ × 2½ × ¼ in.	Long leg down	0	2.88	2.88	54	120	Designed for flexure	6	
	U_1U_2	2C	−18,750	2 angles 3½ × 2½ × ¼ in.	"	0	2.88	2.88	54	120	"		
	U_2U_3	5D	−17,650	2 angles 3½ × 2½ × ¼ in.	"	0	2.88	2.88	54	120	"		
	U_3U_4	6E	−16,490	2 angles 3½ × 2½ × ¼ in.	"	0	2.88	2.88	54	120	"		7
Lower Chord	L_0L_1	1L	+17,760	2 angles 2 × 2 × ¼ in.	Long Leg up	2	1.88	1.50	110		+27,000	5	
	L_1L_2	3L	+15,240	2 angles 2 × 2 × ¼ in.	"	4	1.88	1.12	110		+20,200	2	4
	L_2L_3	7L	+10,160	2 angles 2½ × 2 × ¼ in.	"	4	2.12	1.36	207		+24,500	4	3
Web	U_1L_1	1–2	− 2,280	1 angle 2 × 2 × ¼ in.		0	0.94	0.705	77	120	− 3,930[a]	3	
	U_2L_1	2–3	+ 2,540	1 angle 2 × 2 × ¼ in.		1	0.94	0.52	111		+ 9,350	3	
	U_2L_2	3–4	− 4,570	2 angles 2 × 2 × ¼ in.		0	1.88	1.88	99	120	−22,000	4	
	U_2M_1	4–5	+ 2,540	1 angle 2 × 2 × ¼ in.		1	0.94	0.52	111		+ 9,350	3	
	U_3M_1	5–6	− 2,280	1 angle 2 × 2 × ¼ in.		0	0.94	0.705	77	120	− 3,930[a]	3	
	U_4M_1	6–7	+ 7,620	2 angles 2 × 2 × ¼ in.		2	1.88	1.5	141		+27,000		7
	M_1L_2	4–7	+ 5,080	2 angles 2 × 2 × ¼ in.		2	1.88	1.5	141		+27,000	5	
Special Members	Purlin			5-in., 10-lb. I-beam									2
	Sag-rod		+ 2,270	5/8-in. non-upset rod			0.307	0.156			+ 2,800		
	Diagonal Bracing			1 angle 2 × 2 × ¼ in.		1	0.94	0.52	189		+ 9,350		3
	Strut		Negligibly small	1 angle 3 × 2½ × ¼ in. riveted to a 4-in., 5.4-lb. channel	Short leg down		2.87		200	200	−16,100		3

[a] 3930 with eccentricity; 9500 without.

Double angle members of symmetrical section will be used in all except the lightest stressed members. Single angle members are objectionable for they tend to twist the truss by producing an eccentric force at the joint. All end connections will be designed wherever reasonable to develop the full strengths of the members. This procedure is required by nearly all specifications although many designers use only enough rivets to develop the calculated stress for light members. The saving is usually about one rivet in each connection which hardly seems justified when one considers the added strength and rigidity that can be obtained by the added cost of a few rivets.

TABLE 28

VALUES OF ⅝-IN. RIVETS

Type	Working Stress (lb. per sq. in.)			Rivet Values (lb.)			
	Shear	Single Bearing	Double Bearing	Single Shear	Double Shear	Single Bearing ¼-in. Metal	Double Bearing 5⁄16-in. Metal
Power driven shop rivets	13,500	24,000	30,000	4140	8280	3750	5860
Hand driven field rivets or unfinished bolts	10,000	16,000	20,000	3070	6140	2500	3910

171h. Design of Lower Chord Members. The lower chord will be field spliced at L_2 and L_3 and, accordingly, L_0L_1 and L_1L_2 may be made of one section and L_2L_3 of another. Member L_0L_1 has rivet holes deducted from the vertical legs only. Members L_1L_2 and L_2L_3 have rivet holes deducted from both legs to allow for the splice connection. Members L_0L_1 and L_1L_2 are 5 ft.-7½ in. long (67½ in.) while member L_2L_3 is 13 ft.-6 in. (162 in.) long.

Member. L_0L_1.

Maximum stress = 17,760 lb. tension.

Minimum section. Two angles 2 × 2 × ¼ in. placed back to back on opposite sides of a single 5⁄16-in. gusset.

Net effective section. Two holes must be deducted since connections are only to the vertical legs. The full net area is considered effective for angles placed in this manner. (Spec. 85.)

Net area = 2 × 0.94 − 2 × 0.19 = 1.50 sq. in.

Value of member = 1.50 × 18,000 = 27,000 lb.

Radius of gyration about the horizontal axis = 0.61.

Slenderness ratio = 67.5 ÷ 0.61 = 110.

End connection. Rivets are in double bearing on the 5⁄16-in. gusset. Rivet value = 5860 lb. 27,000 ÷ 5860 = 5 rivets.

Member L_1L_2.

Maximum stress = 15,240 lb. tension.

Minimum section. Two 2 × 2 × ¼-in. angles.

Net effective section. Four holes must be deducted from the gross area because of the splice connection. The full net area is considered effective as for the member L_0L_1.

Net area = 2 × 0.94 − 4 × 0.19 = 1.12 sq. in.

Value of member = 1.12 × 18,000 = 20,200 lb.

Slenderness ratio. Same as L_0L_1.

End connection. The end connection for this member will be worked out when the joint L_2 is designed.

REMARKS. Since two $2 \times 2 \times \frac{1}{4}$-in. angles are satisfactory for both L_0L_1 and L_1L_2, this section will be used for the double-length member.

Member L_2L_3.

Maximum stress = 10,160 lb. tension.

Unsupported length = 162 in. This member usually is controlled by the requirement that its L/r value about the horizontal axis shall not exceed 200. A member of this stiffness will not sag.

Minimum radius of gyration about a horizontal axis = $162 \div 200 = 0.81$.

Required minimum section. Two $2\frac{1}{2} \times 2 \times \frac{1}{4}$-in. angles placed with the 2-in. legs outstanding furnish a radius of gyration of 0.78 about the horizontal axis. This angle will be accepted since the next larger angle is $3 \times 2\frac{1}{2} \times \frac{1}{4}$ in. which far exceeds the requirement in both strength and stiffness.

Net area = $2 \times 1.06 - 4 \times 0.19 = 1.36$ sq. in.

Value of member = $1.36 \times 18{,}000 = 24{,}500$ lb.

End connection. The end connection for this member will be worked out when the joint L_2 is designed.

171 i. Design of the Upper Chord. The entire upper chord will be made of one section. Any saving that might be made possible by changing the sizes of the members would be overbalanced by the cost of a splice and by a loss in the lateral stiffness of the structure.

Length for Buckling. In a vertical plane the length for possible buckling is a panel distance or 5.03 ft. The top chord is supported in its own inclined plane by the purlins. Accordingly, the unsupported length in this plane is the purlin spacing or a maximum of 4 ft.

Trial Section. Try two $3\frac{1}{2} \times 2\frac{1}{2} \times \frac{1}{4}$-in. angles with the $3\frac{1}{2}$-in. legs turned downward. Gross area is 2.88 sq. in. Gross moment of inertia about the horizontal axis is 3.6. Extreme fiber distances are 1.11 and 2.39 in.

Allowable Stresses. At the joint U_2 where the fiber stress may become a maximum, the member can buckle laterally over an unsupported length of 36.75 in. The radius of gyration for the angles placed $\frac{5}{16}$ in. back to back is 1.06 in.

$$f = \frac{18{,}000}{1 + \dfrac{36.75^2}{18{,}000 \times 1.06^2}} = 16{,}800 \text{ lb. per sq. in.}$$

Use the maximum allowable value of 15,000 lb. per sq. in. Midway between panel points the allowable stress is determined from an unsupported length of 60.37 in. between joints and a value of r of 1.12 in.

$$f = \frac{18{,}000}{1 + \dfrac{60.37^2}{18{,}000 \times 1.12^2}} = 15{,}500 \text{ lb. per sq. in.}$$

Use the maximum allowable value of 15,000 lb. per sq. in.

Location of Maximum Stress. From the purlin loads and the purlin spacing as shown in Fig. 161, it is evident that the theoretical maximum moment occurs at L_0. The maximum direct stress occurs in U_1L_0, but the stress in U_1U_2 is nearly as great and, hence, the combined stress in U_1U_2 will also be studied. It is important to note that negative moment over a support produces a larger compressive fiber stress than an equal positive moment at the center of the span because the extreme fiber distance to the compression edge is much greater for negative moment.

Bending Moments. In continuous beams the bending moments may be computed by balancing moments as described in Vol. 2, *Theory of Modern Steel Structures.* The moment curve of Fig. 161 is an approximate one obtained by balancing moments with two-place numerals. It is satisfactory for use in design except that the moments at the joints may be reduced to the moments near the ends of the gusset plates. Apply the factor $8/9$ from Fig. 161 because $DL + SL$ controls.

Negative moment at L_0 = 19,200 × $8/9$ = 17,100 in-lb. (unreduced).

Negative moment at U_2 = 13,400 × $8/9$ = 11,900 in-lb. (unreduced).

Maximum moment under any load = 9100 × $8/9$ = 8100 in-lb.

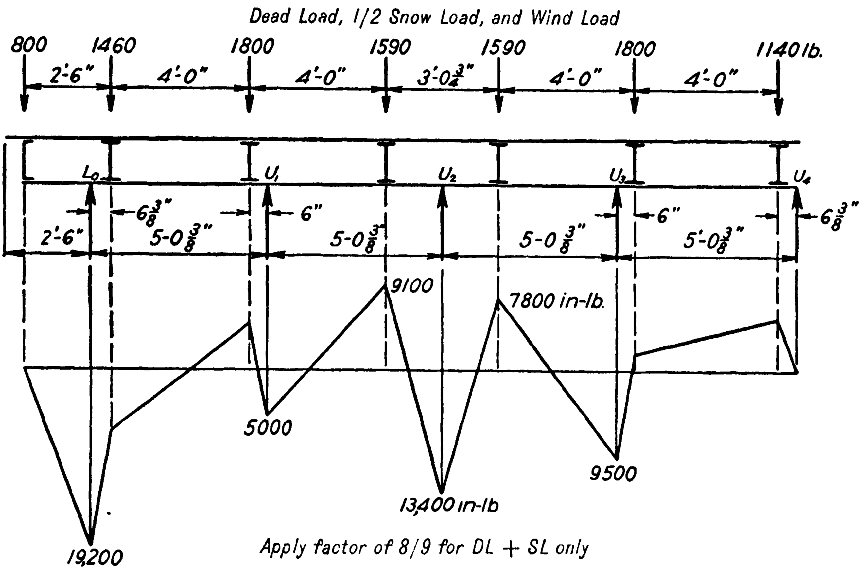


FIG. 161. UPPER CHORD MOMENTS.

Combined Stresses, $P/A + Mc/I$.

At L_0 the combined stress $= \dfrac{19{,}890}{2.88} + \dfrac{17{,}100 \times 2.39}{3.6} = 18{,}300$ lb. per sq. in.

However, this is a theoretical stress that is reduced to a safe value by the influence of the gusset plate.

At U_2 the combined stress $= \dfrac{18{,}750}{2.88} + \dfrac{11{,}900 \times 2.39}{3.6} = 14{,}400$ lb. per sq. in.

Again, the influence of the gusset will greatly reduce the stress.

Under the purlin in the panel U_1U_2 the combined stress is

$$\frac{18{,}750}{2.88} + \frac{8100 \times 1.11}{3.6} = 9000 \text{ lb. per sq. in.}$$

Note the use of the smaller extreme fiber distance to the most heavily stressed fiber directly under the purlin. This stress may be increased if the purlin connection is made by loose bolts that do not fill the holes. Nevertheless, it will still be within safe limits.

End Connection. The rivets are in double bearing on the $\frac{5}{16}$-in. gusset.

Rivet value = 5860 lb. The maximum direct stress is 19,890 lb. (U_1L_0).

Number of rivets = 19,890 ÷ 5860 = 4 rivets. However, the connection for U_1L_0 must also resist bending caused by the purlin loads. The number of rivets will be increased to 6 in order to fill out the gusset plate and provide moment resistance.

171j. Selecting Web Members. Web members will consist of single or double angles according to the magnitude of the stress and the length of the member.

Members M_1L_2 and U_4M_1.

Maximum stress = 7620 lb. tension.

Minimum section. Two angles 2 × 2 × $\frac{1}{4}$ in.

Net area for one hole deducted from each angle = 2 × 0.94 − 2 × 0.19 = 1.5 sq. in.

Value of member = 1.50 × 18,000 = 27,000 lb.

Slenderness ratio = 11.25 × 12 ÷ 0.96 = 141 (take r about axis of symmetry since L is the double length). This slenderness ratio indicates that vibration will not occur.

End connection. Rivets at L_2 are in double bearing on the $\frac{5}{16}$-in. gusset. Rivet value = 5860 lb. 27,000 ÷ 5860 = 5 rivets required.

Members U_2L_1 and U_2M_1.

Maximum stress = 2540 lb. tension.

Minimum section. One angle 2 × 2 × $\frac{1}{4}$ in. (Actually, double angles are used, Fig. 171.)

Effective area = (0.94/2 − 0.19) + 0.94/4 = 0.52 sq. in. Effective area is net area of connected leg plus one half of gross area of unconnected leg. (Spec. 85.)

Value of member = 0.52 × 18,000 = 9350 (eccentricity neglected; see U_1L_1).

Slenderness ratio = 67.5 ÷ 0.61 = 111 (applies for the member U_2M_1; take r about the horizontal axis).

End connection. Rivets are in single bearing on the $\frac{1}{4}$-in. leg. Rivet value = 3750 lb. 9350 ÷ 3750 = 3 rivets required.

Members U_1L_1 and U_3M_1.

Maximum stress = 2280 lb. compression.

Minimum section. One angle 2 × 2 × $\frac{1}{4}$ in.

Effective area = 0.75 × 0.94 = 0.705 sq. in.

Slenderness ratio = 30.2 ÷ 0.39 = 77.4 (using minimum r).

$$\text{Allowable stress} = \frac{18{,}000}{1 + \dfrac{77.4^2}{18{,}000}} = 13{,}500 \text{ lb. per sq. in.}$$

Value of member = 13,500 × 0.705 = 9500 lb. without eccentricity.

Eccentricity of load = 0.59 in.

Allowable eccentric load.

$$P_e = \frac{fA}{1 + ec/r^2} = \frac{13{,}500 \times 0.94}{1 + (0.59 \times 1.41/0.61^2)} = 3930 \text{ lb.}$$

See equation (5), p. 238.

End connection. Rivets are in single bearing on the $\frac{1}{4}$-in. leg. Rivet value = 3750 lb. 3930 ÷ 3750 = 2 rivets required; 3 rivets are used as the minimum connection.

Member U_2L_2.

Maximum stress = 4570 lb. compression.

Minimum section. Slenderness ratio requires two minimum angles, 2 × 2 × $\frac{1}{4}$ in.

Gross area = 2 × 0.94 = 1.88 sq. in.

Slenderness ratio = 60.2 ÷ 0.61 = 98.7.

$$\text{Allowable stress} = \frac{18{,}000}{1 + \dfrac{98.7^2}{18{,}000}} = 11{,}700 \text{ lb. per sq. in.}$$

Value of member = 11,700 × 1.88 = 22,000 lb.

End connection. Rivets are in double bearing on the $\frac{5}{16}$-in. gusset. Rivet value 5860 lb. 22,000 ÷ 5860 = 4 rivets required.

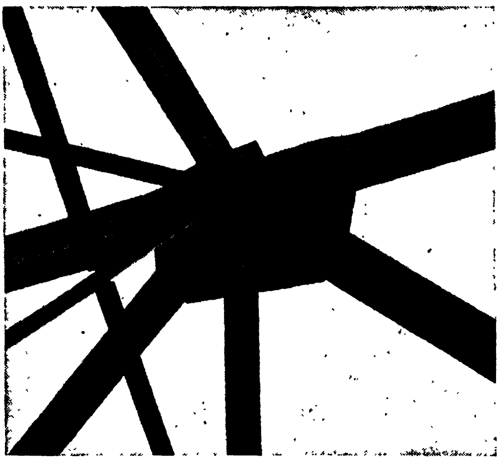


FIG. 162. INTERIOR JOINT OF BUILDING TRUSS WITH LATERAL TRUSS CONNECTION.

REMARKS. Single angle members were used for U_1L_1, U_3M_1, U_2L_1 and U_2M_1. It is common practice to design the sub-struts U_1L_1 and U_3M_1 as single angles, but the tension members U_2L_1, and U_2M_1 are usually made of double angles. This is the arrangement shown on the truss drawing, Fig. 171. For a truss of much longer span or for one designed for heavy loads, all members should be of symmetrical sections.

171k. Design of Joints. The number of rivets in the end of each member has been determined on the assumption that all rivets are power driven shop rivets. Since this building will need but few field connections, and field rivets require power equipment, it may be desirable to use unfinished bolts for field connections. (Spec. 31.) A shear of 10,000 lb. per sq. in. and a single bearing value of 16,000 lb. per sq. in. are allowed for rough bolts. In order to haul the trusses to the site by truck, it is necessary to divide the truss into three parts: left half, right half, and the member L_2L_3. Field connections are required at U_4, L_2 and L_3. The *minimum number of rivets* in any connection will be set at three although *AISC* specifications permit the use of two rivets in light structures. (Spec. 23.)

Joints U_1 and U_3. This joint is shown in Fig. 163. Clearly, the 3 rivets in the member U_3M_1 must be balanced by at least 3 rivets through the top chord and gusset plate. These rivets are spaced 3 in. apart in order to obtain a reasonable moment resistance. Purlins placed between joints, and other causes, produce moments at the joints.

Joint L_1. The gusset plate must be riveted to the bottom chord with sufficient rivets to resist the maximum possible combined horizontal thrust of the members U_1L_1 and U_2L_1. (See Fig. 164.) Of course, $\Sigma V = 0$.

Value of U_1L_1 = 3930 lb.
Horizontal component of U_1L_1 = 3930 × 0.45 = 1770 lb.
Value of U_2L_1 = 9350 lb.
Horizontal component of U_2L_1 = 9350 × 0.60 = 5600 lb.
Total thrust = 5600 + 1770 = 7370 lb.

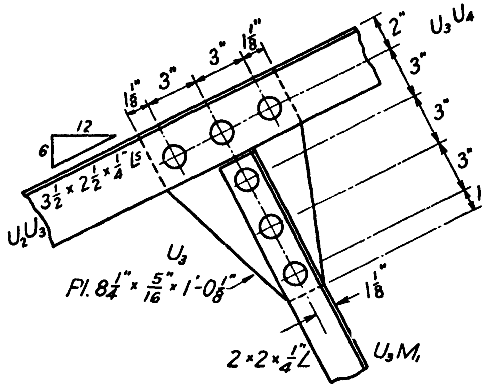


FIG. 163. DETAIL AT U_1 OR U_3.

Connection. Rivets are in bearing on the $\frac{5}{16}$-in. gusset. Rivet value = 5860 lb. 7370 ÷ 5860 = 2 rivets required. The minimum number of 3 rivets will be used.

Joint M_1. The gusset plate must be riveted to the member U_4L_2 for exactly the same resistance that was found necessary at the joint L_1. (See Fig. 165.)

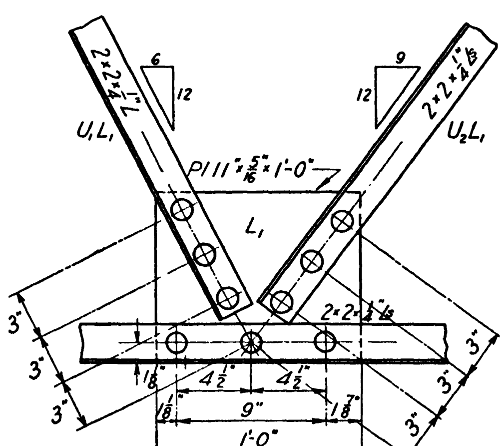


FIG. 164. DETAIL AT L_1.

Joint U_2. The summation of stresses in the members U_2L_1, U_2L_2, and U_2M_1, taken parallel to the upper chord is zero (see Fig. 166). The summation of stress for these members taken perpendicular to the upper chord shows a resultant normal thrust of 2280 lb. which is balanced by the downward thrust of the loads at this joint. One rivet would

carry this load, but the detail shown in Fig. 166 is arranged to meet the common requirement that the rivet spacing shall not exceed 6 in. Many building codes now permit a wider spacing of rivets.

Joint L_2. Specifications have often recommended that chord members be spliced at some distance away from the joints of a truss. The reason is that a splice at a joint makes use of the gusset plate as a splice plate. This use is objectionable if the gusset plate itself is not carefully analyzed and designed to function as a splice plate. Of course, the joint splice does save rivets and extra shop work and is therefore economical. Accordingly, we will give due care to the analysis of stresses in the gusset plate and make use of it as a splice plate. Its analysis will be considered under the heading *Checking the Gussets* on the following page.

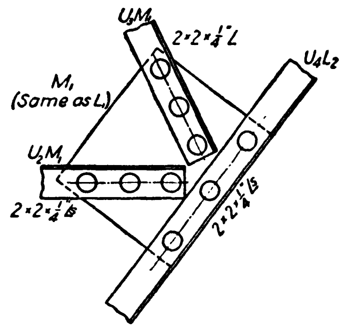


FIG. 165. DETAIL AT M_1.

The design of this joint involves the design of a simple splice in a tension member. The lower chord member is broken here and is spliced by means of the gusset plate and a horizontal splice plate. (See Fig. 167.) This splice plate will also be used as the connection plate for a longitudinal strut.

Value of the lower chord member L_1L_2 = 20,200 lb.; L_2L_3 = 24,500 lb.

Placing rivets. The greatest value that can be obtained from the lower splice plate is the value of the outstanding horizontal legs of the lighter member, which happens to be equal to 10,000 lb.

Member L_1L_2. Place 2 shop rivets through the vertical legs and 4 field bolts through the horizontal legs.

Value of connection = 2 × 5860 + 4 × 2500 = 21,700 lb. (A ⅝-in. shop rivet at 30,000 lb. per sq. in. bearing on a 5⁄16-in. plate = 5860 lb.; a ⅝-in. field bolt at 16,000 lb. per sq. in. bearing on ¼-in. material = 2500 lb.)

Member L_2L_3. Place 4 shop rivets through the horizontal legs and 3 field bolts through the vertical legs. Value of member = 24,500 lb.

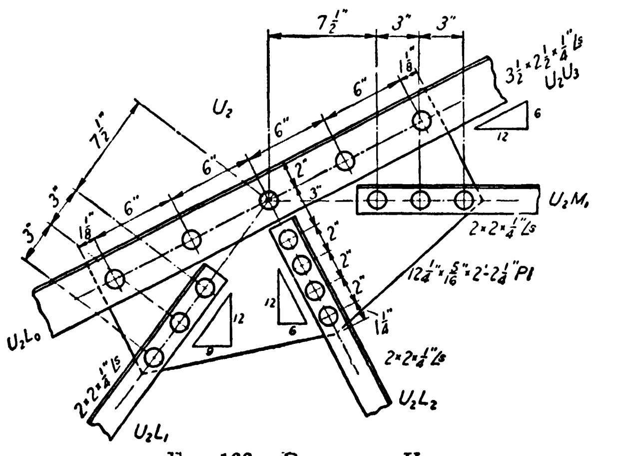


FIG. 166. DETAIL AT U_2.

Value of connection = 4 × 2500 + 3 × 3910 = 21,700 lb. (A ⅝-in. shop rivet at a reduced value of only 16,000 lb. per sq. in. bearing on ¼-in. material = 2500 lb.; a ⅝-in. field bolt at 20,000 lb. per sq. in. double bearing on a $\frac{5}{16}$-in. plate = 3910 lb.) *The 4 shop rivets are credited with the bearing value of field bolts* because the other end of the plate is connected with only an equal number of field bolts. The connection value is slightly less than the value of the member, but it is considered satisfactory since the 2½-in. leg of L_2L_3 was used only for stiffness, its stress actually being smaller than the stress in L_1L_2.

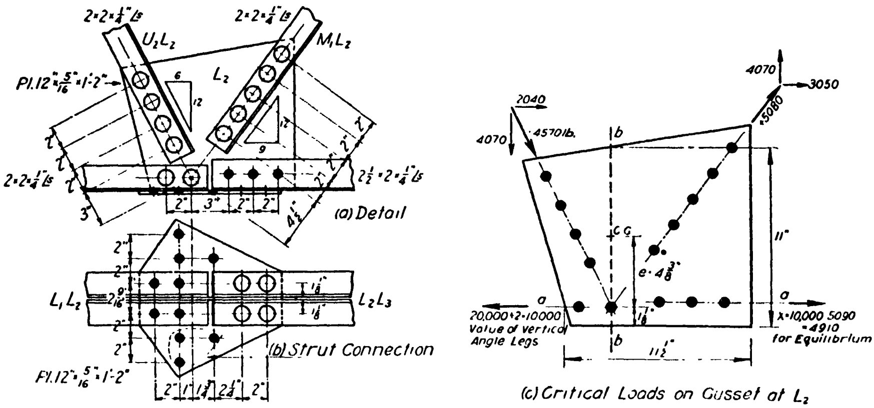


FIG. 167. DETAIL AT L_2—TENSION SPLICE.

Checking the Gussets. The gusset plates of a truss resist direct stress, shear, and flexure. Usually, $\frac{5}{16}$-in. gussets are adequate for a light truss, but, for safety we will check the gusset at L_2 shown in Fig. 167. The diagonal forces shown in Fig. 167(*c*) are maximum web stresses. The shear along the section *a-a* cannot exceed the sum of the horizontal components of these stresses, or 5090 lb. Along *b-b* the shear is 4070 lb.

$$\text{Unit shear on } a\text{-}a = \tfrac{3}{2} \times 5090 \div (\tfrac{5}{16} \times 11\tfrac{1}{2}) = 2100 \text{ lb. per sq. in.}$$

$$\text{Unit shear on } b\text{-}b = \tfrac{3}{2} \times 4070 \div (\tfrac{5}{16} \times 11) = 1770 \text{ lb. per sq. in.}$$

(For a rectangular section the maximum unit shear is $\frac{3}{2}$ times the average.)

Direct Stress and Flexure. Since this gusset forms only a part of the lower chord splice, an assumption must be made if its analysis is to be isolated from the analysis of the horizontal splice plate. This assumption is that the splice plate joins the horizontal angle legs and that the gusset splices the vertical legs of the chord angles. Therefore, we show a force of 10,000 lb. acting to the left as the maximum force that can be delivered to the gusset by L_1L_2. Since four maximum stresses would not be in equilibrium, the stress in L_2L_3 is computed by statics to be 10,000 − 5090 = 4910 lb. Hence, on the section *b-b* we have a force of 4910 + 3050 = 7960 lb. applied at an eccentricity *e* of 4⅜ in. from the mid-height of the section. (The fact that the chord stresses are not exactly in line will be neglected here.)

$$\frac{P}{A} = 7960 \div (11 \times \tfrac{5}{16}) = 2300 \text{ lb. per sq. in.}$$

$$\frac{Mc}{I} = \frac{7960 \times 4\tfrac{3}{8} \times 5\tfrac{1}{2}}{\tfrac{1}{12} \times \tfrac{5}{16} \times 11^3} = 5500 \text{ lb. per sq. in.}$$

$$\text{Total} = 7800 \text{ lb. per sq. in.}$$

This estimated stress would be increased by a consideration of the fact that there is a rivet hole on the section *b-b*, but it is evident that the gusset is understressed in tension, compression, and shear. If the stresses had appeared serious, we might have decided that it would also be necessary to analyze the entire splice (gusset plus horizontal splice plate) as a T-section. See *DS*4. Of course, such stress calculations are merely reasonable estimates because the beam formula does not apply accurately to deep sections. Also, there are other cross-sections than *a-a* or *b-b* that might control, and other load conditions might prove critical. These should be studied.

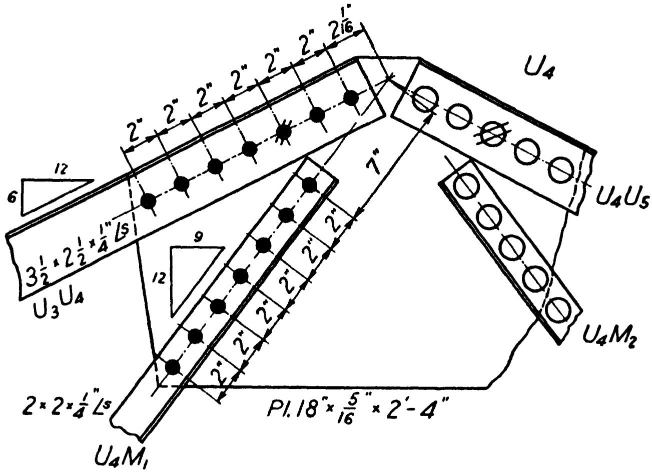


FIG. 168. DETAIL AT U_4.

Joint U_4. This joint will be designed for field bolts since it is desirable to make it symmetrical. (See Fig. 168.)

Value of member U_4M_1 = 27,000 lb.

Value of field bolts in bearing on the $\frac{5}{16}$-in. plate at 20,000 lb. per sq. in. = 3910 lb.

Number of bolts required = 27,000 ÷ 3910 = 7 bolts.

Value of member U_3U_4. This member was designed for compression plus bending. Direct compression = 16,490 lb.

Number of bolts required = 16,490 ÷ 3910 = 5 bolts. This number will be increased to 7 to provide excess resistance to the moment produced in the upper chord by the purlins. These bolts would not develop the value of the member in direct compression, but this is not necessary because the member will always have to resist flexure.

Joint L_0. The number of rivets required in the lower chord member L_0L_1 has already been determined to be 5, and the number through the upper chord member U_1L_0 has been set at 6. (See Fig. 169.)

End reaction caused by dead load = 4050 lb.

" " " by snow load = 7400 lb.

" " " by wind load = 2790 lb.

Maximum end reaction for shoe design = 4050 + 7400 = 11,450 lb.

Rivets through shoe angles = 11,450 ÷ 5860 = 2 rivets. Use 3 rivets as shown in the detail, Fig. 169. Use two 3 × 3 × ⅜-in. shoe angles, 8 in. long.

Bearing on masonry = 11,450 ÷ (8 × 6.37) = 225 lb. per sq. in. (The bearing plate is 8 in. long by 6⅜ in. wide.) This is sufficiently low for bearing on a good brick wall.

Bearing plate. The thickness is determined by flexure as a cantilever.

The thickness of angles should not be considered as a part of the depth of the bearing plate because the rivets joining the two together will not be designed for horizontal shear.

$$\text{Moment} = \frac{225 \times 3^2}{2} = 1020 \text{ in-lb. per in.}$$

$$\text{Section modulus} = \frac{1020}{18{,}000} = 0.057 = d^2/6.$$

$$\text{Depth} = d = \sqrt{6 \times 0.057} = 0.59 \text{ in.}$$

Use a ⅝-in. bearing plate riveted with 4 countersunk rivets to the ⅜-in. angle legs.

Allowance for expansion. (100°F. is ample.) $36 \times 12 \times 100 \times 0.0000065 = 0.28$ in.

Slotted holes. Use a slot of length equal to the diameter of the anchor bolt plus about twice the expected expansion to allow for poor setting of the bolt in the masonry. A slot ⅞ × 1¼ in. will be used to accommodate a ¾-in. anchor bolt.

Location of shoe. Place the shoe with its center directly under the intersection of the lower and upper chords so that there will be no moment of eccentricity introduced by the vertical reaction.

Bed plate on masonry. A bed plate, 10 in. × 12 in., is bolted to the masonry to provide a surface upon which the shoe can slide. The thickness of this plate is also made ⅝ in.

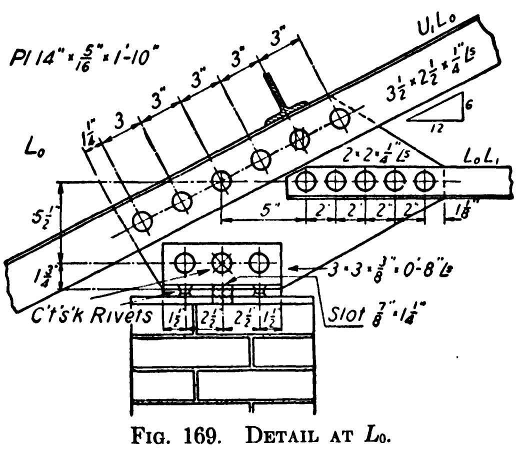


FIG. 169. DETAIL AT L_0.

171. Design of Diagonal Bracing. Diagonal bracing will be used in the plane of the upper chords. The purlins will perform the function of struts. Two pairs of crossing diagonals will be used in each outside bay on each side of the roof. These diagonals brace the trusses together in pairs, and the action of the purlins as continuous struts completes the necessary upper chord bracing. The purpose of this bracing is to stiffen the structure since the diagonals have no calculated stresses. There will be no diagonal bracing used in the plane of the lower chords for this small structure. If vibration due to heavy machinery had been anticipated, such extra bracing could readily have been added. Diagonal bracing in a vertical plane along the centerline or along the sloping plane from U_4 to L_2 would be useful during erection and such bracing resists wind pressure on the end of the building.

Size of Angles. Minimum angles $2 \times 2 \times \frac{1}{4}$ in. will be used.

Slenderness ratio. The angles will be clamped near their points of crossing to a purlin. (Clamps require no holes and do not weaken the purlins.) Accordingly, their unsupported length is $\frac{1}{2}\sqrt{10^2 + 16.25^2}) = 9.6$ ft. $L/r = \dfrac{9.6 \times 12}{0.61} = 189$ (r is taken for the horizontal axis since the value of L/r is computed to determine the tendency toward sagging; $L/r < 250$).

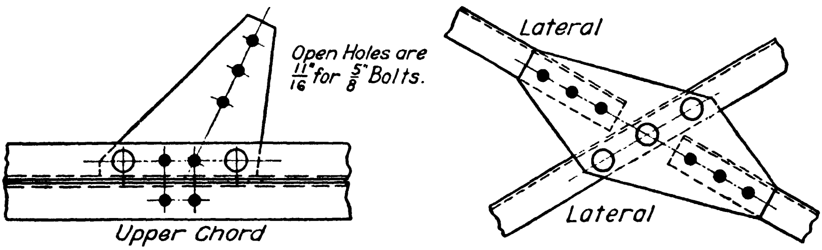


FIG. 170. DETAILS OF LATERAL BRACING FOR THE TRUSS OF FIG. 160.

Use of flats. Angles will project downward and interfere with the insulation board that is to be placed against the bottom flanges of the purlins. Laterals composed of $2\frac{1}{2} \times \frac{1}{4}$-in. flats would overcome this difficulty. Such laterals should be clamped or wired to each purlin to prevent sagging.

End Connections. The diagonals should be connected to the undersides of $\frac{1}{4}$-in. lateral plates riveted to the undersides of the top chords of the roof trusses. Three $\frac{5}{8}$-in. bolts will be used at each connection even though the calculated wind stresses would be negligible. Where the angle diagonals cross each other, one must be cut and spliced with a $\frac{1}{4}$-in. plate which is also riveted to the continuous diagonal. Details for the lateral bracing are shown in Fig. 170.

Vertical Bracing. It is desirable to introduce diagonal bracing at least in the end bays between pairs of trusses for squaring the structure during erection. Pairs of crossing diagonals ($2 \times 2 \times \frac{1}{4}$-in. angles) should be used in a vertical plane along the midspan of the trusses. The calculated stresses in the diagonals caused by wind on the end of the building will be small. Two $\frac{5}{8}$-in. bolts are ample for the end connections of the diagonals.

171m. Design of Struts. Two continuous struts will brace the lower chords of the roof trusses together at the joints L_2 and L_3. No diagonal bracing is needed in the plane of the lower chords for a small roof truss.

Stiffness. The maximum value of L/r permitted for secondary compression members is 200 (Spec. 13). Therefore, $r = \dfrac{16.25 \times 12}{200} = 0.98$.

Section Used. A $3 \times 2\frac{1}{2} \times \frac{1}{4}$-in. angle riveted to a 4-in., 5.4-lb. channel makes a strut having a minimum radius of gyration of 0.99. The weight is 9.9 lb. per ft. The gross area is 2.87 sq. in. The allowable stress is 5600 lb. per sq. in.

Value of member $= 2.87 \times 5600 = 16{,}100$ lb.

End connection. The strut should be bolted to the connection plate at each truss joint with at least three $\frac{5}{8}$-in. field bolts.

171n. Detailing the Structure. Reasonably complete design details of this roof are given in Fig. 171. Instructions for structural detailing are given in § 219. A few important items will be mentioned here.

Stitch rivets. These are used with washer fills for all double angle members. The maximum spacing of stitch rivets in tension members is 3 ft.-6 in. In compression members stitch rivets are placed close enough so that the L/r of one angle between rivets will not exceed ¾ of the L/r of the member as a whole. (Spec. 37.) This requirement is fulfilled by a maximum spacing of about 2 ft.

Rivet spacing at joints. The detailing is made to conform with the required spacing as given in Specs. 34, 38, and 39.

Dimensions. All rivets are located and all dimensions are complete from center to center of joints. Each group of partial dimensions is covered by an overall dimension.

Minimum connections. *Three rivets* are used here for the minimum riveted connection. As stated in Spec. 23, connections with only 2 rivets are reasonably common for light structures. The design presented here is a *first-class* structure.

Joints. All gage lines intersect at the joints. At the shoes, the gage lines of the chords intersect over the center of the reaction.

Field and shop rivets. All field rivets or field bolts are indicated by showing open holes (black). Rivet heads and open holes are made slightly smaller than their true scale for better appearance on the drawing.

Slope of members. Slopes are given beside the members for the convenience of the shop detailer in laying out the plates.

Sizes. Outside scaled dimensions of plates and the lengths of all members are given on the plate for the purpose of ordering material.

Notes. Unusual details as well as any special requirement of construction are best covered by lettered notes on the sheet.

171o. Final Computation of Dead Weight. The dead weight will be computed per bay and will then be divided by 730, the number of square feet of roof surface per bay.

Purlins. 14 purlins 16.25 ft. long at 10 lb. per ft. = 2270 lb. This is equivalent to a weight of 3.1 lb. per sq. ft. of roof surface.

Bracing.

Diagonals. 8 pieces 19.1 ft. long at 3.2 lb. per ft.	= 490 lb.
Struts. 2 pieces 16.25 ft. long at 9.9 lb. per ft.	= 322 lb.
Total	= 812 lb.

Weight of bracing per sq. ft. of roof surface = 812 ÷ 730 = 1.1 lb.

Truss.

Top chord.	Two 3½ × 2½ × ¼-in. angles; 45.0 ft. at 9.8 lb.	=	441 lb.
Bottom chord.	Two 2 × 2 × ¼-in. angles; 22.1 ft. at 6.4 lb.	=	141 lb.
" "	Two 2½ × 2 × ¼-in. angles; 13.5 ft. at 7.2 lb.	=	98 lb.
Web.	Two 2 × 2 × ¼-in. angles; 30.2 ft. at 6.4 lb.	=	193 lb.
"	One 2 × 2 × ¼-in. angle; 28.6 ft. at 3.2 lb.	=	92 lb.
	Truss members	=	965 lb.
Gussets.	About 20 sq. ft. of $\frac{5}{16}$-in. plate; 20 × 12.5	=	250 lb.
Rivet heads, lateral plates, ring fills, etc. (Details add about 10% to weight of members)		=	95 lb.
	Total weight of one truss	=	1310 lb.

$$\text{Weight of trusses per sq. ft. of roof surface} = \frac{1310}{730} = 1.8 \text{ lb.}$$

Roof Covering and Insulation. 4.5 lb. per sq. ft. of roof surface.

Total Dead Weight. 3.1 + 1.1 + 1.8 + 4.5 = 10.5 lb. per sq. ft. of roof surface.

Estimated Dead Weight. 11.0 lb. per sq. ft. of roof surface.

REMARKS. The weight estimate used in computing dead load stresses was close enough to the computed weight so that a revision in dead load stresses is unnecessary. The drawing of this roof truss, Fig. 171, gives the design details. This is a student drawing, but, a similar commercial drawing, Fig. 171*a*, is presented for comparison. It would be possible to fabricate the truss from such a sheet, but it is more common to produce one or more sheets of shop details. The shop details show each member and each plate separately with all shop dimensions given. When shop details are to be made up, it is not necessary to show all dimensions on the general drawing of the truss.

WELDED TRUSS DESIGN

172. Design of a Welded Roof Truss for a Gymnasium.

PROBLEM. Redesign the roof truss from § 171 for fabrication and erection by arc welding.

Working Stresses. Use the same working stresses for main members that were used in the riveted design, § 171. Shear on the roots of fillet welds will be limited to 11,300 lb. per sq. in. to agree with the low allowable stresses for truss members.

Comments. The stress analysis does not need to be performed again and many of the members will be used unchanged from the riveted design. Because of the small space available for welding, it is found to be impossible to develop the full strengths of the web members at the welded joints. The lengths of weld shown on the detail drawing (Fig. 172) are in each case capable of developing more than twice the actual stress in the web member, and this is thought to be sufficient. The chords are welded to produce a resistance equal to the value of the member.

Type of Welded Design. The top chord and bottom chord angles will be turned with legs in the form of a *U-section* or channel section. The web members will be of smaller angles, also with legs turned in, and they will fit into the U-sections of the chords. The upper chord will have 2½-in. in-turned angle legs tack welded together to form a 5-in. channel. With ¼-in. angles, the inside width will be 4½ in. The bottom chord must also have 2½-in. in-turned angle legs. The web members will all have 2-in. in-turned angle legs tack welded ⅜ in. apart. These angles will then be 4⅜ in. back to back, which allows ⅛ in. of clearance when they are inserted into the chords.

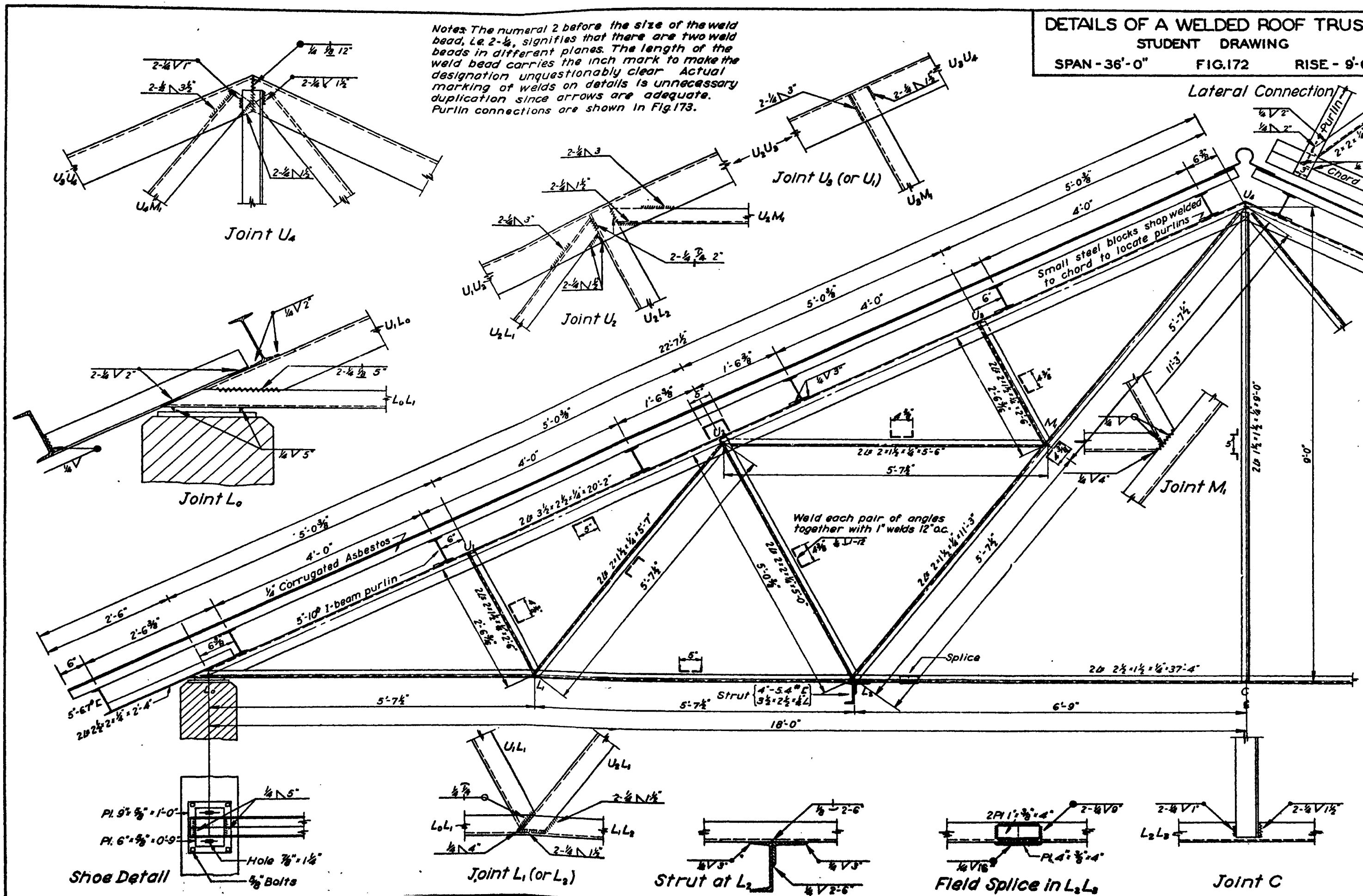
Minimum Angles. For welded design there seems to be no need to limit the minimum angle leg to 2 in. This limitation is necessary in riveted work because a 2-in. leg is required to hold a ⅝-in. rivet which is the smallest structural rivet in common use. Accordingly, 1½-in. angle legs will be allowed in the welded design where they are adequate to meet the required slenderness ratio. A minimum thickness of ¼-in. should be maintained in both riveted and welded work.

172a. Design of Members.

Top Chord. The upper chord will have exactly the same section as the riveted structure, that is, two angles $3\frac{1}{2} \times 2\frac{1}{2} \times \frac{1}{4}$ in. However, the short legs will be turned in and tack welded together while in contact.

Bottom Chord. The entire lower chord will be made of $2\frac{1}{2} \times 1\frac{1}{2} \times \frac{1}{4}$-in. angles. The long legs will be turned in and tack welded together while in contact. In order to use this section, the center member L_2L_3 will have to be supported at the center by a hanger, as shown in Fig. 172.

Gross area of angles $= 2 \times 0.94 = 1.88$ sq. in.
Maximum stress $(L_0L_1) = 17{,}760$ lb. tension.
Value of member $= 1.88 \times 18{,}000 = 33{,}800$ lb.


DETAILS OF A WELDED ROOF TRUSS
STUDENT DRAWING
SPAN - 36'-0"
FIG.172
RISE - 9'-0"
Notes: The numeral 2 before the size of the weld bead, i.e. 2-¼, signifies that there are two weld beads in different planes. The length of the weld bead carries the inch mark to make the designation unquestionably clear. Actual marking of welds on details is unnecessary duplication since arrows are adequate. Purlin connections are shown in Fig. 173.
Joint U4
Joint U2
Joint U3 (or U1)
Lateral Connection
Purlin
Chord
Joint L0
Joint M1
¼" Corrugated Asbestos
5"-10# I-beam purlin
Small steel blocks shop welded to chord to locate purlins
Weld each pair of angles together with 1" welds 12" o.c.
Splice
Strut
18'-0"
9'-0"
22'-7½"
Shoe Detail
Hole ⅞" x 1¼"
⅝" Bolts
Joint L1 (or L2)
Strut at L2
Field Splice in L2L3
Joint C

Slenderness ratio (L_2L_3) = 81 ÷ 0.41 = 198.
Value of field splice = 17 × 2000 = 34,000 lb.
(Use 17 in. of ¼-in. weld on each side of joint).

Tension Web Members. The highest stressed web member is U_4M_1. A similar member (U_2M_1) has the greatest horizontal length, 5 ft.–7½ in. Two 2 × 1½ × ¼-in. angles will be used if satisfactory. Long legs are turned in and tack welded together at a spacing of ⅜ in. apart.

Gross area of angles = 2 × 0.81 = 1.62 sq. in.
Maximum stress (U_4M_1) = 7620 lb. tension.
Value of member = 1.62 × 18,000 = 29,100 lb.
Slenderness ratio (U_2M_1) = 67.5 ÷ 0.43 = 157.
Value of welded connection = 10 × 2000 = 20,000 lb. (10 in. of ¼-in. fillet)

Sub-Struts, U_1L_1 and U_3M_1. These members will be made of two 2 × 1½ × ¼-in. angles.

Gross area of angles = 2 × 0.81 = 1.62 sq. in.
Maximum stress = 2280 lb. compression.
Slenderness ratio = 30.2 ÷ 0.43 = 70.

$$\text{Allowable stress} = \frac{18{,}000}{1 + \dfrac{70^2}{18{,}000}} = 14{,}100 \text{ lb. per sq. in.}$$

Value of member = 1.62 × 14,100 = 22,900 lb. (eccentricity neglected)
Value of welded connection = 9 × 2000 = 18,000 lb. (9 in. of ¼-in. fillet)

Web Compression Member, U_2L_2. The L/r requirement controls the design of this member. Two 2 × 2 × ¼-in. angles are required as in the riveted structure.

Gross area of angles = 2 × 0.94 = 1.88 sq. in.
Maximum stress = 4570 lb.
Slenderness ratio = 60.4 ÷ 0.61 = 99.

$$\text{Allowable stress} = \frac{18{,}000}{1 + \dfrac{99^2}{18{,}000}} = 11{,}700 \text{ lb. per sq. in.}$$

Value of member = 11,700 × 1.88 = 22,000 lb.
Value of welded connection = 11 × 2000 = 22,000 lb. (11 in. of ¼-in. fillet)

172b. Design of Joints. The design of joints is made clear by a study of the sheet of welded details, Fig. 172. This sheet should be studied in connection with the standard welding symbols for draftsmen adopted by the American Welding Society and given in Fig. 56. The system used here is to weld one web member to the chord, and then *to weld the second web member to the first and possibly also to the chord.* This detail has the advantage of permitting a considerable amount of weld to be placed in a small space. It also seems reasonable that the transfer of stress is better effected by this arrangement than by one where both diagonals are welded only to the chord. The reason is that a part of the stress passes from one web member into the other without going through the chord. Of course, we must consider the matter of stress transfer in order to proportion the lengths of weld properly. There must be sufficient length of weld connecting the two web members to the chord to take care of the change in chord stress and to transfer any load acting normal to the chord into the web members. The weld between the web

members must be satisfactory to care for the direct transfer of stress, or else the excess stress must be considered to be transferred from one web member to the other through the chord.

Joints U_1 and U_3. Sub-struts are welded to the upper chord with 9 in. of ¼-in. fillet.

Joints L_1 and L_2. The tension diagonals U_2L_1 and M_1L_2 are welded to the chord with 10 in. of ¼-in. fillet, and the members U_1L_1 and U_2L_2 are welded to the tension diagonals and to the chord with 10 in. of ¼-in. fillet.

Joint M_1. The diagonal U_2M_1 is welded to the back of member $U_4M_1L_2$ with 4 in. of ¼-in. fillet. The sub-strut U_3M_1 is welded to the back of $U_4M_1L_2$ with 6½ in. of ¼-in. fillet, and U_2M_1 is welded to U_3M_1 and to $U_4M_1L_2$ with 6½ in. of ¼-in. fillet.

Joint U_2. The tension diagonals U_2M_1 and U_2L_1 are each welded to the upper chord with 9 in. of ¼-in. fillet. Then the compression member U_2L_2 is butt welded to the diagonals (diagonals and struts are separated ¼ in.) and the same weld connects the strut U_2L_2 to the chord. The combined length of these butt welds is 8 in. There are two extra 1½-in. fillet welds connecting U_2L_2 to the chord.

Joint C. The hanger U_4C is welded on the outside of the bottom chord, and also of the top chord, with 5 in. of ¼-in. fillet.

Joint U_4. The diagonal U_4M_1 is welded to the upper chord with 10 in. of ¼-in. fillet. The two sections of the upper chord are butt welded together all around the inside with a ¼-in. flush butt weld for a length of 12 in. The members should be separated about ⅛ in. before welding. This joint is capable of developing the strengths of the members in compression.

Joint L_0. The upper and lower chords are butt welded together at L_0 with two 5-in. lengths of butt welds. The members should be separated at least ⅛ in. before welding to obtain a good butt weld through the entire thickness of the metal. This joint will be strong enough to develop the full resistances of the members.

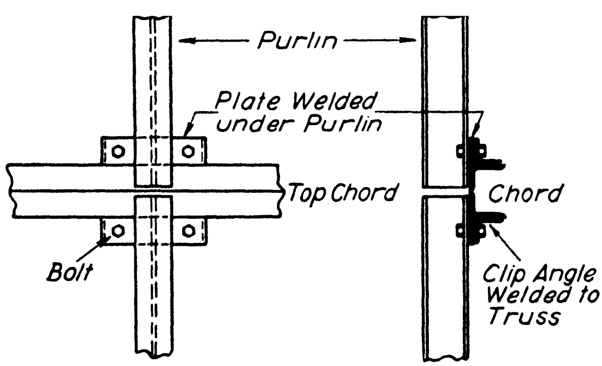


FIG. 173. PURLIN CONNECTION FOR THE TRUSS OF FIG. 160.

172c. End Bearing and Bracing. The *end bearing* for this truss is extremely simple. A sole plate, 6 × ⅝ × 9 in. is welded to the bottom of the lower chord. The cantilever projection is but 2 in. and the ⅝-in. plate was found satisfactory for a 3-in. projection in the riveted design. The bearing pressure is 11,450 ÷ (6 × 9) = 212 lb. per sq. in., slightly less than in the riveted design. The base plate is made 9 × ⅝ × 12 in. Details of slotted holes, etc., are similar to those used in the riveted design.

Diagonal Bracing and Struts. The lateral bracing and struts may be the same sections that were used in the riveted structure. The struts will be welded to the lower

chords with 6 in. of ¼-in. fillet at each connection. The diagonal bracing will be welded to the bottom flanges of alternate purlins, connections being made directly to the truss chords if possible. At least a 4-in. length of ¼-in. fillet should be used at each lateral connection. A detail is shown on the drawing, Fig. 172. Welded diagonals will need to be 3 in. longer than the riveted laterals of Fig. 171. A welded plate connection similar to the riveted detail of Fig. 170 will be used where the laterals intersect. Longitudinal bracing may have bolted connections.

Purlins. The purlins are unchanged from the riveted design. They will be welded directly to the upper chords with 6 in. of ¼-in. fillet per connection. A special arrangement must be made for purlins that act as members of the upper chord system of bracing. These purlins must be bolted to each truss during erection in order to square the building. If convenient, bolt holes may be punched through the purlin flange; but, to save the cost of carrying the purlins to a punch, it is preferable to weld on small plates previously punched to match with similar plates welded to the upper chord of the truss. The detail for this truss, using plates and clip angles, is shown in Fig. 173.

172d. Weight of the Welded Roof Truss.

Top chord. Two 3½ × 2½ × ¼-in. angles; 40.2 ft. at 9.8 lb.	=	394 lb.
Bottom chord. Two 2½ × 1½ × ¼-in. angles; 37.4 ft. at 6.4 lb.	=	240 lb.
Web. Two 2 × 1½ × ¼-in. angles; 54.9 ft. at 5.6 lb.	=	308 lb.
Two 2 × 2 × ¼-in. angles; 10.0 ft. at 6.4 lb.	=	64 lb.
Hanger. Two 1½ × 1½ × ¼-in. angles; 9.0 ft. at 4.7 lb.	=	43 lb.
Eave extension. Two 2 × 2 × ¼-in. angles; 4.7 ft. at 6.4 lb.	=	30 lb.
Lateral plates. 4 × ¼ × 6 in.; 6 plates at 2 lb.	=	12 lb
Total weight of one truss	=	1091 lb.

Remarks. Although the weight of weld metal has not been included here, it is evident that the weight of the welded truss would be less than the weight of the riveted truss designed in § 171. The weight of the riveted truss was 1215 lb., not including the weight of rivet heads. The saving in weight is of small importance and might readily be overbalanced by a slightly greater cost of welding over riveting. No attempt should be made to form an opinion as to the relative costs of welded and riveted structures from the information obtained from these designs. The comparison would not be valuable since these trusses contain too many members of minimum section where stress did not control the design. The welded structure would show to greater advantage if the full gross sections of its tension members could be utilized. Present practice indicates that the saving in weight obtained by welding about compensates for the greater cost of welding over riveting and places the two structures on a competitive basis. A great deal depends upon the designer. A clever designer can arrange for many parts of a welded structure (purlins for instance) to be shipped directly from the mill or warehouse to the job, thus saving the shop cost. Where such methods are employed, the welded job may show considerable economy.

PROBLEMS

204. Redesign the roof of § 171 to carry a wind pressure of 30 lb. per sq. ft. on the vertical surface and a snow load of 40 lb. per sq. ft. on the horizontal surface or 30 lb. per sq. ft. of roof surface for a roof of ¼ pitch. Use riveted construction and *AISC* working stresses.

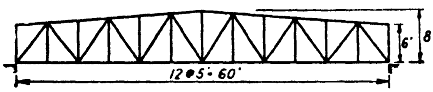
205. Redesign the welded roof truss of § 172 using split beam sections for the chord members. Compare the weight with the weight for the design with U-section chords.

206. Redesign the roof truss of Problem 204 using welded construction and U-section chords. Follow *AISC* working stresses. See § 55.

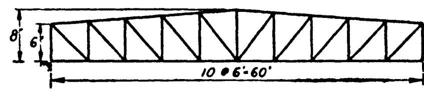
207. Redesign the roof truss of Problem 204 using welded construction and split beams for chord sections. Compare the weight with the design obtained in Problem 206 Use *AISC* specifications and highest *AWS* working stresses.

208. Design a riveted roof truss for a gymnasium similar to the type designed in § 171, but for these changed conditions: Outside dimensions of the building = 70 × 140 ft. Wind load = 30 lb. per sq. ft. of vertical surface. Snow load = 5 lb. per sq. ft. of roof surface. Use ⅜-in. corrugated asbestos roofing which permits a purlin spacing of 6 ft. Keep the distance between roof trusses under 20 ft. Allow for diagonal bracing in the planes of both the upper and lower chords. Use a Fink truss of ⅓ pitch designed for *AREA* working stresses.

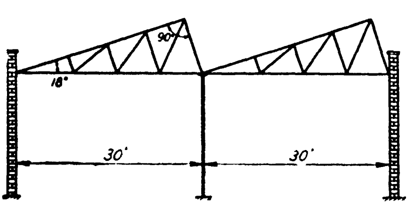
209. Redesign the roof truss of Problem 208 for welded construction. Use your own judgment in the selection of type of section for the chords.



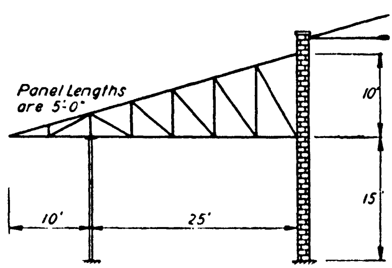
PROBLEM 210.



PROBLEM 212.



PROBLEM 213.



PROBLEM 215.

210. Design a flat roof to span 60 ft. and to carry a total vertical load of 75 lb. per sq. ft. of horizontal projection. This load includes the weight of snow, of concrete slab, and of waterproof covering. Use the Warren type of roof truss of riveted construction as shown in the illustration. Place purlins only at the panel points. Space trusses 20 ft. apart. A field splice is required in order to reduce the length to 40 ft. for shipping. Assume that the roof is protected from the wind by a breast wall and that wind pressure may be neglected. Use 1941 *AASHO* working stresses.

211. Redesign the roof truss of Problem 210 for welded fabrication. Follow *AWS* specifications; use the highest working stresses allowed for ductile welds.

212. Design a welded roof truss of Pratt type to replace the riveted Warren truss of Problem 210. The type of truss is illustrated. The purlin spacing is limited to 5 ft.-0 in. in order to use a 2½-in. precast roof slab.

213. Design a riveted saw-tooth roof where each truss has a span of 30 ft. The total vertical load is 40 lb. per sq. ft. of horizontal projection for the combined effect of wind and snow. Place the trusses 16 ft. apart. Outlines of the roof trusses are as illustrated. Use a ⅜-in. corrugated asbestos roof and keep the purlin spacing under 6 ft. In laying out the truss, place one set of diagonal members perpendicular to the slope of the roof. Use any city building code.

214. Redesign the roof truss of Problem 213 for fabrication by arc welding.

215. Design a light roof to shelter the bleachers of an athletic field. The bleachers are placed beside a gymnasium building so that the roof must slope only in one direction. Details are shown in the illustration. Place the trusses 18 ft. apart. Design for a total vertical load of 15 lb. per sq. ft. of horizontal projection to care for the combined effect of snow and wind. Design the purlins, roof truss, and column. Detail the connections to the masonry wall and to the top of the column. Arrange for statically determined reactions and reverse any diagonals found to be in compression. Select riveted or welded construction according to your judgment as to the type of fabrication best fitted to this extremely light structure. Follow *AISC* specifications.

216. Design a riveted roof truss of 80-ft. span and 9-ft. constant depth back to back of flanges to carry a uniform load of 1500 lb. per ft. of truss. Divide the truss into 8 panels of 10 ft. each and use a Pratt web system. It is suggested that the lower chord be made of two angles placed on opposite sides of a single gusset and that the top chord be made of two such angles and a cover plate. Select specifications.

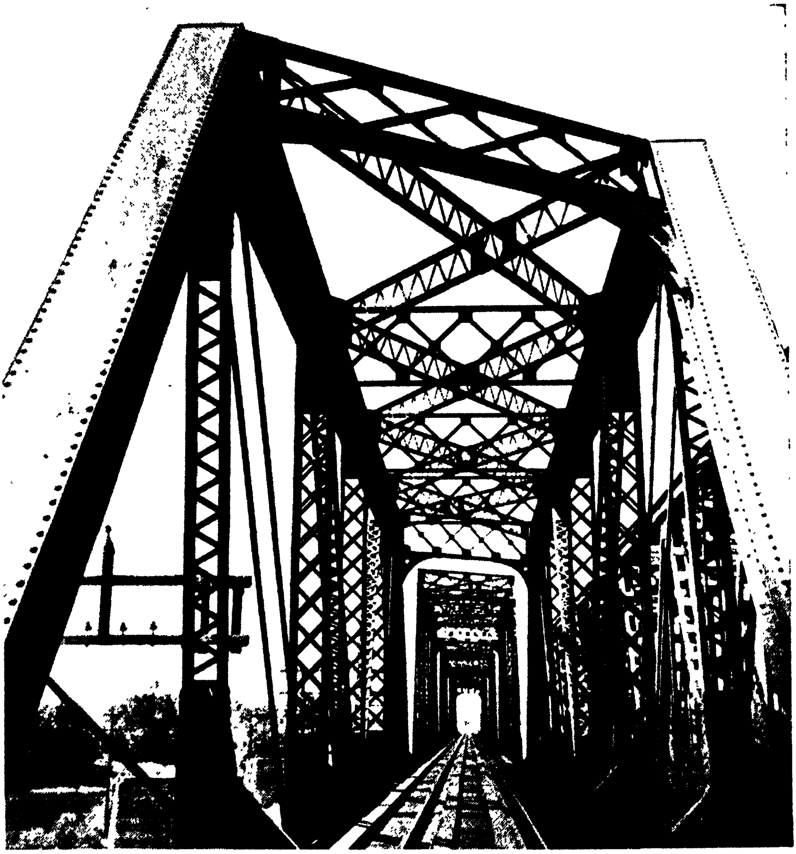
217. Redesign the truss of Problem 216 for fabrication by arc welding It is suggested that the chords be made of wide flange beam sections with the flanges turned vertically. The verticals may be made of angles or of a beam section welded inside of the chords. The diagonals may be made of angles or channels welded on the inside or outside of the chord flanges. Compare the weights of the riveted and welded designs.

173. Other Roof Structures. Roofs for industrial buildings take on new and different forms with each advance in the construction art. Welding has made possible the use of rectangular or polygonal roof arches at low cost. Wide flange beam sections may be split, bent, and rewelded to form structures of almost any shape. The clean-cut modern appearance of indeterminate frames as contrasted to the rather cluttered appearance presented by a roof truss has encouraged architects to make use of such continuous structures. The roof truss, however, is still the most economical structure for medium spans and ordinary roof loading.

CHAPTER 13

DESIGN OF A LOW TRUSS HIGHWAY BRIDGE

174. Low Truss Bridges. These structures have long been in common use as highway spans from 50 ft. to 100 ft. in length. They are of either



Courtesy C. M. St. P. & P. R.R. Co.

FIG. 174. SKEW TRUSSES WITH UPPER CHORD BRACING.

Pratt or Warren type and of parallel or of curved chord construction. There are no sway frames, upper laterals or portal bracing in a low truss bridge. Hence, the floor beams and vertical posts must be fastened to-

gether rigidly to form stiff U-frames that prevent lateral movement of the upper chord. Two types of floor systems are in common use. The floor system with concrete slab, floor beams, and stringers has panel lengths from 12 to 20 ft. and the stringers usually are spaced from 3 to 5 ft. apart. By subdividing a Warren truss, panel lengths may be halved, stringers omitted, and the slab designed to span longitudinally between floor beams. The latter type seems to be preferred. The total weight of steel for the two bridges is not greatly different, but the stringerless bridge is the more rigid structure.

175. Design of a 72-ft. Low Truss Highway Bridge. A low truss bridge will be designed for a span of 72 ft. to carry a 20-ft. concrete pavement.* The live loading is H-20. A stringerless bridge will be selected for maximum rigidity. An arrangement of eight panels at 9 ft.-0 in. seems most

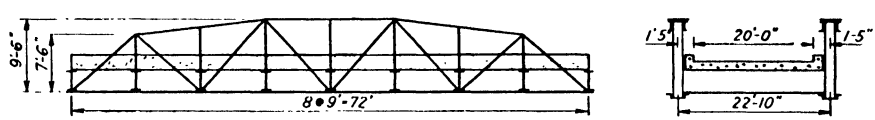


FIG. 175. LOW TRUSS HIGHWAY BRIDGE.

satisfactory. A curved chord truss presents a better appearance and its use may result in a slight economy of material. The height at the center usually is between ⅙ and ⅛ of the span. A center height of 9 ft.-6 in. will be used, the height at the hip joint being 7 ft.-6 in. A Warren truss with verticals meets all of these requirements. A preliminary layout is shown in Fig. 175. In the cross-section, the distance center to center of trusses is given as 22 ft.-10 in. This spacing allows 1 ft.-5 in. from face of curb to center of vertical. The curb is 9 in. wide and the vertical is estimated to be a 10-in. beam section which allows 3 in. clear between the outside of the curb and the face of the vertical. This space is necessary to allow for the lateral overhang of the end post cover plate and to meet the clearance requirement of Fig. 241.

175a. Allowable Stresses — Special Code Requirements.

Tension = 16,000 lb. per sq. in.

$$\text{Compression} = \frac{16{,}000}{1 + \dfrac{(L/r)^2}{13{,}500}} \text{ but not to exceed the value when } L/r = 40.$$

Bending on extreme fiber = 16,000 for beams and plates,
= 24,000 for pins.

Shear on gross section of girder webs = 10,000.

Shear on shop rivets = 12,000. Shear on field rivets = 10,000.

Bearing on pins and shop rivets = 24,000; on field rivets = 20,000.

* A roadway of 22-ft. width is required by 1941 A.A.S.H.O. specifications, p. 418.

Bearing on expansion rollers and rockers in pounds per lineal inch = $600d$.
Steel castings may be stressed to 75% of these values.
Bearing on concrete masonry = 600 lb. per sq. in.
Concrete beams and slabs. f_s = 18,000, f_c = 800 lb. per sq. in. n = 15.

The allowable stresses listed above are considerably lower than those permitted by the American Association of State Highway Officials and by most State Highway Specifications. However, much design work is done for cities and for foreign countries that may have special code requirements. It is therefore desirable for the designer to become familiar with more than one set of allowable stresses. Note, for example, the different working stresses above for *shop* and *field* rivets. Modern codes often allow the same working stresses if the rivets are "power driven." As an exercise, the following design may be revised for 1935 or 1941 *AASHO* working stresses.

175b. Slab Design. The distribution width for a truck wheel is usually specified as a function of S and W where S is the span in feet and W is the width of the tire, that is, 20 in. or 1.67 ft. For example, read Spec. 68 (*AASHO*). However, the effective width through the central section of the slab is limited physically to a maximum of $4\frac{1}{2}$ ft. by the proximity of the adjacent wheel loads as is shown by Fig. 176. To allow

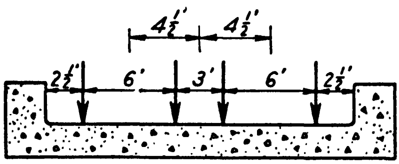


FIG. 176.
EFFECTIVE WIDTH AT CENTER.

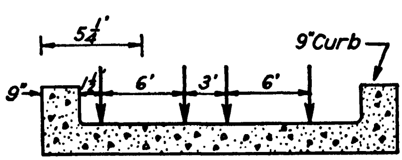


FIG. 177.
EFFECTIVE WIDTH AT EDGE.

for clearance, the center of the outside wheel must be placed 1 ft.-6 in. from the face of the curb. Hence, the effective width at the edge is 5 ft.-3 in. as shown in Fig. 177. The action of the curb as a beam will be neglected except that it is assumed to be able to carry its own weight.

Center Strip. The depth of slab * is estimated at 1 in. per foot of span or 9 in.; 20 lb. per sq. ft. is allowed for a future wearing surface. The maximum *positive moment* † occurs near the center of the end span. Without reference to continuity this moment may be approximated as follows. H-20 loading — Spec. 56.

D.L. moment per ft. $(\frac{1}{16}wL^2) = \frac{1}{16} \times 132.5 \times 9^2 \times 12 =$ 8,100 in-lb.

L.L. " " " $(PL/5) = \dfrac{16{,}000 \times 9 \times 12}{5 \times 4.5}$ = 76,700

Impact " " " $\left(\dfrac{50}{9 + 125} = 37.3\%\right)$ (Spec. 62.) = 28,600

Total = 113,400 in-lb.

The maximum *negative moment* † occurs over the first interior support. It may be approximated as follows.

* Shear may influence slab depth, but we will not consider it here. By 1941 *AASHO* specifications, shear is neglected when slab is designed to resist moment.

† Maximum moment coefficients are subject to revision depending upon positions of expansion joints, type of live loading, and fixation at end of slab. Those coefficients used should not be considered to represent special conditions without investigation.

$$\text{D.L. moment per ft. } (\tfrac{1}{10}wL^2) = \tfrac{1}{10} \times 132.5 \times 9^2 \times 12 = 12{,}900 \text{ in-lb.}$$

$$\text{L.L. " " " } (PL/8) = \frac{16{,}000 \times 9 \times 12}{8 \times 4.5} = 48{,}000$$

$$\text{Impact " " " } \left(\frac{50}{9 + 125} = 37.3\%\right) = \underline{17{,}900}$$

$$\text{Total} = 78{,}800 \text{ in-lb.}$$

$$\text{Positive steel area} = \frac{113{,}400}{18{,}000 \times 0.87 \times 7.5} = 0.96 \text{ sq. in. per ft.}$$

Use ¾-in. ϕ at 5½-in. spacing.

$$\text{Negative steel area} = \frac{78{,}800}{18{,}000 \times 0.87 \times 7.5} = 0.67 \text{ sq. in. per ft.}$$

Use ⅝-in. ϕ at 5½-in. spacing.

$$\text{Maximum compression} = \frac{2 \times 113{,}400}{0.4 \times 0.87 \times 12 \times 7.5^2} = 965 \text{ lb. per sq. in.}$$

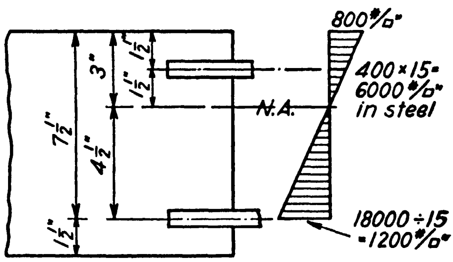


Fig. 178. Stresses at Center of Slab.

The resistance of the negative steel must be considered since the calculated stress exceeds the allowable compressive concrete stress of 800 lb. per sq. in.

Moment resisted by compressive steel. (See Fig. 178.)

$M = 6000 \times 0.67 \times 6.0 = 24{,}000$ in-lb.

Net moment $= 113{,}400 - 24{,}000 = 89{,}400$ in-lb.

$$\text{True compressive stress in concrete} = \frac{2 \times 89{,}400}{0.4 \times 0.87 \times 12 \times 7.5^2} = 760 \text{ lb. per sq. in.}$$

Edge Strip. The depth at the edge can be made about ½ in. less than at the center. Since a crown of 1.0 in. is commonly used, either the center depth must be made ½ in. more than necessary or the additional crown can be produced by varying the thickness of topping. The computations below are for a depth at the edge of 8½ in. and for an effective width of 5 ft.-3 in.

The maximum *positive moment* occurs near the center of the end span. We will approximate its value as follows.

$$\text{D.L. moment per ft. } (\tfrac{1}{16}wL^2) = \tfrac{1}{16} \times 126.5 \times 9^2 \times 12 = 7{,}700 \text{ in-lb.}$$

$$\text{L.L. " " " } (PL/5) = \frac{16{,}000 \times 9 \times 12}{5 \times 5.25} = 65{,}700$$

$$\text{Impact " " " } \left(\frac{50}{9 + 125} = 37.3\%\right) = \underline{24{,}500}$$

$$\text{Total} = 97{,}900 \text{ in-lb.}$$

The maximum *negative moment* occurs over the first interior support. Its approximate value is obtained as follows.

D.L. moment per ft. $(\frac{1}{10}wL^2) = \frac{1}{10} \times 126.5 \times 9^2 \times 12 = 12{,}300$ in-lb.

L.L. " " " $(PL/8) = \dfrac{16{,}000 \times 9 \times 12}{8 \times 5.25} = 41{,}100$

Impact " " " $\left(\dfrac{50}{9 + 125} = 37.3\%\right) = \underline{15{,}300}$

Total $= 68{,}700$ in-lb.

Positive steel area $= \dfrac{97{,}900}{18{,}000 \times 0.87 \times 7.0} = 0.89$ sq. in. per ft.
Use ¾-in. ϕ at 5½-in. spacing.

Negative steel area $= \dfrac{68{,}700}{18{,}000 \times 0.87 \times 7.0} = 0.63$ sq. in. per ft.
Use ⅝-in. ϕ at 5½-in. spacing.

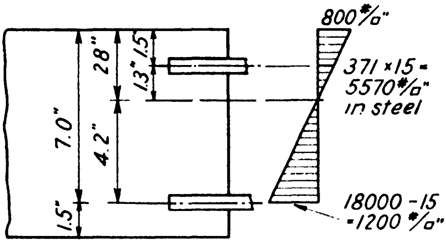


FIG. 179. EDGE STRESSES IN SLAB.

Moment resisted by compressive steel $= 5570 \times 0.67 \times 5.5 = 20{,}600$ in-lb. (See Fig. 179.)

Net moment $= 97{,}900 - 20{,}600 = 77{,}300$ in-lb.

Compression stress in concrete. $f_c = \dfrac{2 \times 77{,}300}{0.4 \times 0.87 \times 12 \times 7^2} = 760$ lb. per sq. in.

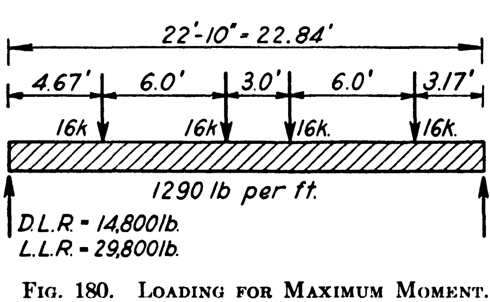


FIG. 180. LOADING FOR MAXIMUM MOMENT.

175c. Design of the Floor Beams. The span of the floor beam must be taken from center to center of the vertical posts, or 22 ft.-10 in. (Spec. 81.) The dead load may be taken as a uniform load over the entire span of 22 ft.-10 in., equal to the weight of the beam itself plus the maximum weight of the slab with topping. This procedure adequately compensates for neglecting the concentrations of load produced by the curb.

The Interior Floor Beam. This beam must be designed to carry the entire weight of the rear wheels of two 20-ton trucks. (Spec. 67 and Spec. 58.)

Maximum moment. The placing of the live load for maximum moment is shown in Fig. 180. The dead load of 1290 lb. per ft. includes an allowance of 100 lb. per ft. for the weight of the floor beam. The maximum moment occurs under the wheel load nearest the center of the span.

$$\text{D.L. moment} = 14{,}800 \times 10.67 - \frac{1290 \times 10.67^2}{2} = 84{,}200 \text{ ft-lb.}$$

$$\text{L.L. ''} = 29{,}800 \times 10.67 - 16{,}000 \times 6 = 222{,}000$$

$$\text{Impact ''} \left(\frac{50}{18 + 125} = 35\%\right) = 77{,}800$$

$$\text{Total} = 384{,}000 \text{ ft-lb.}$$

$$\text{Required section modulus} = \frac{384{,}000 \times 12}{16{,}000} = 288.$$

Selection. A $30WF108$ beam section is the most economical, but a $27WF114$ section will be used to lower the floor level. Either beam furnishes a section modulus of 299.2.

End shear. The allowable end shear on this beam at 10,000 lb. per sq. in. of web area is over two times the maximum end shear of 61,800 lb. This end reaction is found when the outside wheel is so placed that the distance from its center to the face of the curb is 1 ft.-6 in.

Selection of the End Floor Beam. There is a smaller dead load carried by an end floor beam than by an interior floor beam. Considering the effect of continuity which reduces the dead load reaction of the concrete deck to $4/10$ of the reaction for an interior beam, and allowing 100 lb. per ft. for the weight of the beam itself, we find the total dead load to be $0.4 \times 1190 + 100 = 576$ lb. per ft. This is less than 50 per cent of the dead load for an interior beam.

$$\text{D.L. moment } (\tfrac{1}{8}wL^2 \text{ is nearly exact}) \quad \tfrac{1}{8} \times 576 \times 22.83^2 = 37{,}600 \text{ ft-lb.}$$

$$\text{L.L. '' (same as for an interior beam)} = 222{,}000$$

$$\text{Impact ''} \left(\frac{50}{9 + 125} = 37.3\%\right) = 82{,}900$$

$$\text{Total} = 342{,}500 \text{ ft-lb.}$$

$$\text{Section modulus} = \frac{342{,}500 \times 12}{16{,}000} = 257.$$

Section. A $27WF98$ beam section furnishes a modulus of 255.3. This beam is of the same depth as the interior beam, which simplifies details. It fulfills the *AASHO* specifications and will be used. However, many designers prefer to use the same beam selected for the intermediate floor beams. Thus, we allow for an increased impact caused by the roughness which frequently exists at the joint between floor slab and road slab.

175d. Estimate of the Dead Weight of the Bridge.

Weight of concrete deck = 2520×72	= 181,500 lb.
Weight of roadway surface = $20 \times 20 \times 72$	= 28,800
Weight of floor beams = $114 \times 22 \times 7 + 98 \times 22 \times 2$	= 21,800
Weight of trusses and bracing = $L(250 + 4.5L)$ = $72(250 + 4.5 \times 72)$	= 41,400
Weight of handrail (20 lb. per ft.) = 20×72	= 1,500
Total	= 275,000 lb.

Dead load per foot of bridge = $275{,}000 \div 72 = 3800$ lb.

$$\text{Dead load panel concentration on one truss} = \frac{3800 \times 9}{2} = 17{,}100 \text{ lb. or 17.1 kips.}$$

TRUSS DESIGN

175e. Live Loading for Truss Design. The equivalent H-20 loading (two 20-ton trucks) is shown in Fig. 181(a). (Spec. 58.) The uniform load is the same for the determination of shear or moment, but the concentrated load is 18,000 lb. for computing moment and 26,000 lb. for computing shear. Both the uniform and concentrated loads are considered movable and both must be increased for impact.

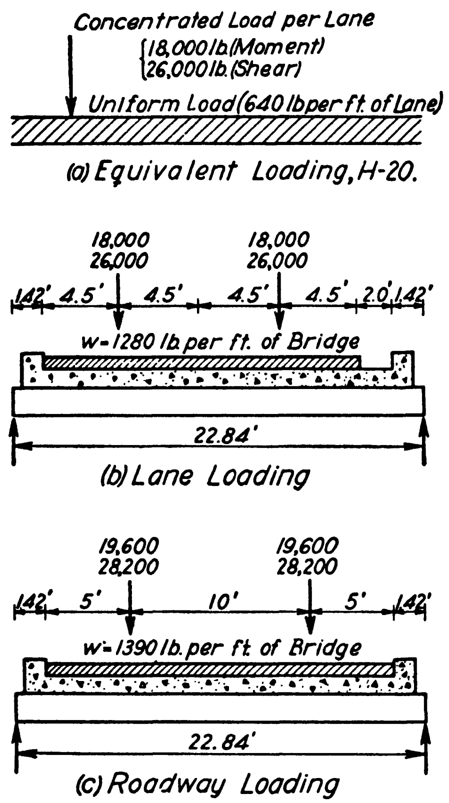


FIG. 181. APPLICATION OF LOADINGS.

Eccentricity of Loading. Since the roadway width of 20 ft. is 2 ft. more than the width of two traffic lanes, eccentricity of loading must be considered. The more serious condition represented by either specification given below must be allowed to control. (Spec. 59.)

1. Each traffic lane loading (9-ft. width) shall be considered as a unit, and the number and position of the loaded lanes shall be such as will produce maximum stress. This specification is met by placing the loads as shown in Fig. 181(b).
2. The roadway shall be considered as loaded over its entire width with a load per foot of width equal to ⅑ of the load of one traffic lane when reduced 1% for each foot of roadway width in excess of 18 ft., but not to exceed 25% reduction. This reduction is 2% for a 20-ft. roadway and the loading is shown in Fig. 181(c).

If we determine the left-hand reaction (load on the truss) from either of the sketches (b) or (c) of Fig. 181, we find that the uniform load per foot of truss becomes 695 lb. and the two concentrations on the truss are 19,600 lb. for moment and 28,200 lb. for shear. This loading is shown in Fig. 182. The panel concentration caused by the uniform load is $695 \times 9 = 6.3$ kips.

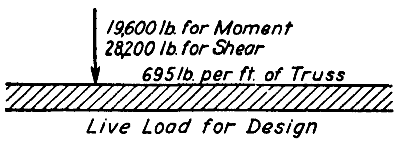


FIG. 182. LOADING ON TRUSS.

175f. Analysis of Stresses. The stress analysis for dead load and live load will not be performed in detail here. When we compute live load stresses, we place both the uniform load and the concentrated load to produce maximum stress. The percentage of impact is determined from the loaded length covered by the uniform load when placed for maximum. (Spec. 62.) The conventional method of loading is to be used. Figure 183 and the accompanying stress table show the dead load stresses, the live load stresses, and the combined stresses.

175g. Wind or Lateral Forces. The wind force on the truss is taken at 30 lb. per sq. ft. on $1\frac{1}{2}$ times the area of the structure as seen in elevation. (Spec. 64.) The truss members, handrail, gussets, etc., cover approximately 40% of the entire elevation of the truss. Taking the average depth at 8.5 ft., we find this area to be 3.4 sq. ft. per ft. of truss. To this value must be added 1.5 sq. ft. to account for the side elevation of slab and curb. The total is 4.9 sq. ft. per foot of truss; this value is increased by 50% to allow for the second truss and curb.

Wind force per foot of truss $= 30 \times 1.5 \times 4.9 = 220$ lb.
Wind force per foot of truss acting on the live load $= \underline{200}$ lb.
Total $= 420$ lb. per ft.

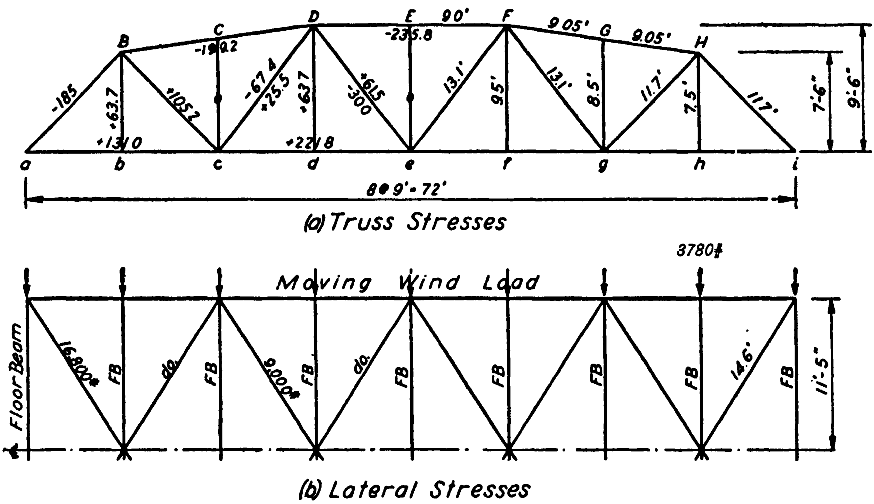


FIG. 183. DESIGN STRESSES FOR HIGHWAY BRIDGE TRUSS.

This value is greater than 50 lb. per sq. ft. on the unloaded structure and hence will control. (Spec. 64.) Wind stresses for design of the lateral system are shown in Fig. 183. These are maximum stresses by the conventional method. No wind stresses are shown for the struts (floor beams) or for the chords (lower chords of truss) since these stresses are negligibly small. Working stresses for main members may be increased 25% when lateral forces are considered. (Spec. 69.) Hence, lateral forces do not affect the design of the lower chords or the floor beams of low truss bridges, even though these members act as parts of the lateral system.

175h. Selection of the Sections for Truss Members. It is desirable that as many members as possible be selected of *beam sections* since these members require no fabrication except cutting to length and punching rivet holes for the end connections. No stay plates, diaphragms, or lacing bars are needed. Ordinarily, it is possible to use beam sections for all web members. The top chord in most pony trusses is built up from two channels and a cover plate. (Spec. 117.) The lower chord can be made of channels, angles, or possibly from a beam section. Beam sections of 8-in., 10-in., and 12-in. depths are available in numerous weights. The 8-in. beams are satisfactory as

TABLE 29

STRESS TABLE FOR TRUSS OF FIG. 183

Member	Dead Load	Live Load			Impact Per Cent	Impact Stress	Combined Stress	Design Stress[b]
		Uniform	Conc.	Total				
ab-bc	+ 71.9	+26.5	+20.6	+47.1	25.4	+12.0	+131.0	+131.0
cd-de	+121.8	+44.8	+34.9	+79.7	25.4	+20.3	+221.8	+221.8
BC-CD	−109.4	−40.3	−31.3	−71.6	25.4	−18.2	−199.2	−199.2
DE	−129.7	−47.5	−37.1	−84.6	25.4	−21.5	−235.8	−235.8
aB	− 93.4	−34.4	−38.6	−73.0	25.4	−18.6	−185.0	−185.0
Bc	+ 47.7	+19.6	+25.3	+44.9	28.0	+12.6	+105.2	+105.2
cD	− 18.6	−12.4	−18.7	−31.1	29.4	− 9.2	− 58.9	− 67.4
cD[c]	− 13.0[a]	+ 5.6	+16.6	+22.2	35.0	+ 7.8	+ 17.0	+ 25.5
De	+ 11.7	+10.9	+19.4	+30.3	31.1	+ 9.5	+ 51.5	+ 61.5
De[c]	+ 8.2[a]	− 6.6	−14.6	−21.2	32.9	− 7.0	− 20.0	− 30.0
Bb&Dd	+ 17.1	+ 6.3	+28.2	+34.5	35.0	+12.1	+ 63.7	+ 63.7

[a]70% of the D.L. stress. (Spec. 82.)
[b]When reversal occurs, 50% of the smaller combined stress is added to each to obtain the design stresses. (Spec. 82.)
[c]Reversal; that is, the truss is live-loaded from the left to attempt to reverse the sign of the design stress.

web members for truss spans up to about 80 ft. if the structure is designed for medium loads, but 10-in. sections will be used in this truss because of the heavy live loading.

175i. Compression Chord Members. *Top Chord Member DE.* The length of the member is 9 ft. or 108 in. The maximum allowable stress for a compression member corresponds to an L/r value of 40 for which the allowable stress becomes 14,300 lb.

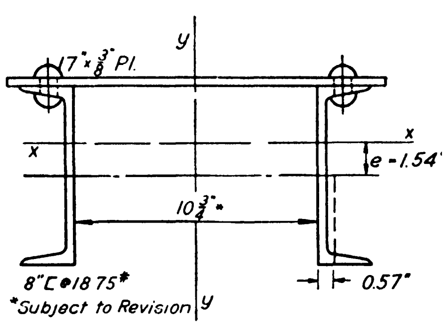


FIG. 184. TOP CHORD *DE*.

per sq. in. Accordingly, the minimum value of r for this member to justify this allowable stress is $108/40 = 2.7$ in. The minimum value of r for the typical upper chord section of two channels and a cover plate is approximately 3/8 of the depth of the channel. Hence, the smallest channel is probably $8/3 \times 2.7 = 7.2$ in. and an 8-in. channel will be tried. The width of cover plate can be varied considerably, but, to provide a section with a high degree of lateral rigidity for H-15 loading, it is satisfactory to allow a *width in inches of 1/10 of the span in feet plus 9 in.* This width may be decreased 1 in. for H-10

loading and should be increased 1 in. for H-20 loading. This rule suggests a 17-in. cover plate here.

$$\text{Area required (gross)} = 235{,}800 \div 14{,}300 = 16.50 \text{ sq. in.}$$

$$\text{Area furnished (gross)} = \begin{cases} \text{cover plate } 17 \times \tfrac{3}{8} \text{ in.} = & 6.37 \text{ sq. in.} \\ 2 \text{ [s; 8 in.-18.75 lb.} = & \underline{10.98} \end{cases}$$

$$\text{Total} = 17.35 \text{ sq. in.}$$

This cross-section is shown in Fig. 184. The distance back to back of channels, 10¾ in., is the nominal depth of a 10-in. beam plus the thickness of two ⅜-in. gussets. This width will have to be varied slightly to care for the actual depth of the beam selected for the web members.

Eccentricity. $e = (6.37 \times 4.19) \div 17.35 = 1.54$ in.

Moment of inertia about the x-x axis.

Plate	$6.37 \times 2.65^2 =$	44.7
Channels	$2(43.7 + 5.49 \times 1.54^2) =$	113.6
	$I_{x-x} =$	158.3

Moment of inertia about the y-y axis.

Plate	$\frac{1}{12} \times \frac{3}{8} \times 17^3 =$	153.5
Channels	$2(2.0 + 5.49 \times 5.95^2) =$	393.5
	$I_{y-y} =$	547.0

Radii of gyration.

$$r_{x-x} = \sqrt{\frac{158.3}{17.35}} = 3.02 \text{ in.}$$

$$r_{y-y} = \sqrt{\frac{547.0}{17.35}} = 5.60 \text{ in.}$$

Note that r_{y-y} is more than 1.5 times r_{x-x}, fulfilling Spec. 117.

Maximum value of $L/r = 9 \times 12/3.02 = 35.7$. Since this value is under 40, the allowable stress is 14,300 lb. per sq. in. and the section is satisfactory.

Top Chord Member BC-CD. This member is identical with the member DE except that a thinner cover, or channels of lighter weight, can be used.

$$\text{Area required (gross)} = 199{,}200 \div 14{,}300 = 13.9 \text{ sq. in.}$$

$$\text{Area furnished (gross)} = \begin{cases} \text{cover plate } 17 \times \tfrac{3}{8} \text{ in.} = & 6.37 \text{ sq. in.} \\ 2 \text{ [s; 8 in.-13.75 lb.} = & \underline{8.04} \end{cases}$$

$$\text{Total} = 14.41 \text{ sq. in.}$$

The section is shown in Fig. 185.

$$I_{x-x} = 134.0; \quad r_{x-x} = 3.05.$$

Again, L/r is less than 40 and the allowable stress is 14,300 lb. per sq. in.

End Post — aB. This member also must be identical with the top chord except for a reduction in weight. The next lighter weight of channel is the 8-in., 11.5-lb. section. The web thickness is only 0.22 in. for this section which is objectionably thin although it is frequently used. It would seem more reasonable to retain the 13.5-lb. channels and to reduce the cover plate to the minimum thickness of $\frac{5}{16}$ in. The effective width of cover is $40t$ plus the width outside of the rivet lines, that is, $40 \times \frac{5}{16} + 3.5 = 16$ in. (Spec. 87.)

Approximate radius of gyration = 3.0, $L/r = \dfrac{11.7 \times 12}{3.0} = 46.8.$

$$\text{Allowable stress} = \frac{16{,}000}{1 + \dfrac{46.8^2}{13{,}500}} = 13{,}750 \text{ lb. per sq. in.}$$

$$\text{Approximate area required} = \frac{185{,}000}{13{,}750} = 13.5 \text{ sq. in.}$$

$$\text{Area furnished} \begin{cases} \text{plate} = 16 \times \tfrac{5}{16} & = 5.00 \text{ (effective)} \\ \text{channels} = 2 \times 4.02 & = \underline{8.04} \end{cases}$$

$$\text{Total} = 13.04 \text{ sq. in.}$$

The section is shown in Fig. 186.

$I_{x-x} = 126.0;\ r_{x-x} = 3.1.$

$$\text{The slenderness ratio} = \frac{11.7 \times 12}{3.1} = 45.2.$$

$$\text{Allowable stress} = \frac{16{,}000}{1 + \dfrac{45.2^2}{13{,}500}} = 13{,}900 \text{ lb. per sq. in.}$$

Capacity of member = 13.04 × 13,900 = 181,000 lb.

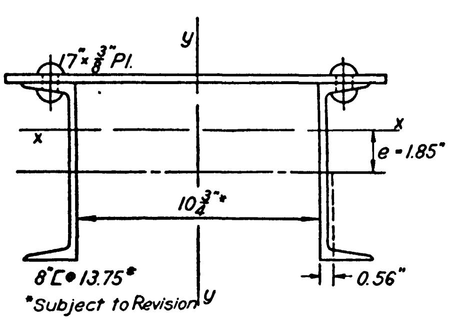


FIG. 185. TOP CHORD *BC-CD*.

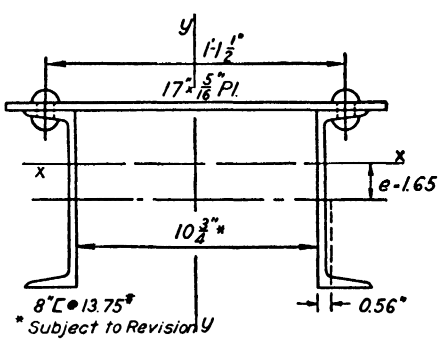


FIG. 186. TRIAL SECTION — END POST.

This is 4000 lb. under the actual load and the member is unsatisfactory. The cover will be increased to ⅜ in. which makes this member identical with the member *BC-CD*.

175j. Diagonal Web Members. *Diagonal Bc.* This is a tension member carrying a stress of 105,200 lb.

Required net area = 105,200 ÷ 16,000 = 6.6 sq. in.

A 10*WF*33 section offers a gross area of 9.71 sq. in.

Net area with four 1-in. holes out of flanges = 9.71 − 4(1.0 × 0.433) = 7.98 sq. in.

A 10*WF*29 section offers a gross area of 8.53 sq. in.

Net area with three 1-in. holes out of flanges = 8.53 − 3(1.0 × 0.50) = 7.03 sq. in. (A deduction of 3 rivet holes is sufficient with proper arrangement of rivet spacing.)

The choice between these sections will depend upon the sections chosen for the other web members. All web members must be approximately of *the same depth.* The depth of the 10*WF*33 beam section is 9.75 in. and the depth of the 10*WF*29 beam is 10.22 in.

Diagonal cD. The design stresses for this member are a compression of 67,400 lb. and a tension of 25,500 lb. The compression stress controls the design. The member will be a 10-in. beam section.

The 10*WF*33 section has a minimum r of 1.94; $L/r = \dfrac{13.1 \times 12}{1.94} = 81.0.$

$$\text{Allowable compressive stress} = \frac{16{,}000}{1 + \dfrac{81^2}{13{,}500}} = 10{,}750 \text{ lb. per sq. in.}$$

$$\text{Actual stress} = \frac{P}{A} = \frac{67{,}400}{9.71} = 6940 \text{ lb. per sq. in.}$$

The 10*WF*29 section has a minimum r of 1.34; $L/r = \dfrac{13.1 \times 12}{1.34} = 117.$

$$\text{Allowable compressive stress} = \frac{16{,}000}{1 + \dfrac{117^2}{13{,}500}} = 7950 \text{ lb. per sq. in.}$$

$$\text{Actual stress} = \frac{P}{A} = \frac{67{,}400}{8.53} = 7900 \text{ lb. per sq. in.}$$

Diagonal De. The design stresses for this member are a tension of 61,500 lb. and a compression of 30,000 lb. Either stress may control the design. A beam section that is satisfactory for tension will be selected and checked for compression.

Required net area = 61,500/16,000 = 3.84 sq. in.

A 10*WF*21 section offers a gross area of 6.19 sq. in.

Net area with four 1-in. holes out of flanges = $6.19 - 4(1.0 \times 0.34) = 4.83$ sq. in.

Minimum radius of gyration = 1.25; $L/r = \dfrac{13.1 \times 12}{1.25} = 126.$

$$\text{Allowable compressive stress} = \frac{16{,}000}{1 + \dfrac{126^2}{13{,}500}} = 7350 \text{ lb. per sq. in.}$$

$$\text{Actual stress} = \frac{P}{A} = \frac{30{,}000}{6.19} = 4840 \text{ lb. per sq. in.}$$

This section is considerably understressed both in tension and in compression, but all special 10-in. beams of lighter weights have narrow flanges (4 in.) which will not carry ⅞-in. rivets.

175k. Tension Chord Members. *Lower Chord Member, ab-bc.* The design stress for this member is a tension of 131,000 lb. A beam section as a bottom chord member does not furnish a simple connection for the lateral bracing and, therefore, will not be considered. Two angles placed outside of the gusset plates with the longer legs turned down form a satisfactory member.

$$\text{Net area required} = \frac{131{,}000}{16{,}000} = 8.2 \text{ sq. in.}$$

Two $6 \times 4 \times \frac{9}{16}$-in. angles furnish a gross area of 10.62 sq. in.

Details will be arranged so that but 2 holes need be deducted from each angle. The full net section is effective. (Spec. 85.)

Net area = 10.62 − 4(1.0 × 0.56) = 8.38 sq. in.

Slenderness ratio $= L/r = \dfrac{9 \times 12}{1.9} = 57$ (under 200, Spec. 84).

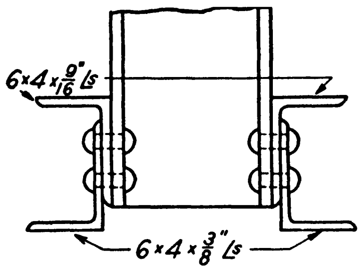


FIG. 187. BOTTOM CHORD *cd-de*.

Lower Chord Member, cd–de. The design stress for this member is a tension of 221,800 lb. A desirable section can be formed of 4 angles, 2 of which are the same as those used for the member *ab-bc*. The angles are placed as shown in Fig. 187.

Net area required $= \dfrac{221{,}800}{16{,}000} = 13.85$ sq. in.

Net area of two 6 × 4 × 9/16-in. angles = 8.38

Difference = 5.47 sq. in.

Two 6 × 4 × 3/8-in. angles furnish a gross area of 7.22 sq. in.

Net area = 7.22 − 4(1.0 × 0.375) = 5.72 sq. in.

175l. Vertical Web Members. *Hangers Bb and Dd.* These members resist a direct tension stress of 63,700 lb. They also must be designed to resist a bending moment at the top of the floor beam produced by a lateral force on the top chord. This force is determined from the expression $150(A + P)$ where A is the cross-sectional area of the top chord in *square inches*, and P is the panel length in *feet*. (Spec. 116.)

Force on top chord = 150(17.35 + 9) = 3950 lb.

The lever arm for this force is the depth of the truss minus the depth of the floor beam and minus the gage distance for the 6-in. angle leg of the lower chord. (The floor beam rests on top of the lower chord.)

Maximum moment = 3950(9.5 × 12 − 27 − 2.25) = 335,000 in-lb.

Section. Try a 10*WF*33 section. Reduce the gross area for 2 holes out of one flange, and, according to the usual procedure in girder design, reduce the moment of inertia by two holes out of each flange.

Net area = 9.71 − 2(1.0 × 0.433) = 8.85 sq. in.

Net moment of inertia = 170.9 − 4(1.0 × 0.433 × 4.65^2) = 133.4.

$$\text{Fiber stress} = \frac{P}{A} + \frac{Mc}{I} = \frac{63{,}700}{8.85} + \frac{335{,}000 \times 4.87}{133.4} = 19{,}500 \text{ lb. per sq. in.}$$

This stress is not excessive since a 25% increase in working stress is allowed when the calculated stress is produced by a combination of D.L., L.L., and lateral forces. (Spec. 69.)

Vertical Members Cc and Ee. These members are not stressed as heavily as the members *Bb* and *Dd*, (in fact, they carry no calculated direct stress) but, to simplify details, the same sections are used.

175m. Remarks on the Selection of Web Members. All web members should be approximately of the same depth in order to avoid the use of *fillers*. Hence, a 10WF29 section (10¼-in. depth) cannot be used for a diagonal when the 10WF33 section (9¾-in. depth) is required for the vertical hangers. Either the verticals would have to be 10WF45 sections (10⅛-in. depth) or else they could be made of built-up sections, (10 × 5/16-in. web and 4 angles 4 × 3 × 5/16 in.). A better arrangement is to use the

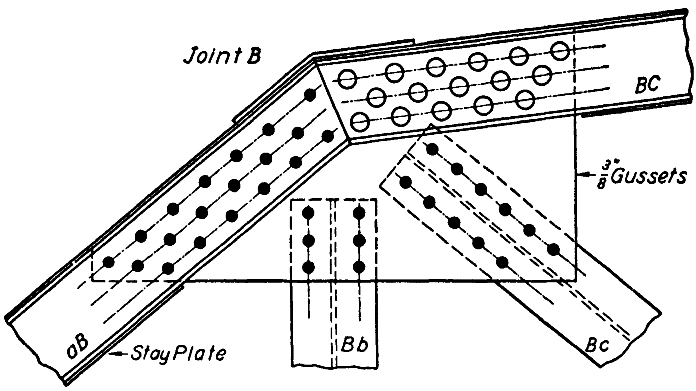


FIG. 188. DETAIL AT THE JOINT B.

10WF33 section for all the diagonals and verticals. The 10WF21 section (9⅞-in. depth) could be used for the diagonal Dc, but the 10WF33 section is shown on the drawing. Gusset plates will be placed 9⅞ in. apart, which leaves ⅛-in. clearance for easy insertion of all web members. The influence of this change upon the design calculations 's negligible.

175n. Lateral Diagonals. The maximum stress in a lateral diagonal is 16,800 lb. tension. The minimum angle allowed is 3 in. by 2½ in. (Spec. 113.)

$$\text{Net area required} = 16{,}800 \div 16{,}000 = 1.05 \text{ sq. in.}$$

$$\text{Net effective area of a } 3\tfrac{1}{2} \times 3\tfrac{1}{2} \times \tfrac{5}{16}\text{-in. angle} = \frac{2.09}{4} + \frac{2.09}{2} - (1.0 \times 0.312)$$

$$= 1.25 \text{ sq. in.} \quad \text{(Spec. 85.)}$$

Slenderness ratio = 14.6 × 12 ÷ 1.08 = 163. (Laterals are connected to floor beams at their intersection.) **Note use of r about a horizontal axis. Explain.**

This slenderness ratio is not excessive for a tension member. Hence, there will be no objectionable sagging of the laterals.

175o. Design of the Joints. The rivets will be ⅞ in. The top chord will be field spliced at the joints B and D. The lower chord is field spliced just to the right of the joint c. This splice could be placed to the left of the joint c more conveniently, but the center member would need to be more than 40 ft. in length, which is the usual length of a freight car. Gusset plates are to be shop riveted to the chords. The riveted joints will be designed to develop the *full strengths of the members*. (Spec. 98.) Gusset plates will be of the minimum thickness allowed — ⅜ in. (Spec. 86.)

Joint B. Each member meeting here must be connected to the gusset plate for its full value.

Member *aB*. All rivets pass through the channel webs, and bearing on the web (0.303 in. thick) controls the design (field rivets).

Value of a ⅞-in. field rivet in bearing on a 0.303-in. web = 5300 lb.
Value of member = 14.41 × 13,800 = 199,000 lb.
Number of rivets = 199,000 ÷ 5300 = 38 rivets.

Member *BC*. Bearing on the web again controls the design (shop rivets).

Value of ⅞-in. rivet in bearing on a 0.303-in. web = 6370 lb.
Value of member = 14.41 × 14,300 = 206,000 lb.
Number of rivets = 206,000 ÷ 6370 = 32 rivets.

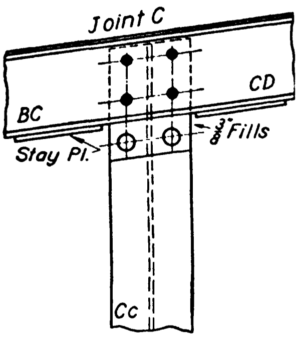


FIG. 189. DETAIL AT THE JOINT *C*.

Member *Bc*. Single shear controls the design (field rivets).

Value of rivet = 6010 lb.
Net value of member = 7.98 × 16,000 = 128,000 lb.
Number of rivets = 128,000 ÷ 6010 = 22 rivets.

Member *Bb*. This member is designed for both tension and flexure. Hence, the connection needs to be designed for the direct tension only (63,700 lb.).

Rivet value = 6010 lb. as for *Bc*.
Number of rivets = 63,700 ÷ 6010 = 11 rivets (12 rivets are used).

Details. The arrangement of rivets at the joint is shown in Fig. 188.

Joint C. There are no calculated stresses to be resisted at this joint. The detail shown in Fig. 189 forms a satisfactory connection.

Joint D. The field connection for the member *CD* can be obtained by increasing the number of shop rivets in *BC* at the joint *B* by 20 per cent (39 rivets).

Member *DE*. Single shear controls the design (shop rivets at 7220 lb. per rivet).

Value of the member = 17.35 × 14,300 = 248,000 lb.
Number of rivets = 248,000 ÷ 7220 = 34 rivets.

Member *cD*. Single shear controls the design (field rivets).

Value of member = 10,750 × 9.71 = 104,500 lb.
Number of rivets = 104,500 ÷ 6010 = 18 rivets. This number of rivets is more than adequate to meet the requirements of Spec. 82.

Member *De*. Single shear controls the design (field rivets).

Value of member. The end connection must develop the sum of the tension and compression stresses for any member which undergoes reversal, that is, 51,500 + 20,000 = 71,500 lb. (Spec. 82.)

Number of rivets = 71,500 ÷ 6010 = 12 rivets. However, 14 rivets are shown in Fig. 190 as required by the shape of the gusset.

Member *Dd*. Same as *Bb* at the joint *B* — 12 rivets are used.
Details. The arrangement of rivets at the joint *D* is shown in Fig. 190.

Joint c. This joint will be considered in connection with the splice in the lower chord. The 6 × 4 × ⁹⁄₁₆-in. angles are continuous for the full length of the lower chord and will be spliced immediately to the right of the joint *c*. The 6 × 4 × ⅜-in.

angles are required through the four center panels only; they will be riveted to the gusset plates at c for their full value.

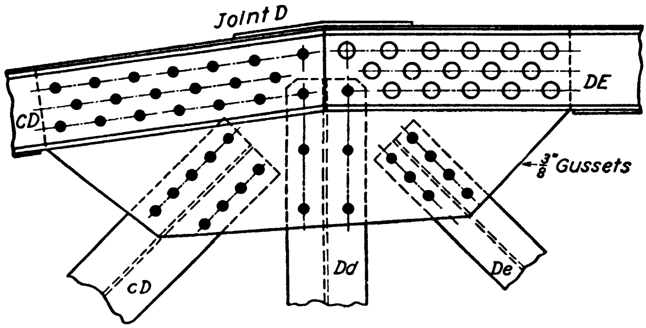


FIG. 190. DETAIL AT THE JOINT D.

Member *cd*. These rivets act in single shear and they must be field rivets.

Net value of two $6 \times 4 \times \frac{3}{8}$-in. angles $= 5.72 \times 16{,}000 = 91{,}500$ lb.

Number of rivets $= 91{,}500 \div 6010 = 16$ rivets into the gusset plates.

Additional rivets. The 4 extra *shop* rivets shown through the gusset in Fig. 191 are used to fill out the length of the gusset and to form a connection to the lower chord that will resist the sum of the horizontal components of the diagonal web members. The 14 rivets connecting the lower chord to *each gusset* are adequate to resist the horizontal components of the diagonal members. The diagonals, considered together, have 20 rivets through each gusset plate. Approximate horizontal component $= 0.7 \times 20 = 14$ rivets.

Splice in the $6 \times 4 \times \frac{9}{16}$-in. Angles. A $\frac{3}{8}$-in. plate, $18\frac{1}{2}$ in. wide placed across the 4-in. legs offers more than enough area for splicing them. Two $5\frac{1}{2} \times \frac{5}{8}$-in. plates placed inside of the 6-in. legs complete the splice. All rivets are field driven and act in single shear. (See Fig. 191.)

Net value of two $6 \times 4 \times \frac{9}{16}$-in. angles $= 8.38 \times 16{,}000 = 134{,}000$ lb.

Total number of field rivets $= 134{,}000 \div 6010 = 22.3$. Use 23 rivets.

Rivets through 6-in. legs $= \frac{6}{10} \times 23 = 14$.

Rivets through 4-in. legs $= \frac{4}{10} \times 23 = 10$.

Details. The arrangement of rivets at the joint c and in the lower chord splice is shown in Fig. 191.

Joint e. Fourteen rivets are placed in each diagonal member. These diagonals cannot operate simultaneously at maximum capacity (one in tension and the other in compression) but such an assumption is on the safe side. Hence, the number of field rivets required to attach each gusset plate to the lower chord will be $14 \times 9/13.1 = 10$ rivets. As shown in Fig. 192, we have used 8 shop rivets and 2 field rivets.

Gusset Plate Design. The gusset plates function in transferring stress from one member to another. Double gussets for short spans are usually $\frac{3}{8}$ in. thick. Much thicker gussets are necessary for long heavy trusses. As a minimum study we will check the vertical shear and the flexural stress for the gusset at the joint c. (See Fig. 191.) Evidently, it is conservative to check the gusset for applied forces equal to the values of the riveted connections. For convenience, the controlling rivets (groups *Bc* and *cd*) are given values of 6010 lb., the single shear value of a field rivet. The forces to the right in Fig. 191(b) control maximum moment and those shown to the left produce

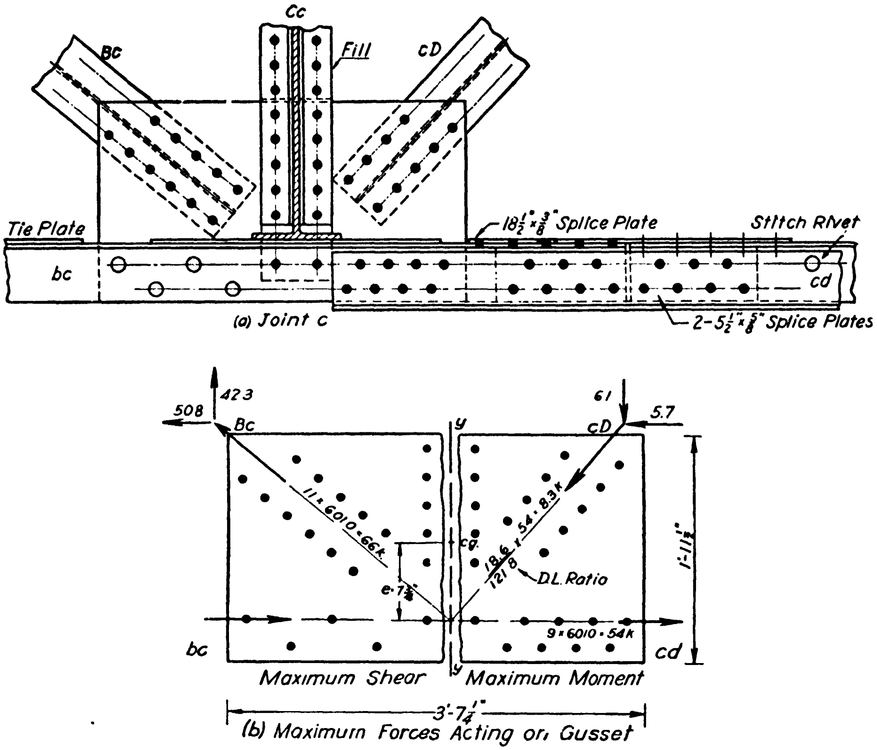


FIG. 191. TRUSS JOINT *c* AND CHORD SPLICE.

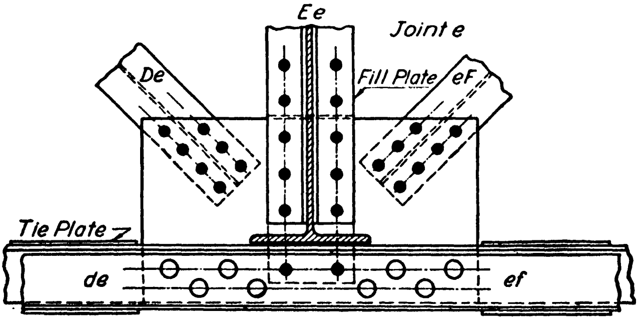


FIG. 192. DETAIL AT THE JOINT *e*.

maximum shear. The stresses for the diagonal *cD* and for the chord *cd* would be produced by a *uniform loading*. For the forces from Fig. 191(*b*) we have

Maximum unit shear on section *y-y* (parabolic variation) =

$$1.5 \times 42{,}300 \div (0.375 \times 23.5) = 7{,}200 \text{ lb. per sq. in.}$$

$$\text{Unit direct stress on } y\text{-}y = (54{,}000 - 5700) \div (0.375 \times 23.5) = 5{,}500 \text{ lb. per sq. in.}$$

$$\text{Unit flexural stress on } y\text{-}y = \frac{48{,}300 \times 7.75 \times 11.75}{\frac{1}{12} \times 0.375 \times 23.5^3} = 10{,}800$$

$$\text{Total} = 16{,}300 \text{ lb. per sq. in.}$$

Even though a net section along the vertical line of rivets had been used, the unit stress would still have been reasonable. There seems to be no possibility that any combination of applied forces would overstress this gusset since the forces used are much larger than the actual maximum stresses in the members. There may be some question as to the proper choice of eccentricity *e* in Fig. 191(*b*). However, since the upper line of rivets was taken as the working center line of the lower chord, it seemed proper to measure the eccentricity from this line.

175p. Floor-Beam Connections. The end floor-beam connection resists vertical shear alone while the connection for an interior beam must resist both shear and moment. The connection for an interior beam will be designed first, and if it is not excessively heavy, the same connection will be used on the end floor beam to simplify details.

Interior Floor-Beam Connection.

Maximum end reaction = 61,800 lb.

Lateral force at C.G. of top chord = 3950 lb. (Spec. 116.)

Moment about mid-height of floor beam = M = 3950 × 98.0 = 387,000 in-lb.

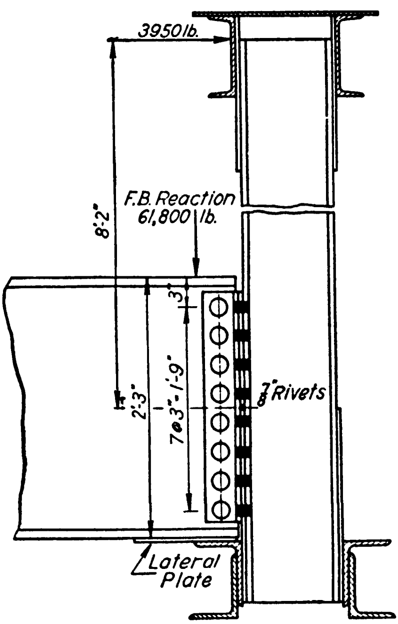


Fig. 193. Floor-Beam Connection.

Trial connection. The standard end connection shown in Fig. 193 will be tried.

Vertical shear per rivet through web of beam = 61,800 ÷ 8 = 7720 lb.

Moment of inertia of rivet group about its center (each rivet is considered as a unit area) = $2(1.5^2 + 4.5^2 + 7.5^2 + 10.5^2) = 378$.

$$\text{Horizontal shear caused by flexure} = \frac{Mc}{I} = \frac{387{,}000 \times 10.5}{378} = 10{,}700 \text{ lb. per rivet.}$$

Direct horizontal shear = 3950 ÷ 8 = 500 lb. per rivet.

Resultant rivet shear. (See Fig. 194.)

$$R = \sqrt{11{,}200^2 + 7720^2} = 13{,}600 \text{ lb. per rivet.}$$

Allowable rivet shear. These are shop rivets acting in bearing on the 0.57-in. beam web. Their allowable shear is 12,000 lb. per rivet, which may be increased 25% to 15,000 lb. for the effect of combined loads. It should be realized that the lateral force

really acting at the top chord is much smaller than 3950 lb. Hence, the computed rivet shear is a measure of *stiffness* rather than *strength*.

Connection to the Vertical Post. The rivets connecting the clip angles to the vertical post act in single shear to resist the end reaction of the floor beam and in tension to resist the moment produced by the lateral force on the top chord. Since all hot driven rivets carry a high initial tension, which was considered by those who determined the working stress in shear, *it is unnecessary to consider combined stresses*. However, both the shear and tension must be kept within their respective allowable limits.

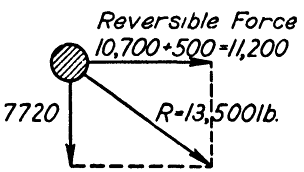


FIG. 194. RESULTANT RIVET SHEAR.

Single shear per rivet = 61,800 ÷ 16 = 3860 lb.

Effective cross-section for flexure. Use sectional area of ⅞-in. tension rivets above the neutral axis and bearing area of 4-in. angle legs below. The neutral axis is located at the center of gravity of the cross-section by trial. (See Fig. 195).

Moment of inertia.

$$\text{Bearing area} = 2(\tfrac{1}{3} \times 4 \times 4.5^3) = 243 \text{ in.}^4$$

$$\text{Rivet areas} = 2 \times 0.6(3^2 + 6^2 + 9^2 + 12^2 + 15^2 + 18^2) = 981$$

$$\text{Total } I = 1224 \text{ in.}^4$$

Bending moment about N.A. = 3950 × 105.5 = 416,000 in-lb.

$$\text{Tension stress in upper rivet} = \frac{Mc}{I} = \frac{416{,}000 \times 18}{1224} = 6100 \text{ lb. per sq. in.}$$

Allowable tension. Rivet tension is limited to one half of the working stress in single shear or 5000 lb. per sq. in. for field rivets. (Spec. 97.) However, this working stress may be increased to 6250 lb. per sq. in. for combined loading which includes the effect of lateral forces.

The standard end connection (shown in Fig. 193) is satisfactory and will be used. The angles used will be 4 × 3½ × ⅝ in., 2 ft.-0 in. long. This thickness of metal is required to resist flexure caused by the pull of the tension rivets. The same detail will be used for all floor-beam connections. Observation will show that most designers extend the connection angles above the top of the floor beam. This detail makes possible the use of thinner connection angles.

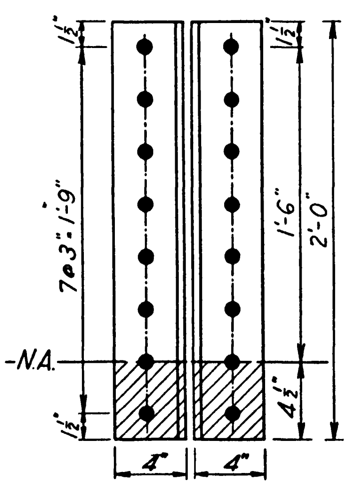


FIG. 195. FLOOR-BEAM CONNECTION TO POST.

175q. End Bearing and Joint *a*. The number of rivets required in the members *aB*, *ab*, and in the floor-beam connection can be determined from previous computations. A short piece of 10*WF*33 beam section placed between the gussets opposite the floor-beam connection serves the purpose of a *diaphragm* and also assists the gussets to provide a sufficient thickness of metal for bearing on the pin. Study the drawing, Fig. 196, and the photograph, Fig. 197.

Selection of the Pin. Gross end reaction is computed for a dead load of 1900 lb. per ft. of truss, a uniform live load of 695 lb. per ft. of truss, and a concentrated

live load of 28,200 lb. The impact percentage is 25.4. Hence, $R = 1900 \times 36 + 1.254(28{,}200 + 695 \times 36) = 135{,}400$ lb.

Shear in the pin is one half of the gross end reaction = 135,400 ÷ 2 = 67,700 lb.
Average unit shear in a 3½-in. pin = 67,700 ÷ 9.6 = 7050 lb. per sq. in.
Bending moment in pin = 67,700 × 1.31 = 88,800 in-lb. (See Fig. 196.)

$$\text{Fiber stress in pin} = \frac{Mc}{I} = \frac{88{,}800 \times 1.75}{0.049 \times 3.5^4} = 21{,}100 \text{ lb. per sq. in.}$$

Bearing on pin = 67,700 ÷ (3.5 × 0.875) = 22,200 lb. per sq. in. (See Fig. 196.)
The 3½-in. pin is not overstressed and will be used. (See page 309.)

Rocker.

Allowable bearing (600*d*; Spec. 70) = 600 × 16 = 9600 lb. per lineal in.
Length of rocker = 135,400 ÷ 9600 = 14.1 in. A length of 21 in. is used to provide ample space for anchor bolts. (See Fig. 196).

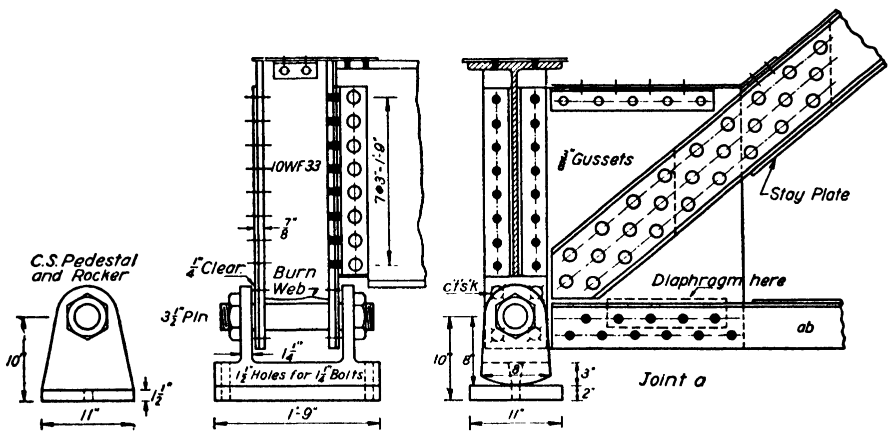


Fig. 196. Rocker Detail at the Reaction.

Moment in pillow. When the ends of a beam uniformly loaded overhang by 35% of the clear span, as in Fig. 196, the maximum moment is $wL^2/16$.

$$wL^2/16 = \frac{135{,}400}{21.0} \times \frac{12.37^2}{16} = 61{,}500 \text{ in-lb.}$$

Bending stress in rectangular beam 8 in. wide and 2 in. thick. $f = \dfrac{61{,}500 \times 1.0 \times 12}{8 \times 2^3}$ = 11,500 lb. per sq. in. (12,000 is allowable for cast steel, see § 175a.) Hence, the stress in the pillow (lower part of rocker casting) is satisfactory since its moment of inertia is considerably greater than that of the 8-in. × 2-in. rectangle.

Standards. The vertical standards as shown in Fig. 196 are made 1¼ in. thick. The allowable stresses for cast steel are ¾ of the allowable stresses for rolled steel. (Spec. 71.) Hence, a thickness of 1¼ in. is ample since a ⅞-in. thickness of rolled steel was found satisfactory for bearing on the pin. The standard is far understressed as a column.

Bearing Block.

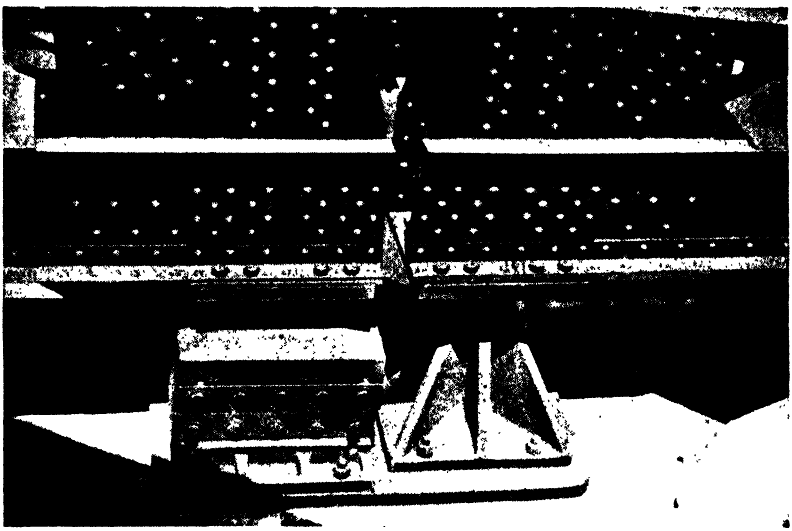
Width required for bearing on concrete masonry = 135,400 ÷ (600 × 21) = 10.8 in. Use 11 in. as shown in Fig. 196.

Thickness required for flexure. Bending moment at center line $= \frac{135,400}{2} \times \frac{5.5}{2} = 187,000$ in-lb.

Section modulus = 187,000 ÷ 16,000 = 11.7.

Required thickness. $\frac{1}{6}bd^2 = 11.7$; $d = \sqrt{\frac{6 \times 11.7}{(21.0 - 3.0)}} = 1.98$ in.

Use a rolled steel plate; 21 in. × 11 in. × 2 in.



Courtesy C. M. St. P. & P. R.R. Co.

Fig. 197. Expansion Support at Left, Fixed Pedestal at Right.

Fixed End. The fixed pedestal is similar to the rocker except that its height must be equal to the height of the rocker plus the thickness of the base plate. It must provide a bearing area 21 in. × 11 in. A thickness of 1½ in. is found to be satisfactory for the flat base with stiffeners. The standards are made 1¼ in. thick. (See Fig 196.)

175r. Lateral Connections.

Maximum stress in a lateral = 16,800 lb.

Value of member = 1.25 × 16,000 = 20,000 lb. with unincreased working stress.

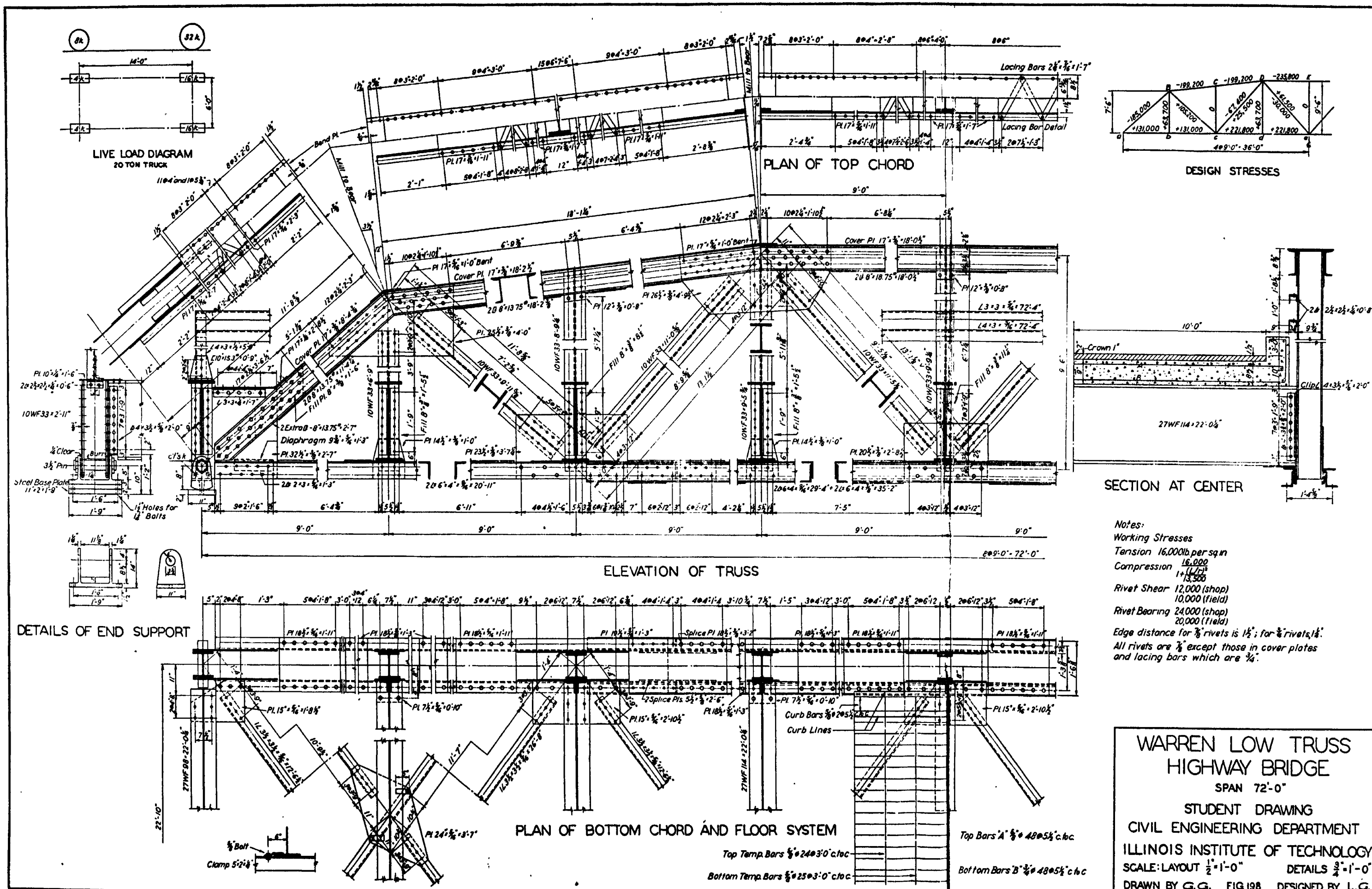
Value of a ⅞-in. field rivet in bearing on the 5/16-in. angle leg = 5470 lb. with unincreased working stress. Since stresses for the member and also for the rivets are not increased for wind allowance, the design of the connection will be correct.

Number of rivets = 20,000 ÷ 5470 = 4 rivets.

It is not always considered necessary to develop the full value of laterals and other secondary members where excess area is used to provide stiffness. Many designers would use 3 rivets here. (Spec. 113.)

Lateral plates. 5/16-in. plates are used throughout.

175s. Truss Details. Details for this truss are shown in Fig. 198. Spacing of rivets, edge distances, and other such details are arranged to conform to the specifications of the American Association of State Highway Officials. A few details need special consideration. Instructions for structural detailing are given in § 219.



Notes:
Working Stresses
Tension 16,000 lb. per sq. in.
Compression $\frac{16,000}{1+\frac{(L/r)^2}{13,500}}$
Rivet Shear 12,000 (shop)
10,000 (field)
Rivet Bearing 24,000 (shop)
20,000 (field)
Edge distance for ⅞" rivets is 1½"; for ¾" rivets 1¼".
All rivets are ⅞" except those in cover plates and lacing bars which are ¾".

WARREN LOW TRUSS
HIGHWAY BRIDGE
SPAN 72'-0"
STUDENT DRAWING
CIVIL ENGINEERING DEPARTMENT
ILLINOIS INSTITUTE OF TECHNOLOGY
SCALE: LAYOUT ½" = 1'-0" DETAILS ¾" = 1'-0"
DRAWN BY G.G. FIG. 198 DESIGNED BY L.G.

Stay Plates. The thickness of stay plates, to avoid possible buckling, cannot exceed $\frac{1}{50}$ of the distance between rivet lines. (Spec. 103.) The maximum distance between rivet lines occurs on the lower chord and is $15\frac{5}{8}$ in. $t = \dfrac{15.6}{50} = \frac{5}{16}$ in. The lengths of stay plates must conform to Spec. 103.

Lacing Bars. Single lacing can be used on the upper chord since the distance between rivet lines is but $13\frac{5}{8}$ in. (Spec. 104 limits this distance to less than 15 in. for single lacing.)

Shear to be resisted for the member *DE*. (Spec. 104.)

$$V = \frac{P}{100}\left(\frac{100}{L/r + 10} + \frac{L/r}{100}\right) = \frac{248{,}000}{100}\left(\frac{100}{108/5.6 + 10} + \frac{108/5.6}{100}\right) = 8900 \text{ lb.}$$

Shear per lacing bar for single lacing = 8900/2 = 4450 lb. (Spec. 104.)
Stress per bar (placed at 60° with member) = 4450/0.866 = 5150 lb.

$$\text{Thickness of bar } (\tfrac{1}{40} \text{ of length}) = \frac{13.62}{0.866 \times 40} = 0.39 \text{ in. Use } \tfrac{7}{16} \text{ in.}$$

$$\text{Slenderness ratio } L/r = \frac{13.62}{0.866 \times 0.29 \times 0.44} = 124.$$

$$\text{Allowable compression} = \frac{16{,}000}{1 + \dfrac{124^2}{13{,}500}} = 7470 \text{ lb. per sq. in.}$$

$$\text{Actual stress } (2\tfrac{1}{4} \times \tfrac{7}{16}\text{-in. bar}) = \frac{5150}{2.25 \times 0.44} = 5200 \text{ lb. per sq. in.}$$

End connection. One $\frac{3}{4}$-in. shop rivet has a value of 5300 lb. in single shear. Use one $\frac{3}{4}$-in rivet at each connection.

Slenderness ratio for channel between connections = 13.62 ÷ (0.866 × 0.60) = 26. (Spec. 104.)

175t. Net Section of Lower Chord. Only 2 rivet holes were deducted from each lower chord angle. (There are 3 gage lines spaced for clearance as shown in Fig. 199.) The rivet stagger must be arranged so that this condition will be fulfilled. The deduction is controlled by the formula $X = 1 - \dfrac{s^2}{4gh}$. (Spec. 106.)

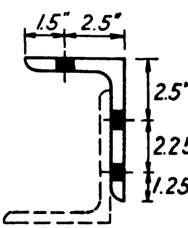


FIG. 199. SPECIAL GAGES.

Assume the stagger to be 2 in.	Assume the stagger to be $2\frac{1}{2}$ in.
$X = 1 - \dfrac{2^2}{4 \times 5 \times 1.0} = 0.80$	$X = 1 - \dfrac{2.5^2}{4 \times 5 \times 1.0} = 0.69$
$X = 1 - \dfrac{2^2}{4 \times 2.25 \times 1.0} = \underline{0.55}$	$X = 1 - \dfrac{2.5^2}{4 \times 2.25 \times 1.0} = \underline{0.30}$
Total deduction = 1.35 + 1 = 2.35 holes.	Total deduction = 0.99 + 1 = 1.99 holes.

A stagger of $2\frac{1}{2}$-in. is satisfactory and will be used between the first 2 rivets. The stagger can then be decreased to 2 in., since the total stress on the net section has been reduced by the value of the first rivet. This arrangement applies at the joint *a* as may be seen in Fig. 198 where a diaphragm necessitates rivet holes on three gage lines. At the joints *b* and *d* the stagger is shown as $1\frac{1}{2}$ in. Actually, it would be desirable to increase this value to 2 in. since the floor-beam connection requires three gage lines to be used in *one* of the chord angles.

PROBLEMS

218. Redesign the highway bridge truss of § 175 for the standard working stresses of the American Association of State Highway Officials as given in Spec. 70.

219. Redesign the highway bridge truss of § 175 for *H*-15 loading and working stresses taken from the 1941 *AASHO* specifications.

220. Design a highway bridge truss to meet these requirements: (*a*) low truss type, (*b*) span 80 ft.-3 in., (*c*) clear roadway width 24 ft.-0 in., (*d*) *H*-20 loading, (*e*) *AASHO* specifications and working stresses from § 216, (*f*) roadway to be of untopped concrete supported on stringers.

221. Design a light industrial roadway bridge of low truss type for *H*-10 loading and a 65-ft. span. The roadway width is 18 ft.-0 in. The flooring is of planks 3 in. thick. Use a horizontal upper chord and a floor system with stringers. Use the working stresses allowed by the *AISC* specifications but follow the *AASHO* requirements in other respects as given in § 216.

222. Set up general conditions regarding the terrain of a stream crossing and design a low truss highway bridge to fit these conditions. Follow your state highway specifications or other local codes.

176. Conclusions Regarding Truss Bridge Design. The low truss type of highway bridge was chosen to illustrate the general problem of truss bridge design because it can be presented rather briefly. The through truss bridge is used for longer spans. It has more members, but its design is not particularly more complicated. The only other problems involved are the design of the upper laterals, the portal, and the sway frames. Consideration of the railway truss brings up the complication of locomotive loadings, but its design introduces few other problems. A pin connected truss is considerably different from the riveted truss designed here. However, this difference is primarily in the specialized problems of pin packing and of pin-plate design considered in Chapter 4. Welded trusses are becoming more common and they have already been given consideration as building structures where their use has been entirely accepted. If the student carries out the design of a low truss bridge in all of its details, he will have met with most of the common problems of steel bridge design.

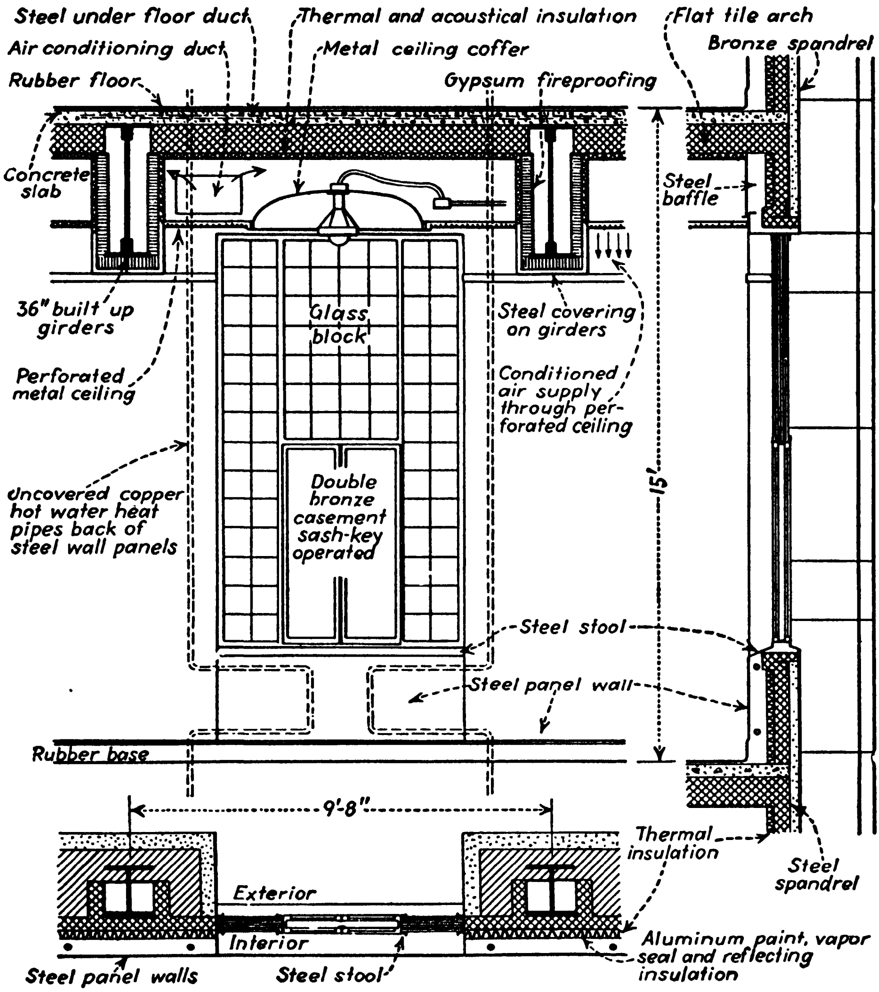
CHAPTER 14

OFFICE BUILDINGS

177. Tier Construction. The skyscraper is the obvious example of tier arrangement, but modern design employs this construction for buildings of all heights. The older development of wall bearing construction practically disappeared with the introduction of modern architecture which employs glass or other *thin wall construction* that is unable to support its own weight for a height above one story. All loads must be supported on the steel frame consisting of beams, girders, and columns. As soon as the height of the building exceeds the normal length of a column (two stories or three stories at most), we have to splice one column on top of another, and we have then produced tier construction. Except for the wind-stress problem, the same considerations will apply to the four or five-story building as to one which is twenty or thirty stories in height. Some of these problems that affect the structural design are as follows. What spacing of columns will prove economical? Are wall columns to be permitted or are the floors to cantilever out beyond the columns? How are the floors to be framed? What arrangement of elevators and stairs is needed? What methods of *fireproofing* are to be used? What heavy machinery needs to be supported? These and similar questions are matters of structural and of functional design. They may be answered from experience or by analysis and study of solutions found by other engineers for other structures. Since most young engineers lack experience, the following group of examples are introduced to help guide the beginner who is faced with the layout of a tier building. These examples have been reported by Engineering News-Record as instances of good structural design and good functional arrangement.

EXAMPLES OF FUNCTIONAL ARRANGEMENT

178. Seven-Story Office Building. This structure was built for the Bankers Life Company of Des Moines. Exterior trim is of granite and limestone, casement windows and exterior doors are of bronze. The windows are set in large panels of glass brick as shown in Fig. 200. Interior finish is of enameled steel sheets. The building is insulated and air conditioned. Heating is by wall pipes encircling the windows. The forced ventilation is sufficient to change the air once in eleven minutes, cool air



Courtesy Engineering News-Record.

FIG. 200. EXTERIOR WALL DETAILS FOR AN OFFICE BUILDING.

Steel panel interior walls, perforated metal ceilings, bare copper hot water heat pipes and recessed lights are new building features. Walls and ceilings are also well insulated against heat and sound. The wall panel illustrated is typical of the exterior construction.

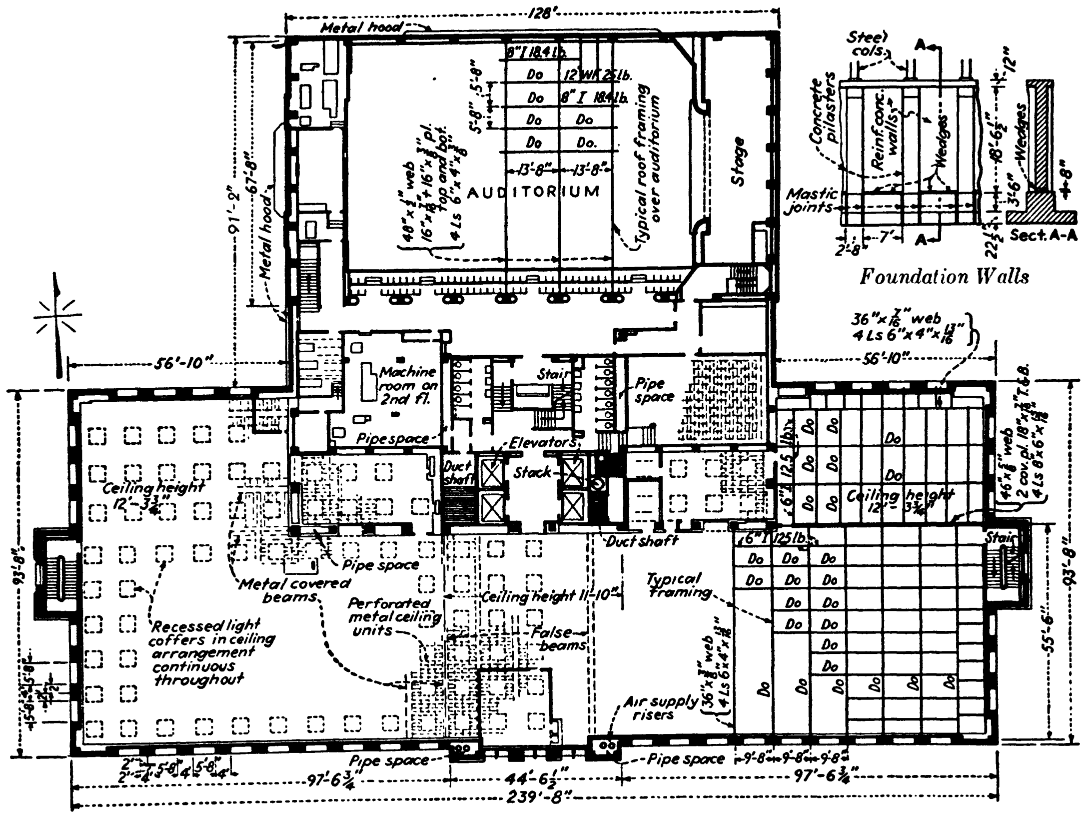
being used winter and summer to equalize the heat released by the artificial lighting and by the occupants themselves. The artificial lights are depressed in the ceiling, fixtures being at 10-ft. centers. Some of these features are evident in Fig. 200.

The plan shown in Fig. 201 is typical of the six working floors. It furnishes a main building 94 ft. by 240 ft. which is six stories high (the seventh floor being of small area) and a single-story auditorium 91 ft. by 128 ft. at the rear. The basement is used for service and storage of equipment, ground floor for dead filing and mailing, first floor for entrances and live files, second to fifth floors for clerical areas, sixth floor for executive offices, and the partial seventh floor for the directors' room and the necessary mechanical plant.

The important part of the plan of Fig. 201 is the wide open clerical areas where columns are entirely avoided. This U-shaped area on each floor is 93 ft. by 239 ft., the minimum clear span being 53 ft. and the open area being 16,000 sq. ft. *without column interference.* This requirement was set because of the company's experience that columns interfered with efficient planning and arrangement of its large clerical departments. The concentration of elevators, stair wells, ducts, and shafts in a small area represents efficient planning. The ceiling height of 12 ft.-3¾ in. for the clerical areas follows the modern practice of using reasonably low ceilings even for very large spans.

The Steel Structure. There being no interior columns, the structure consists merely of wall columns into which the main girders frame. Floor joists are flush with the tops of the girders and are attached at right angles to them. The omission of the usual interior columns made heavy girders necessary because they must span 55 ft. These girders, spaced 9 ft.-8 in. apart, are 36-in. riveted sections. *Holes are cut through the girder webs* at about the quarter points of the span to allow openings for air ducts, pipes and conduits. These holes are one half the depth of the web and the web is reinforced around each hole. Columns and joists are of standard rolled sections. The joists, spaced about 5½ ft. apart, support flat tile arches that form the floor.

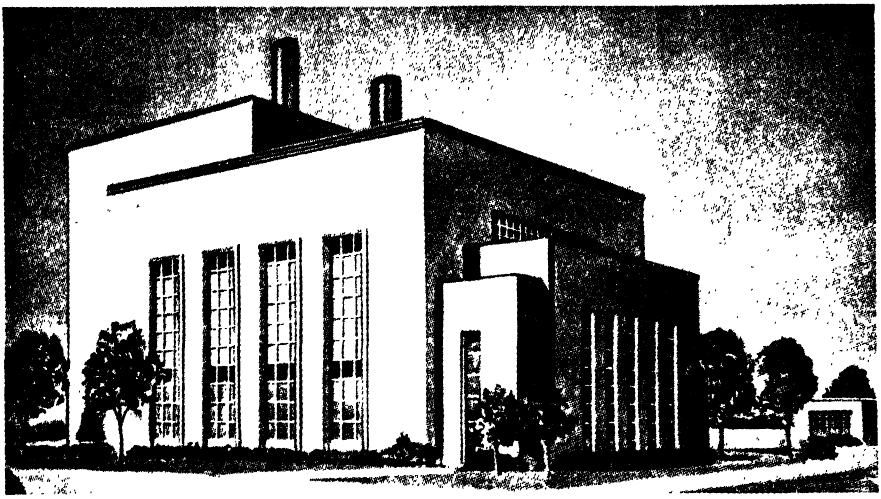
The entire frame could have been designed for simple-span beam action. However, for economy, the designers produced *rigid connections* between girders and columns and tied the joists together across the girders (with welded top tie plates) to resist negative moments. The reduction in positive moment was used as a proper excuse for reducing sections. This was of particular advantage in decreasing girder deflections, which were naturally found to be serious for such long spans. This building illustrates a combination of good functional design and the most modern conceptions of good structural design in steel. The total weight of structural steel was 3500 tons.



Courtesy Engineering News-Record.

FIG. 201. PLAN OF BUILDING ARRANGED FOR LARGE CLERICAL OFFICES TO BE FREE OF COLUMNS.

Foundations. Since the only columns supporting the building were around the outside walls and along one interior face, the foundation problem was rather unique. The footing material was variable and, therefore, a unit foundation was considered necessary. The outside basement walls were reinforced to act as distribution walls across the tops of the column footings. Cross walls tied the entire foundation structure into a single unit that was found to be more economical than the use of piling without such unit construction.

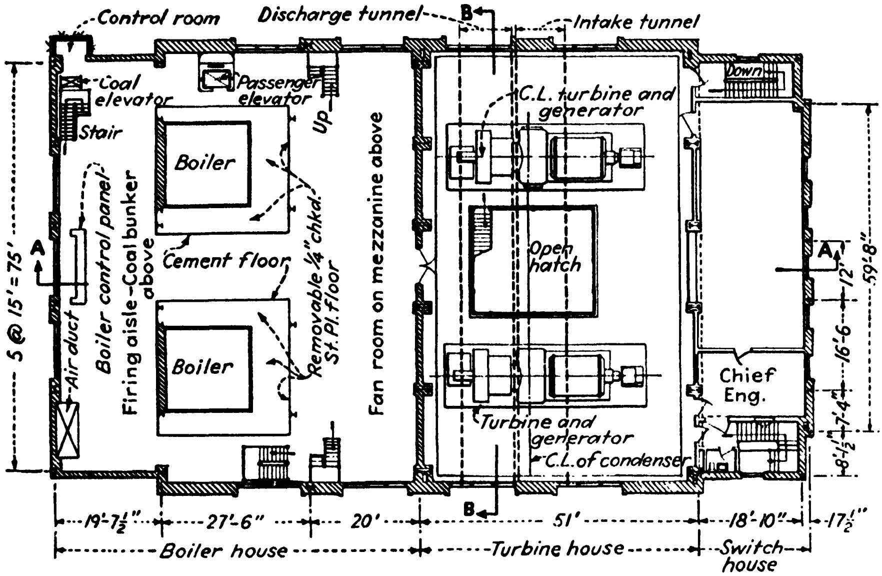


Courtesy Eng. News-Record.

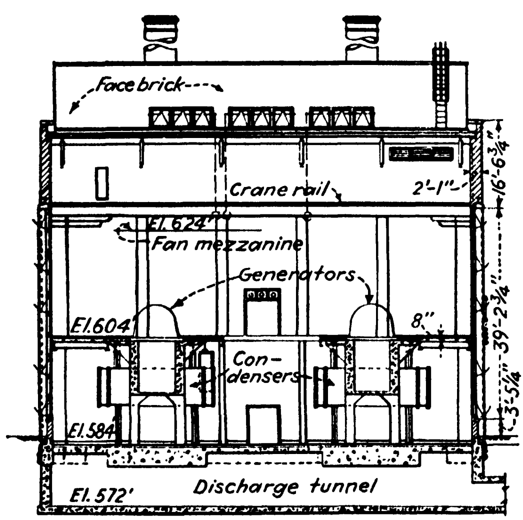
FIG. 202. POWER PLANT BUILDING FUNCTIONALLY DESIGNED.

Architect's sketch of power plant, with roof levels from high to low corresponding to boiler house, turbine house and switch house. Small structure at right is screen house over intake tunnel.

179. Power Plant Building. An architecturally pleasing power plant building with good structural design characteristics, built for the city of Holland, Michigan, is shown in Fig. 202. This building attracted wide attention because of the care given to appearance and to function. *The three subdivisions of function* — boiler room, turbine room, and switch room are recognized in the plan (Fig. 203) and also in elevation by different heights of roof. An operating floor carries through the three subdivisions of the building at one level, that is, 20 ft. above the ground floor. Turbines and boilers are located on this level. Half way from the operating floor to the ground is a mezzanine floor that extends through the boiler house and the switch house and through a part of the turbine house. This may be observed on Section *A-A* of Fig. 203. It serves for toilets, showers, and locker facilities and provides space for the forced draft fans. The



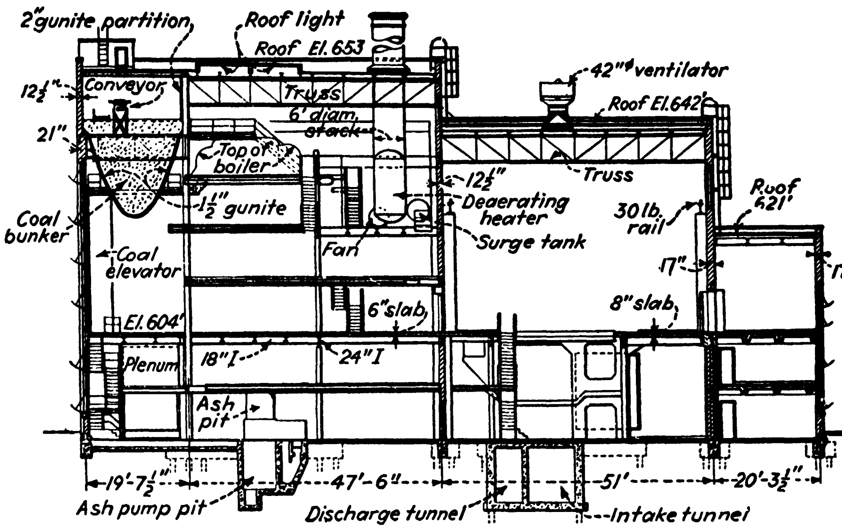
Plan View



Section B-B

Courtesy Engineering News-Record.

FIG. 203. FLOOR PLAN AND SECTIONS OF 15,000 K-W POWER PLANT FOR THE CITY OF HOLLAND, MICHIGAN.



Section A-A

FIG. 203. CONCLUDED.

ground floor has the ash removal apparatus and pumps in the boiler house, the condensers in the turbine room, and the batteries in the switch house. Above the main operating floor in the turbine room, there is a 30-ton overhead crane. This crane will handle turbine parts or other equipment that may be delivered by truck to the ground floor and then lifted through a large open hatch to the operating floor of this room. The space above the operating floor of the boiler room is filled with fans, tanks, heaters, stacks, and a coal bunker; several levels of floors, with stairs and an elevator, are provided for service. It is significant that the coal bunker is maintained below atmospheric pressure (by a simple connection to the furnace air duct) to avoid the flying dust problem so evident in the usual boiler house.

The Structure. Columns occur in five main lines across the building as may be seen in the upper or plan view of Fig. 203. A sixth partial line of columns occurs along the exterior face of the switch house. There are six columns in each main line and 18-in. I-beams are framed between these lines of columns on the operating floor. Joists at about 6-ft. spacing parallel to the lines of columns frame between the main beams to support the 6-in. concrete floor slab. Operating floors at higher levels in the boiler room are of open steel grating, 1½ in. thick. The successive roof spans are roughly 20 ft., 47 ft., 51 ft., and 20 ft. The two shorter spans are framed with roof beams, but the longer ones require trusses. These trusses, like the main 18-in. floor beams, are 15 ft. apart.

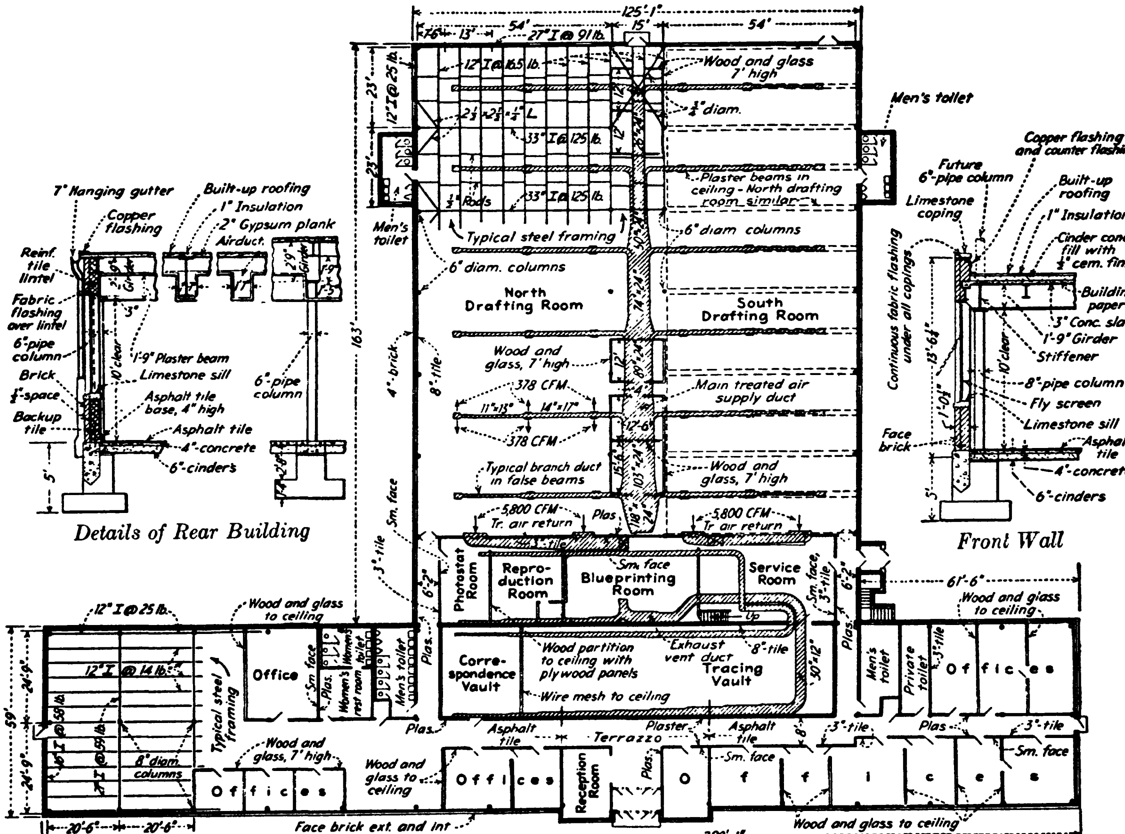
Architectural Details. Outside walls are of buff face brick with darker brick spandrels below the windows. A vitreous but unglazed tile varying

from buff to red is used throughout the interior. Exterior trim around doors and windows is a cream colored stone. Windows and doors are metal. Floor finish in the boiler house is troweled concrete with a metallic hardener. A tile floor is used in the turbine house. The switch house floor and some other areas are terrazzo. Roofing is tar and gravel (4 ply) on a 1-in. cork board insulation, which, in turn, is supported by precast concrete tile. The modern appearance of the building (Fig. 202) achieved by the contrast of long vertical and horizontal lines and by a total lack of adornment is exceptionally pleasing.

180. Engineering Drafting Offices. A building functionally designed for engineering office work was constructed by the Pullman Standard Car Mfg. Co. in Chicago. This structure of T-shaped plan is 289 ft. by 222 ft. as may be seen in Fig. 204. The long narrow office section at the front of the building is 289 ft. by 59 ft. This section is one story high but it may be extended to two stories when the need develops. The two rear drafting rooms are each 54 ft. by 163 ft. separated by an aisle of 15-ft. width outlined by the interior columns. Each drafting room is unencumbered by columns. In addition, there is a small basement 24 ft. by 35 ft. and a penthouse 24 ft. by 32½ ft. where the machinery for ventilation and air conditioning is housed.

The final interior arrangement of the drafting rooms developed naturally from two paramount requirements: (1) excellent lighting to avoid eyestrain, and (2) humidity control to make accurate work on vellum drawings possible. Vellum stretches and shrinks under moisture change. Hence, scale drawings are accurate only when moisture changes are prevented. Consideration was given to saw-tooth construction with natural lighting, but it was decided that a uniform light intensity of 35–40 foot-candles was needed and could only be obtained by artificial lighting of the indirect type. Naturally, then, a flat roof was chosen in preference to the less modern saw-tooth style. Because of the low ceiling (10 ft. clear) there arose the problems of dissipating the heat from the artificial lighting and of providing proper ventilation. A modified air conditioning system was designed to accomplish three results, that is, to ventilate the drafting rooms, to cool the air sufficiently to absorb the heat from the lights, and also to control humidity. The duct system is visible in the plan of Fig. 204. The use of indirect lighting led to a plastered interior for the drafting room in order to maintain reasonable economy in the use of electricity. Thus, *function* dictated all important elements of the building.

Details of Construction. Walls are 4-in. brick backed up by 8-in. tile around the drafting room and by an equal thickness of brick in the office section. Glass block bands provide natural light in the offices. Floors are of 4-in. concrete on a 6-in. cinder fill. An asphalt tile flooring (3/16 in.) is used in the drafting rooms and offices; terrazzo was chosen for the



Courtesy Engineering News-Record.

FIG. 204. PLAN OF PULLMAN'S ENGINEERING BUILDING.

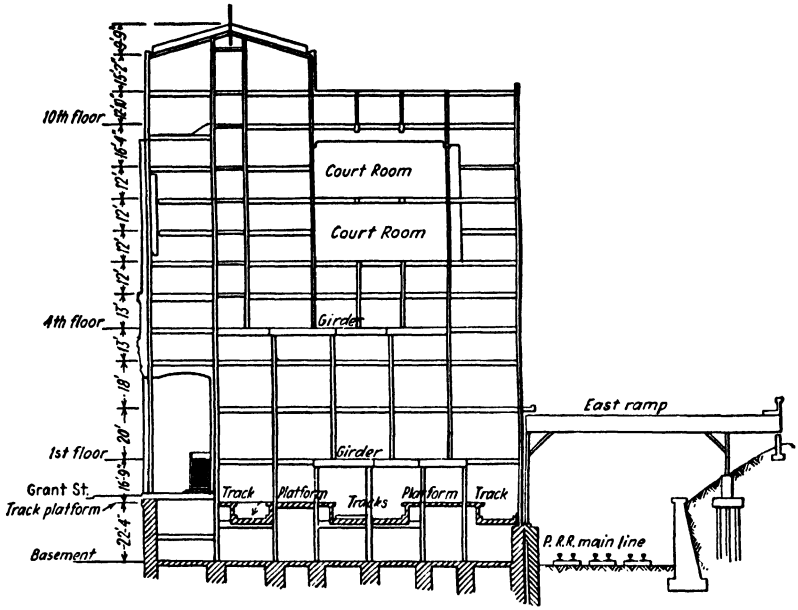
washrooms and vestibules while finished concrete was considered adequate for the service rooms and vaults. Main partitions are of hollow tile; temporary ones are of wood and glass.

The walls carry no roof loading. Circular pipe columns about 23 ft. apart support the 33-in., 125-lb. roof girders that span 54 ft. across the drafting rooms. There is no rigid-frame action in this building. The roof construction consists of beams perpendicular to the roof girders at 6-ft. spacing supporting 2-in. gypsum planks. Over the office section, the ceiling consists of a 4-in. concrete slab that will serve as a floor for a future second story. All roofing is built up of tar and roof paper placed over a 1-in. insulation board. Interior finish throughout is simple but effective in appearance. Steelwork, ceilings, and exposed piping are all painted a uniform cream color. Hence, they blend together so that the piping is hardly noticed. Office interiors of red face brick, aluminum venetian blinds, and varied floor tile are colorful and attractive. The drafting room walls and ceiling are painted a light cream color *to reflect light* properly. The drafting room provides space for 200 draftsmen with one large drawing table and one half of another table per man. The actual drafting space is about 75 sq. ft. per draftsman. The overall space including offices and service rooms is 190 sq. ft. per draftsman.

181. Post Office and Federal Building. The Pittsburgh Post Office and Federal Building, a ten-story tier structure, is worth study because of the *irregular column layout* that varies from story to story necessitating *girders for column transfer* and the use of shallow girders for this purpose to save story height.

The upper floors provide working offices and also several court rooms that required open floors without column interference. The court rooms have a height of two stories and a floor area 44 ft. by 50 ft. each. They are evident in the elevation of Fig. 205. The floors from the first to the third were used by the post office and a regularity of column spacing was considered necessary. Since this spacing could not match the column spacing in the upper floors, where a narrow corridor between rows of offices was required, *the 4th floor girder had to be designed for column transfers.* Again at the railroad track level (below the first floor) a special " crazy " column arrangement was necessary to support the skewed railroad spur tracks that entered the building. Accordingly, we find *a column transfer girder at the 1st floor line.*

Shallow girders had to be used at the 1st floor level. They are of the types shown in Fig. 206. In order to provide standard railroad head room, it was necessary to allow 20 ft.-11 in. above the top of rail even with the minimum permissible girder which was 37¼ in. deep. Some girders carrying columns that supported ten stories required type I girders with cover plates 2¼ in. thick. These column loads reached 900 tons and the girder



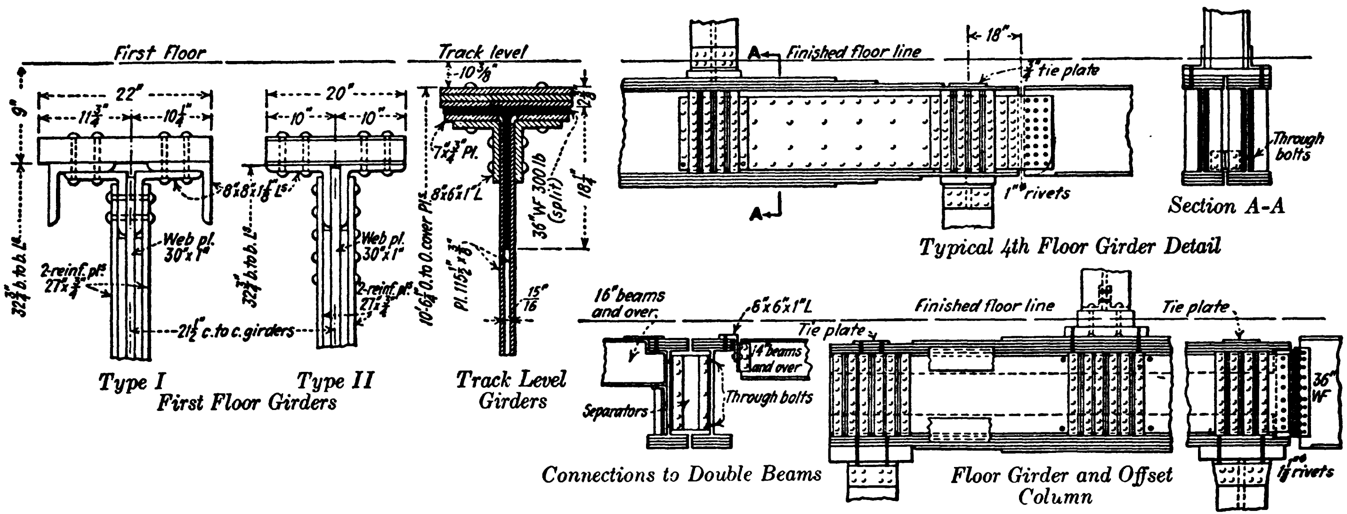
Courtesy Engineering News-Record.

FIG. 205. SECTION THROUGH POST OFFICE BUILDING AT PITTSBURGH.

moments were as much as 2050 ft-tons. Type II girders even with 3-in cover plates were able to resist only 1730 ft-tons. At the 4th floor level the solution was much the same but was obtained with lighter girder sections because the maximum column load was 610 tons and the largest girder moment was 1470 ft-tons.

Some special 50-ft. girders were required to span the columns and railroad tracks over the trucking space in the basement. These are special girders (marked *track level girders* in Fig. 206) with split beam Tees inserted in the flanges. These girders of 10-ft. depth resist bending moments of 8920 ft-tons. A column load of 1072 tons had to be supported. Girders designed to act in pairs were riveted together with stiff connections.

Soundproofing. Since the railroad tracks were to be supported by the building columns, a serious problem developed in soundproofing the building. The solution was a simple one. Cross beams spaced about 5 ft. apart supported a concrete slab of trough section 16 in. thick. Within this trough were placed three 1-in. layers of cork. The trough was then filled with another reinforced concrete slab, also of about 16-in. thickness, that supported the track. The inside slab was designed to transfer the wheel loads to the cork with a resultant unit pressure of only 10 lb. per sq. in. The inner slab was keyed to the outer slab or trough with concrete teeth bearing in cork insulated depressions. *Sound waves* were thus insulated from the building.



Courtesy Engineering News-Record.

Fig. 206. Girder Details for the Building of Fig. 205.

The shallow girders of Types I and II are used in pairs at the first and fourth floors for column transfer. The deep track-level girders are built up with split beam Tees as a part of their flanges. They are of 10-ft. depth.

Design and Construction Details

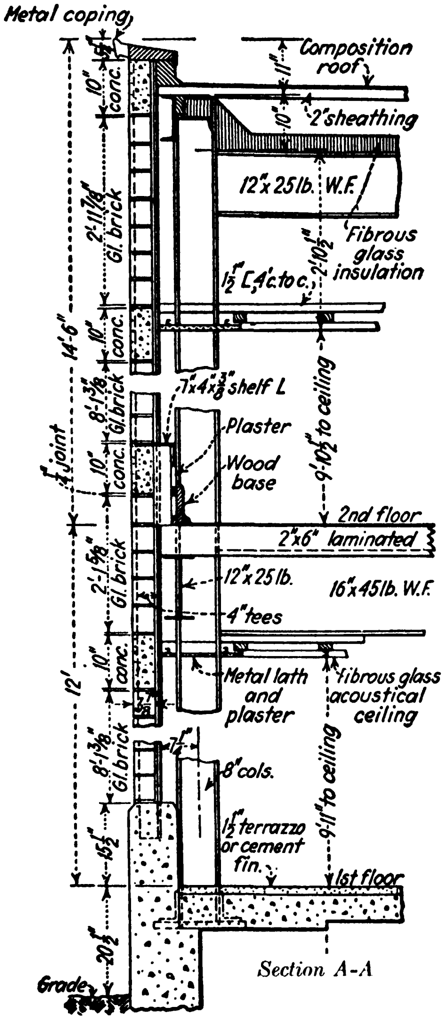
182. Glass-Block Walls. Glass blocks may be obtained in all sizes up to 12 in. × 12 in. The nominal thickness is 4 in.; the actual thickness being 3⅞ in. They are made by casting two open rectangular or square dishes. These glass dishes are then fused together after their rims are first dipped in molten metal. Since glass blocks have become a standard feature of modern construction, both for industrial and office buildings, some data on their strength are necessary. The following data were reported by Engineering News-Record from the results of tests made at Purdue University.

Compression Tests. "Single blocks, tested in compression with a gypsum cap, cracked first at 1078 lb. per sq. in. and carried a maximum load of 2036 lb. per sq. in. With a cement and plaster of paris cap, the first crack occurred at 685 lb. per sq. in., and a maximum load of 828 lb. per sq. in. was carried. These are averages for six blocks. Piers five blocks high with 3⁄16-in. rodded mortar joints (1 part masonry cement and 2½ parts sand) cracked first at 405 lb. and carried a maximum load of 478 lb. per sq. in. Panels three blocks wide and nine blocks high gave crack and maximum load values of 325 and 400 lb. per sq. in. respectively; there was little difference whether the joints were deep rodded or flush. These data refer to the load per square inch of *actual compressive area.*

"*Lateral Strength.* Panels about 7 × 8 ft. in size, held between brick piers and sills, were subjected to lateral pressure through an inflated bag held against the wall. There was no visual failure until a pressure of 120 lb. per sq. ft. had been attained, after which the panel failed by *shearing of a joint* near the top. Deflection at the center of the panel was about ¼ in. A steel sash panel of the same size deflected ¼ in. for a pressure of only 5 lb. per sq. ft.

"*Heat Transmission.* Conclusions as to the insulating value (based upon a panel containing 190 sq. in. of glass and 26 sq. in. of mortar) were that the glass block transmits 69.3 per cent less heat than common steel sash, and that it does not permit inside surface condensation when subjected to ordinary temperature and humidity conditions. Also, that the glass-block wall is equivalent in insulating value to a 16-in. plain brick wall, an 8-in. brick wall furred with wood lath and plaster, or a 16-in. concrete wall furred with lath and plaster. The tests showed that the block has an average coefficient of heat transmission of 0.29 B.t.u. per sq. ft. per hour per degree F. difference in temperature; walls of 8-in. solid brick and 12-in. concrete block have coefficients of 0.32. In a companion series of tests on solar-heat penetration, the glass block eliminated 54.6 per cent of the available solar heat, while steel sash eliminated only 20.4 per cent.

"*Transmission of Light.* Blocks with various types of rib and prism designs were tested under a 50 foot-candle light. For one design, 86.5 per cent of the light was transmitted; with another, 78.5 per cent was transmitted, whereas a ribbed wire-mesh plate glass transmitted 77.5 per cent and a plate glass with a pebble-dash surface transmitted 88.3 per cent. Expressed in another way, 1.3 sq. ft. of the block used in the building is equivalent to 1 sq. ft. of ordinary window pane. One face design transmitted only 11.7 per cent of the light falling on it; such blocks are used where high light intensities are not desirable."



Courtesy Engineering News-Record.

FIG. 207. WALL SECTION SHOWING THE SUPPORT OF GLASS-BLOCK PANELS.

Support of Glass-Block Panels. Glass-block walls or panels may be supported like ordinary brick walls. Spandrel sections used in one building are shown in Fig. 207. The exterior walls consist of concrete and of glass block. Combinations of glass block with spandrels of stone, brick, terra cotta tile, artificial marble, and polished or enameled metal are quite common. Since glass block expands far less than steel or concrete, frequent expansion joints are needed both vertically and horizontally. Such joints may be of packed glass wool sealed into the joint with mastic. Plaster is usually returned to direct contact with glass block for interior construction.

183. Expanded Metal Framework. The search for economy in light steel construction has led to the use of *expanded metal joists* which are reasonably standard for roof and floor construction. The use of *expanded metal studs* and their grouping into columns by welding is a relative in-

novation. This device makes light steel construction in forms and heights commonly used in timber construction possible. Of course, all joints and connections are welded. The following description of the construction of such a light two-story office building by the Bethlehem Steel Co. is taken from Engineering News-Record. The details of this construction are illustrated by Fig. 208.

Example of Expanded Metal Office Building Frame. The framing of the walls consisted of 4-in. expanded metal studs used singly or in groups of three after fabrication into H-section columns by welding. These stud columns occur at four locations along either side wall, where they are

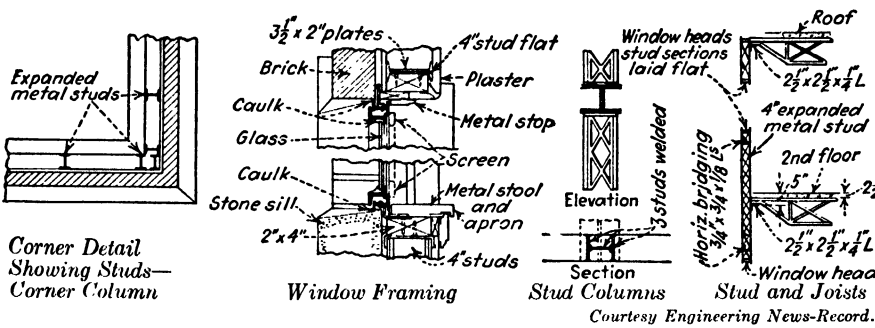


FIG. 208. DETAILS OF BUILDING WITH EXPANDED METAL JOISTS AND STUDS.

required to support the outer ends of the transverse I-beams. Studs are placed at 2-ft. intervals around the building except at the corners which are formed by three closely set studs welded together. Except at the doors and windows, the studs are used in the full two-story length of $22\frac{1}{2}$ ft. Stud sections laid flat comprise the necessary horizontal members of the door and window framing. They are also used at the roof level.

Bracing of the frame is largely accomplished by light bridging in each wall. This consists of three lines of $\frac{3}{4} \times \frac{3}{4} \times \frac{1}{8}$-in. angles threaded through the stud webs and welded to the inside of the exterior upright. These lines of bridging are at the top and bottom of the second story windows and at the top of the first story windows. Where joists frame into walls, as is the case both for end walls and for those portions of the side walls opposite the structural steel framing, $2\frac{1}{2} \times 2\frac{1}{2} \times \frac{1}{4}$-in. shelf angles are used as supports; these angles are welded to the studs and thus serve as additional bracing. In the window bays, V-shaped bridging of $1 \times \frac{1}{8}$-in. flats is used on the exterior faces of the studs between the top of the first and the bottom of the second story windows and between the top of the second story windows and the top of the building frame.

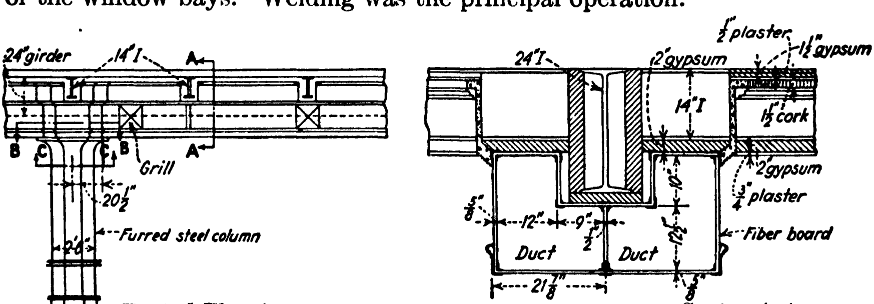
The wall framing is covered outside with paper-backed wire mesh, which is wired to the studs. The single-brick exterior wall is laid up about

½ in. clear of the wire mesh, and this space is flushed full of mortar to assure a good bond. The brick veneer is also anchored by brick wall anchors pushed through the building paper and snapped over the outside flange of each frame stud at every 5th course. By filling the stud spaces with 4 in. of mineral wool held in place by lath and plaster, a wall with insulating properties equivalent to a 12-in. brick wall is said to be provided.

Floor joists are 12 in. deep, spaced 21 in. on centers; they are welded at the supports, and for additional stability they rely on the 2½-in. slag-concrete floor slab. This slab was placed directly on metal-rib lath stretched over the joists and tack welded to them; *a dry mix was used so that it would not run through the lath,* and a very satisfactory floor resulted. A 2-in. terrazzo topping is used in the dispensary room and in the main entrance lobby; elsewhere the slab is merely covered with linoleum. Ceilings are metal lath and plaster in the basement and for the first floor; on the second floor a suspended ceiling of acoustic tile is used.

The concrete roof slab is covered with a 2-in. cinder concrete fill, 1 in. of wood-fiber board, and built-up roofing paper, to provide an insulating effect comparable to that of the walls.

With the exception of a few connection details on the rolled sections, field forces managed the entire fabrication as well as the erection of the building. The expanded metal studs came to the job in stock lengths of 22½ ft. so that cutting was necessary only on those required in the center of the window bays. Welding was the principal operation.



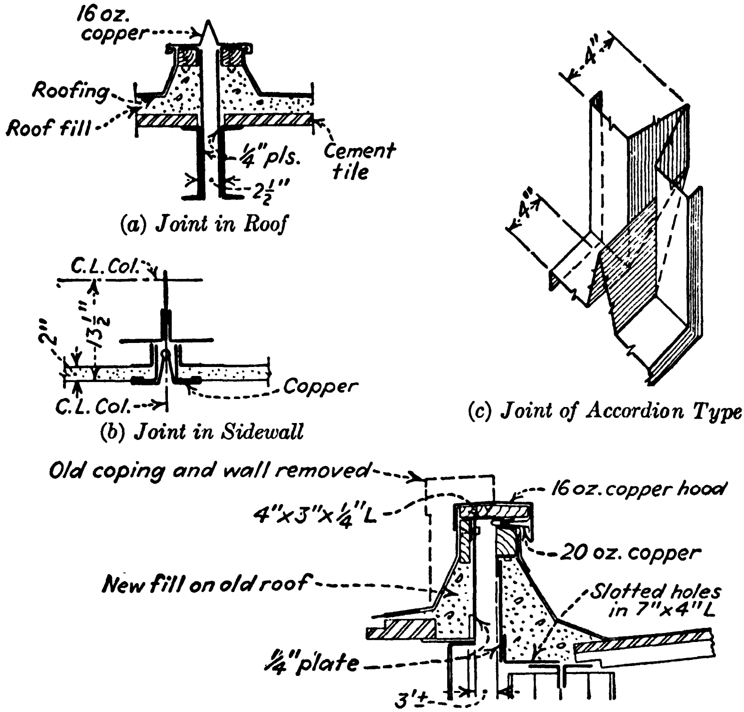
Courtesy Engineering News-Record.

FIG. 209. CONCEALED DUCTS FOR AIR DISTRIBUTION.

184. Ductwork. With the advent of air conditioning as standard practice in office buildings and even for industrial structures, the problem of arranging the ductwork in an inconspicuous manner has become important. If the arrangement of ducts is given little consideration, it may spoil the inside appearance of the structure. The problem is to conceal the horizontal distribution system and also the set of vertical risers that bring the conditioned air from the lower to the upper stories. One solution to this problem is indicated in Fig. 209. The vertical ducts are furred

spaces around the columns. The horizontal distribution ducts are shown as enclosures of the floor girders where they appear as wide girders along the ceiling. This is only one of many possible ways of concealing ductwork. Exposed ductwork when well arranged may be functionally attractive.

185. Expansion Joints. The length of building that may be constructed without an expansion joint is dependent upon the type of construction, the exposure, the difference between inside and outside temper-



(*a*) *Joint in Roof*

(*b*) *Joint in Sidewall*

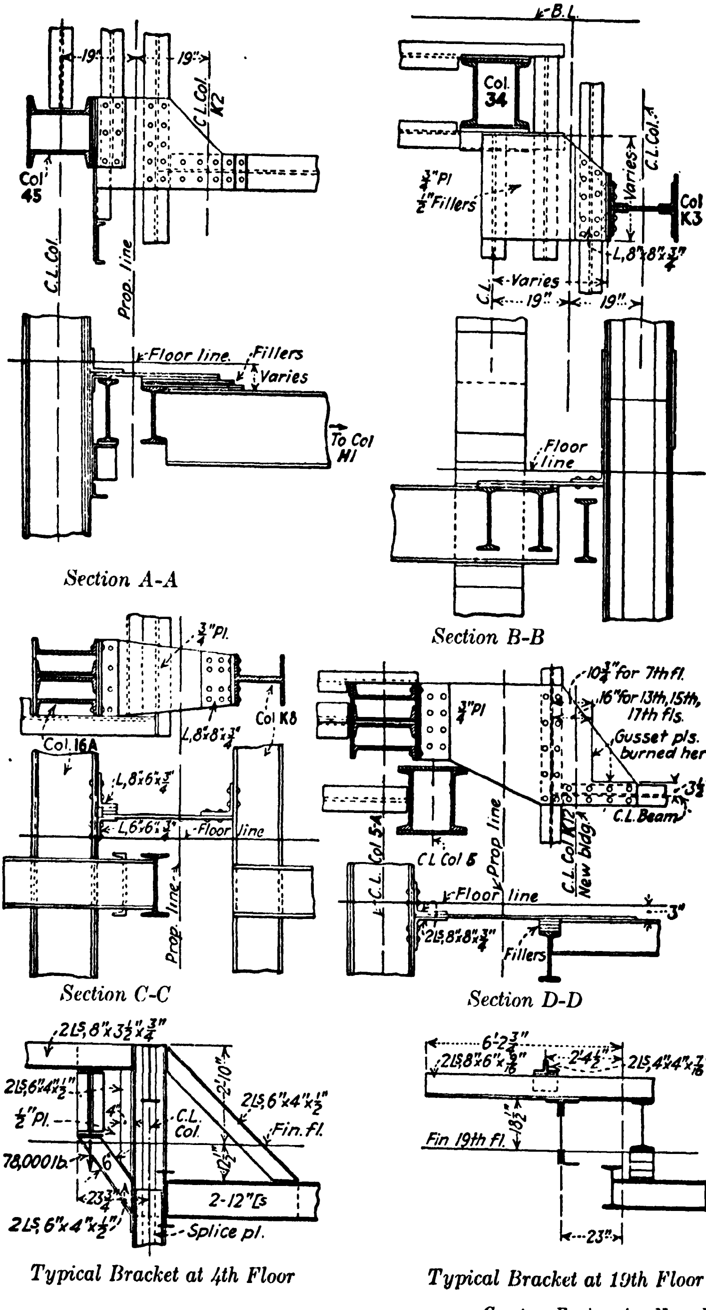
(*c*) *Joint of Accordion Type*

(*d*) *Expansion Joint between New and Old Building*

Courtesy Engineering News-Record.

FIG. 210. EXPANSION JOINTS OF LEAK-PROOF TYPES.

atures, and the possible deflection due to temperature that may be absorbed without damage. A common rule is to provide for a movement of ½ in. for each 100 ft. of length, with expansion joints spaced from 200 to 300 or possibly 400 ft. apart. Brittle materials are subject to damage by expansion even when main sliding joints are placed in the building. For this reason glass block must be provided with open joints packed with fiber glass and sealed with mastic at from 30 to 50 ft. apart. Since some glass block is of pyrex glass with a very low coefficient of expansion, there is the probability of considerable movement between the glass block and the steel frame that supports it.



Courtesy Engineering News-Record

FIG. 211. COMPLEXITY OF GIRDER-TO-COLUMN DETAILS.

In Fig. 210 we find several typical expansion joints illustrated. Sliding joints (*a*) and (*b*) are standard and have been in use for many years. The joint (*c*) is a special accordion design for a corner. It permits a weather-tight expansion joint passing around the corner of a building or across the valley of a roof. The most difficult problem occurs in providing expansion between new and old construction. A solution is illustrated in Fig. 210(*d*). These joints all are designed to eliminate or reduce the maintenance common to joints that depend upon the squeezing out of mastic filler to permit movement. Such filler squeezes out and does permit movement, but an open leaky joint is left when the building contracts.

186. Column and Girder Details. For very regular buildings, the column and girder details may be quite simple. On the other hand, it frequently happens that the structural engineer is furnished with architectural plans necessitating the use of off-set columns and highly irregular floor framing. Girders that miss the columns entirely must be supported upon cantilever brackets. Girders and beams may meet at different levels. Wall and floor supports require two or even three beams closely spaced at the spandrels. These and other irregularities may be seen in the details of Fig. 211. The designer must be prepared to arrange structural details to fit the architectural plan without excessive cost.

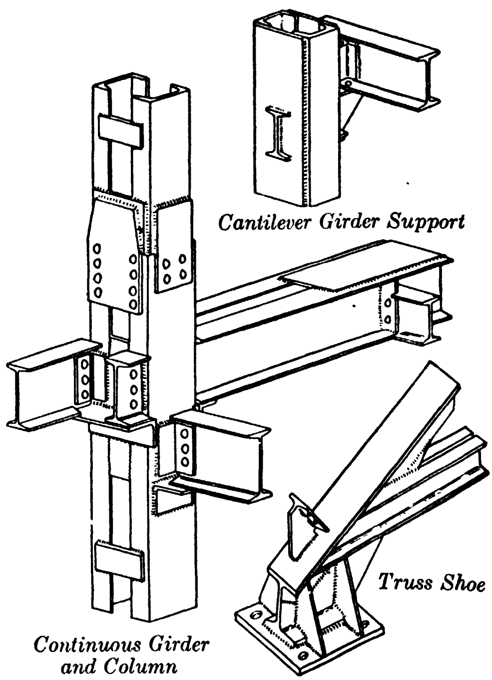
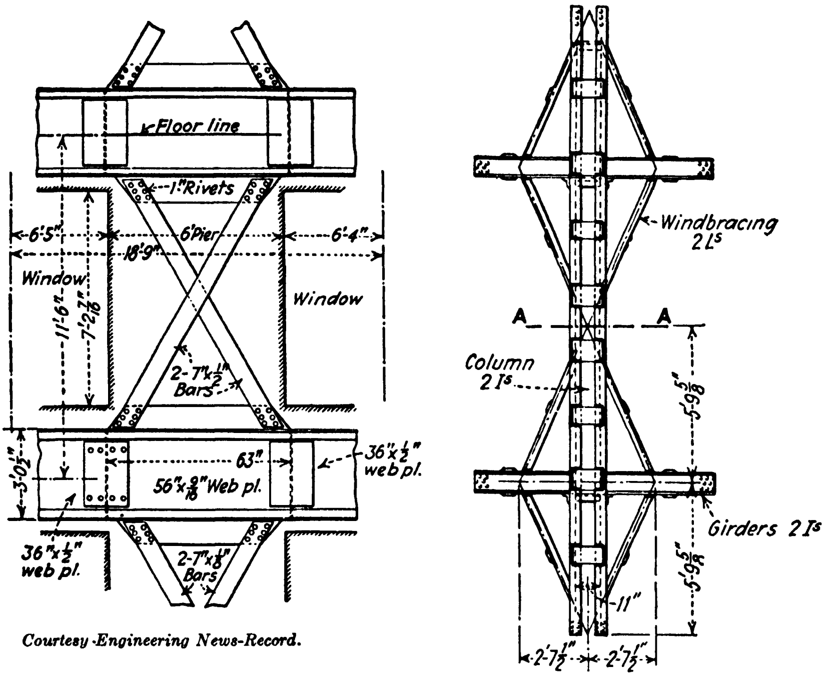


FIG. 212. WELDED COLUMN AND GIRDER DETAILS.

The illustration shows how complex details may be simplified by welding. These details are from a 17-story apartment house in Europe. Note the use of double girders to save head room and the further attempt to reduce girder moments by providing continuity through the columns. The structure was shop welded and field riveted.

Structural welding has simplified the designer's problem greatly. A few details that are clearly simplified by being partially welded are shown in Fig. 212. The modern tendency to produce continuous girders in building construction in order to reduce the design moments is largely possible because of the use of electric arc welding. The cost of riveted details of sufficient rigidity to produce

continuous beam action has usually been considered too costly to be justified for complex structures. This situation has been reversed by the use of welding.



(*a*) *Riveted X-bracing between Stories.* (*b*) *Welded K-Bracing to Columns.*

FIG. 213. COMPARISON OF DIAGONAL WIND BRACING.

187. Wind Bracing. Special details of the wind bracing for tier buildings are shown in Figs. 213 and 214. The *X* or *K*-bracing of Fig. 213 has long been considered the most economical as well as the most rigid type of bracing. It is used in outside walls and around elevator shafts. The introduction of horizontal bands of windows or of glass block entirely around the building eliminates the possibility of using enough *X* or *K*-bracing to stabilize a tower structure. Dependence is then placed either upon short knee braces between columns and girders or upon brackets and knuckle connections as illustrated by Fig. 214. These details are shown riveted but they are often welded instead. If advantage is to be taken of the influence of rigid connections in the reduction of the dead load and live load moments in the girders, a welded structure with wind resistance dependent upon the column-to-girder joints may prove as economical as one with *X* or *K*-bracing. Since welded joints are able to resist wind

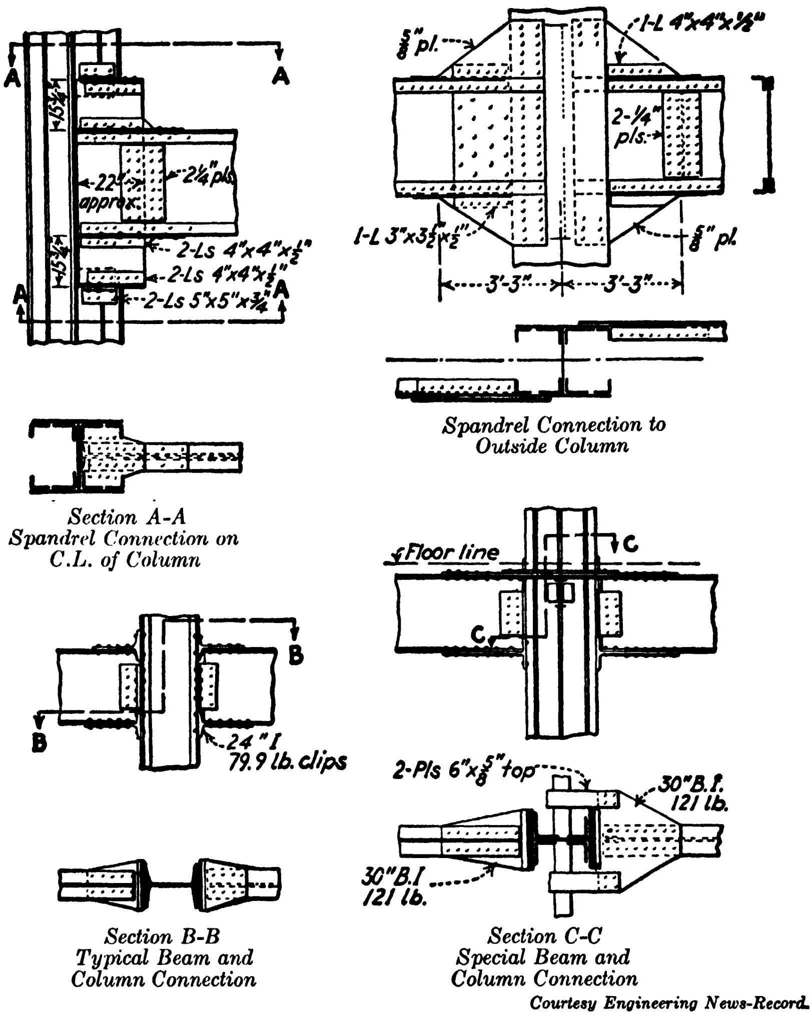


FIG. 214. TYPES OF WIND BRACING CONNECTIONS, UNION TRUST BUILDING, DETROIT.

moment whether the X-bracing is used or not, we can easily understand how economy might be achieved in such a structure by the omission of the X-bracing. Much is dependent upon the regularity or irregularity of the structure. When columns are offset and girders must be supported upon cantilever brackets from the columns, the development of wind resistance through the column-to-girder connections is not as satisfactory as for the regular frame.

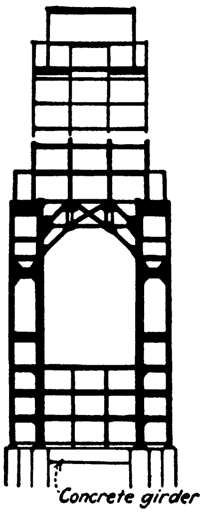


FIG. 215. PORTAL ABOVE BANKING ROOM.

A special problem in wind bracing is illustrated by Fig. 215. Here we have a large portal that not only resists the wind shear but also carries two column loads over an open banking room which is between four and five stories high. Such heavy portal trusses are subject to high secondary stresses because of their *short stiff members.* In the structure illustrated by Fig. 215, the rivets were driven successively as the loads were applied, rivet holes being reamed in the field where necessary. Thus, the final rivets in any joint were not driven until all of the dead load had been applied to the portal truss. It is thought that this method of construction permitted slight *rotations between members* at the joints and in this manner relieved the secondary flexural stresses that would normally have accompanied the application of the dead load.

CHAPTER 15

DESIGN OF A TALL BUILDING

188. Function of the Building. A college of science and engineering must be accommodated on a plot of land 600 by 125 ft. located in the heart of a metropolitan area. The student body is expected to remain constant in number, but it must be possible to change the character of any part of the building to meet the needs of a changing student body. Therefore, floors will be designed so that partitions can be added or removed at will. Thus, classrooms can be changed to laboratories or *vice versa.*

Since the college building is located in a business district of a large city, a reasonably tall building is appropriate and perhaps desirable in order to create an adequate impression upon the community. To distinguish this building from its neighbors it is desirable to surround it with an area of green grass with plantings. The building should preferably not cover more than one third of the land. For a technological center the appropriate architecture is obviously modern. The simple lines of modern industrial buildings and the inexpensiveness of such construction commend themselves. The wide use of glass, both translucent and clear, will provide excellent natural lighting, although proper artificial lighting is needed in the late afternoon for all college classrooms and laboratories. Ducts for artificial ventilation should be provided throughout even though air conditioning is considered necessary for only a few rooms.

189. Structural Form. An analysis shows that 16 floors about 45 ft. by 450 ft. in size are adequate. This plan offers a gross area of 325,000 sq. ft. and a working area in classrooms, offices, laboratories, etc., of about 225,000 sq. ft. In order to preserve the full usefulness of the main area and to permit complete freedom of subdivision of this area for large and small rooms, the service shafts (elevators, stairways, conduits, ducts) are placed on the face of the building and outside of the working area. The architectural plan of Fig. 216 shows this arrangement and also the arrangement of walkways, entrances, and plantings. The service shafts on the front of the building introduce contrasting vertical lines that add interest to the structure. The dimensions on the plan of Fig. 216, which show the building to be 48 ft. by 442 ft., are approximate outside dimensions.

Column Spacing. It would have been desirable for economy to have spaced the columns about *20 ft. apart in both directions.* Actually, the ap-

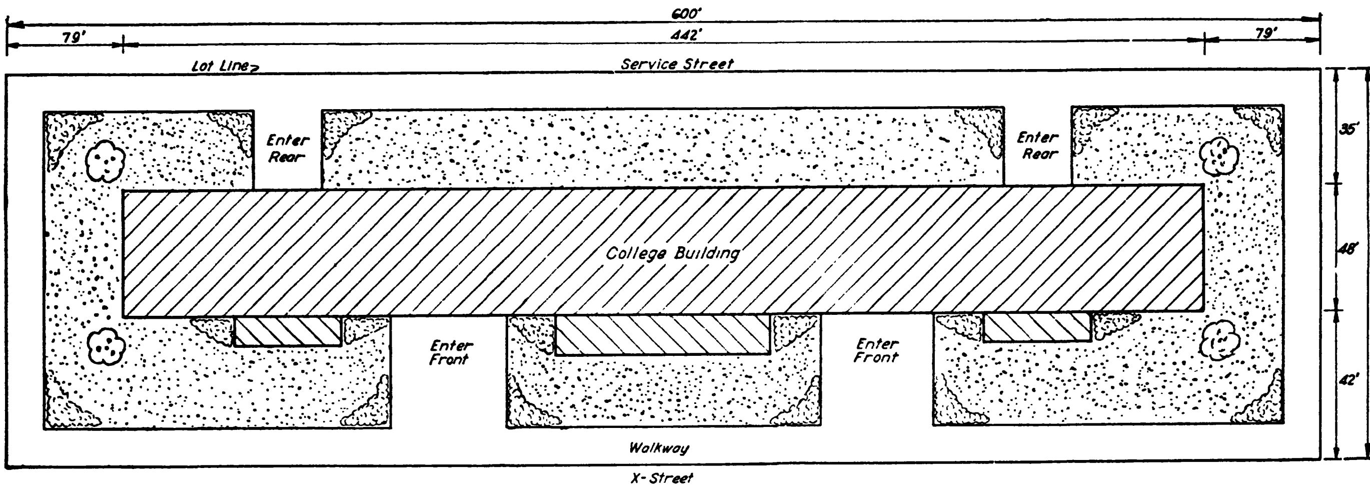


FIG. 216. GROUNDS PLAN SHOWING BUILDING, WALKWAYS, AND PLANTING.

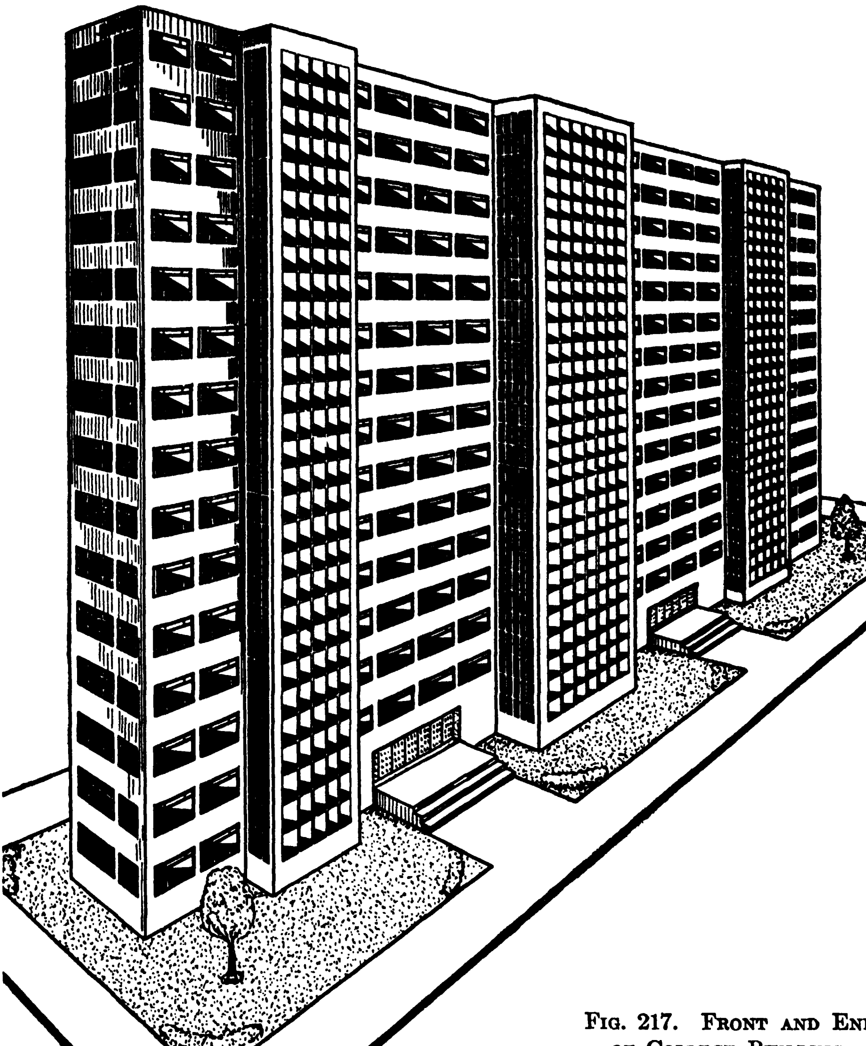
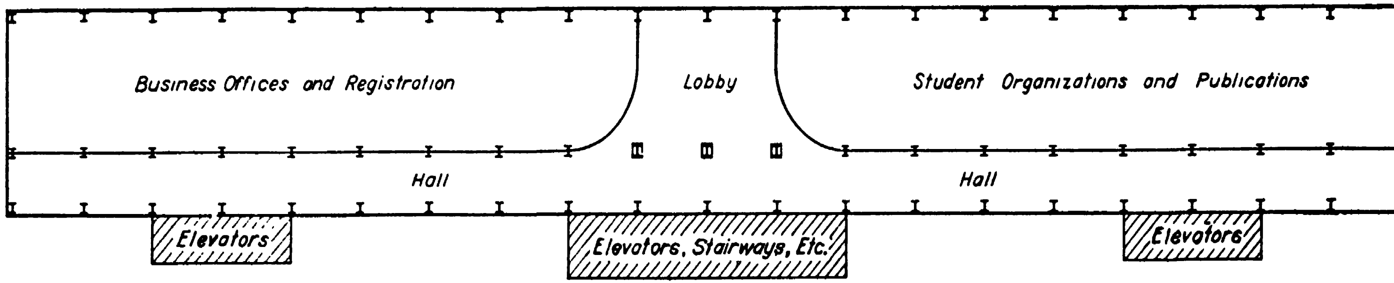
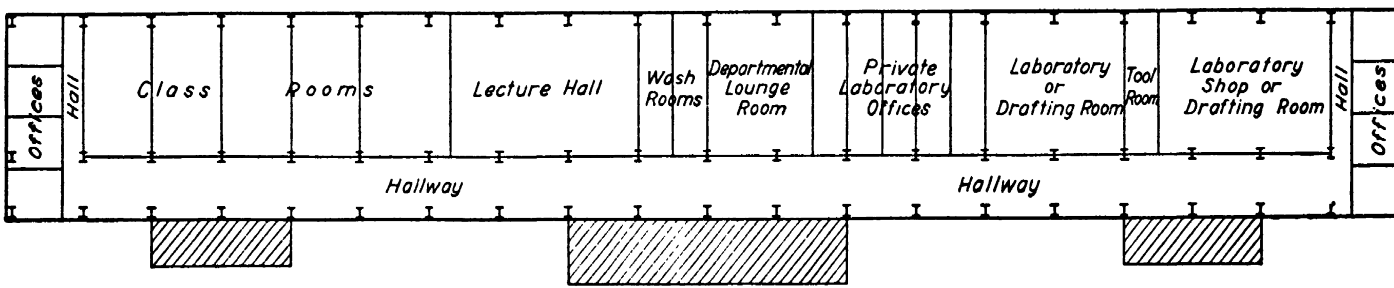


FIG. 217. FRONT AND END OF COLLEGE BUILDING.

proximate spacing shown in the views of Fig. 219 and Fig. 221 is 22 ft. along the building and 13 ft.-8 in. and 32 ft.-4 in. across the building. The spacing along the building might have been reduced, but 22 ft. was considered a desirable average unit for classroom and laboratory areas. The narrow column spacing across the building was fixed by the need for an



(a) Main Floor at Entrances



(b) Typical Floor Plan

FIG. 218. LAYOUT OF FLOORS FOR A COLLEGE BUILDING.

unobstructed corridor (set at 13 ft.-8 in.) to permit rapid movement of students between classes and to give a feeling of spaciousness that would otherwise be entirely lacking in a building where lobbies, plazas, courts, and similar areaways do not exist. The wide spacing of 32 ft.-4 in. between columns in the working area was necessary to permit change in room sizes at any time during the life of the building. If this span had been subdivided by adding a fourth column across the building, there

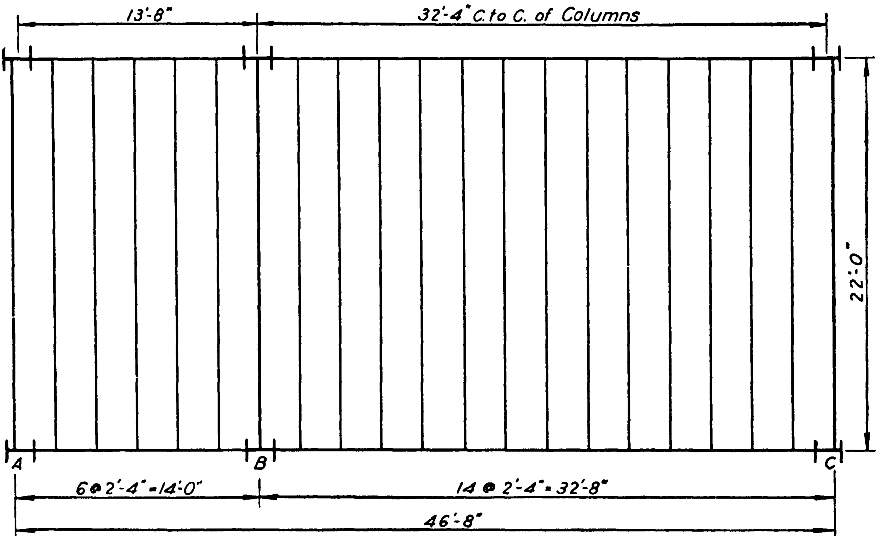


FIG. 219. APPROXIMATE DIMENSIONS OF A FLOOR BAY.

Note: This is a preliminary arrangement that will be revised when the floor details are decided upon.

would have been a line of columns along the centers of all large lecture rooms and laboratories. This restriction was considered to be unacceptable.

Wind Resistance. The choice of column arrangement is functional rather than structural. However, the first duty of the structural engineer is to make the building serve its primary function in as nearly a perfect manner as possible. The structural arrangement is secondary. The design chosen produces a bent of unusual form. Since this narrow building will have but three columns in any wind bent (Fig. 221) and since diagonal bracing or even knee braces are evidently incompatible with the requirement that all partitions across the building must be removable, we may find a rather serious wind-stress problem here. The self-evident solution will be to use *rigid welded connections* between all columns and girders in order to reduce the wind stresses as much as possible. With normal story heights, the *height-width ratio* for this building will approach 5.0. This places it in the class of buildings for which the wind stresses are of sufficient

Courtesy Eng. News-Record.

FIG. 220. TIER BUILDING CONSTRUCTION.

importance to influence column sections and to increase the girder sections greatly.

Story Heights. A common requirement in city building codes is that school buildings shall have a clear ceiling height of 12 ft. This requirement presupposes natural ventilation and might be reduced with forced ventilation. If the over-all story height is set at 14 ft., the clear height

can probably be 12 ft. and still provide space for conduits and pipes below the floor joists and above a suspended ceiling. The design will therefore be started upon the assumption that the building from roof to basement floor will have a height of 224 ft. or 16 stories at 14 ft. each. This is indicated on Fig. 221.

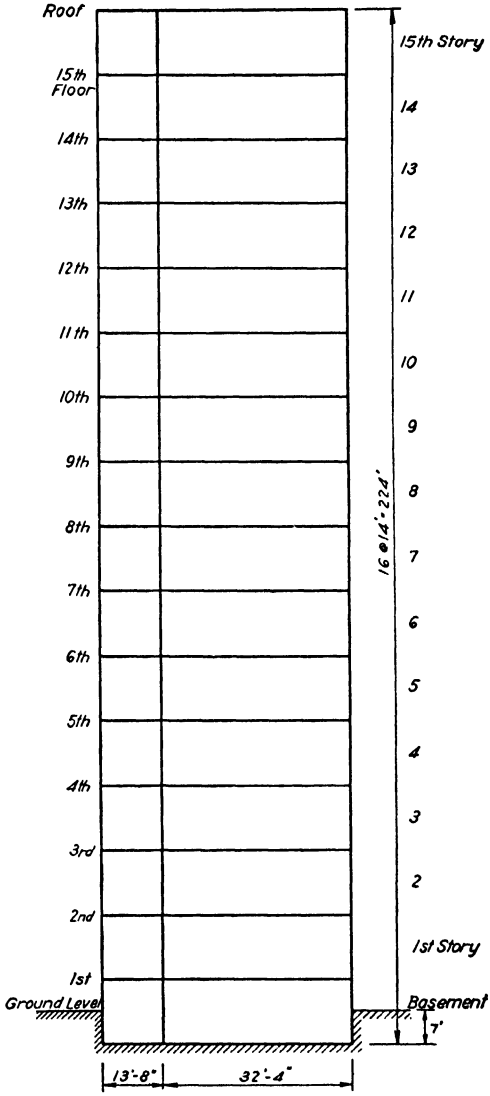


FIG. 221. APPROXIMATE DIMENSIONS OF THE STRUCTURE.

190. A Preliminary Design. Obviously, we cannot devote sufficient space to present a detailed final design for this building. Instead, we will make a preliminary design, including, although perhaps crudely, the influence of dead load, live load, and wind load for the purpose of obtaining a reasonably accurate estimate of the weight of the structural frame. For this purpose, column sections will be selected at two levels, the basement story and the twelfth story, the average weight of section being taken as the mean of these two. Finally, with the calculated dead loads accepted, we will redesign the columns and girders at the basement level by using the methods of analysis presented in Vol. 2, *Theory of Modern Steel Structures*. This final step in the design need not be studied if the student does not have an adequate background. It is purposely separated from the preliminary design for which the analysis requires only a knowledge of *statics*.

Specifications. No single set of specifications will be followed in this design. Building codes vary in every city and it is just as well for us to begin to familiarize ourselves with changing specifications. Those needed will be quoted from time to time.

Working stresses will be 20,000 lb. per sq. in. for steel in tension and $17{,}000 - 0.485(L/r)^2$ for compressive stresses in columns. Reinforced concrete beams may be stressed to 1200 lb. per sq. in. in compression with corresponding working stresses (for 3000-lb. concrete) in shear and bond. No allowance will be made for the added strength provided by concrete encasement of steel members, although some building codes make such provisions. Working stresses will be increased 33⅓ per cent for members designed for wind resistance.

Loadings consist of dead load, live load, and wind pressure. The dead load will be estimated for concrete fireproofing weighing 145 lb. per cu. ft. enclosing all main steel to a depth of at least 2 in. Live loads will consist of a floor loading of 100 lb. per sq. ft. in the corridors and 75 lb. per sq. ft. for classrooms, laboratories, and offices. *Improbability of full live loading* justifies a 15 per cent reduction of live load for all *girders* and also for the columns at the top of the building. Successive live load increments on the *columns* are reduced 5 per cent per story to a minimum value of 50 per cent. The wind pressure is taken at 20 lb. per sq. ft. of exposed wall area.

Materials will be steel for the main frame, reinforced concrete covered with wood flooring for the floor, brick, glass, and glass block for the exterior walls, and gypsum block for the plastered partitions. The partition along the hallway will be of 8-in. thickness weighing 30 lb. per sq. ft. and all other partitions will be of 6-in. thickness weighing 25 lb. per sq. ft. Firewalls may be needed at intervals to meet building code provisions, but this is a special problem that must be solved according to local requirements. Exterior brick walls 13 in. thick weigh 150 lb. per sq. ft. Glass-block panels with glass panes weigh about 20 lb. per sq. ft.

Floor Arrangement

191. Floor Design. The main girders evidently must be framed in the short direction of the building on two spans of 13 ft.-8 in. and 32 ft.-4 in. approximately. These will also be the wind girders and they will therefore be welded to the columns to resist wind moments. Steel beams will be placed in the opposite direction between columns to serve several purposes. *First,* they are needed to act as struts between columns during erection of the frame, *second,* they will carry wall and partition loads, and *third,* they may function also as floor joists.

Concrete Pan Construction. Since fire regulations require 1½ in. of concrete around interior steel beams and joists, it will be more economical to use reinforced concrete joists which are fire proof. An economical alternative might prove to be the use of flat tile floor arches between steel joists. The floor arch then fireproofs the joists. However, the use of steel pans to form reinforced T-beam joists is by far the most common type of construction. It will be chosen in this preliminary design without an actual study of other possible floor systems. For an actual design, careful comparisons of the relative costs of several floor systems would be advisable.

In the classrooms, where the panel dimensions are 22 ft. by 32 ft.-4 in. approximately, the joists clearly ought to span the shorter dimension. In the corridor we face a serious question as to which direction is proper

for the span of the joists. If the joists are spanned the shorter direction from outside wall to inside partition, they will be greatly lightened, but their reactions must then be carried upon light beams of relatively long span (22 ft.). However, if the joists are spanned the longer direction in the corridor, they will rest upon the short heavy wind girders which will be able to carry considerable dead load moment very economically since they will have to resist large wind moments. It should be recalled that wind moment up to 33 per cent of the dead load and live load moment is permitted *without increase of section.* The other factor favoring the long-span joists (22 ft.) is the fact that such joists in the corridor may be duplicates of the joists for the classrooms. Duplication is always economically desirable in reinforced concrete construction. For these reasons the joists in all parts of the building will span the distance of 22 ft. between columns. In practical design this decision would be made upon the basis of calculated weights and costs.

Structural Layout. The exact spacing of columns has not been set nor have the beams and girders been spaced with relation to the columns. These details are worked out on Fig. 222 and Fig. 223. The arrangement of the floor for the use of 20-in. pans with joist stems 8 in. wide requires the dimensions shown. Since the exterior face of the column must be covered by at least 4 in. of brickwork, the column spacing works out to be 13 ft.-8⅝ in. and 32 ft.-4⅝ in. This spacing provides for the deepest 14-in. column, the $14 \times 16WF426$ section, which should be about adequate for this building. If the column must have a greater area than is provided by the standard 14-in. sections, use may be made of web plates that will not deepen the section. Columns are stacked one above the other *on the same center line* so that there is *no eccentricity of loading.*

The spandrel beams are placed as near as possible to the centers of the exterior walls. See Fig. 223. The position shown is arranged to provide 4 in. of brickwork outside of the extreme edge of the beam flange. Thus the spandrel beam is placed 6 in. outside of the center line of the column. This eccentric load will prove useful in balancing to some extent the moment introduced into the column by the live load and dead load acting on the connecting floor girder. It is to be particularly noted, however, that such *moment of eccentricity does not accumulate* from story to story.

192. Joist Design. An estimate must be made of the load per lineal foot of joist for an effective span of 21 ft. This effective span is 1 ft. less than the distance center to center of columns. It allows for the shortening of the joist span by a 12-in. concrete encasement around the wind girder. The joists are estimated to be 28 in. center to center which allows for an 8-in. width of stem and 20-in. pans. We will design a joist for the classroom where the live load is 75 lb. per sq. ft. and a partition load may occur at the mid-point of the span.

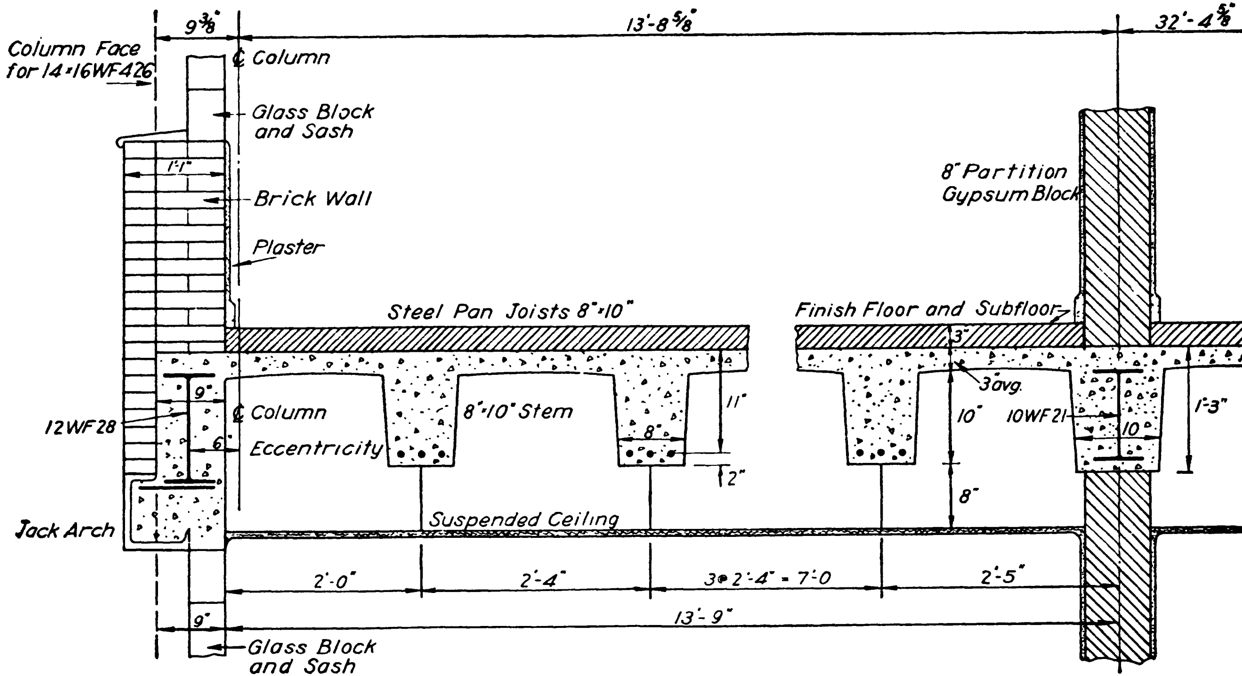


FIG. 222. SECTION THROUGH THE CORRIDOR AT ANY FLOOR.

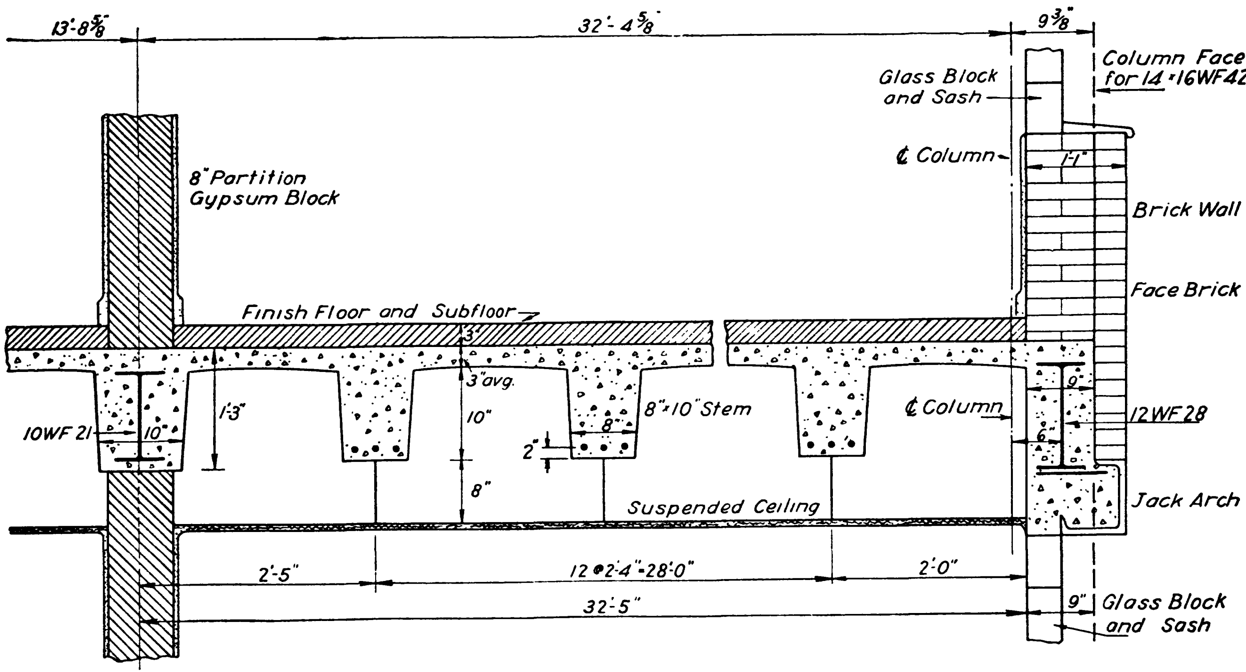


FIG. 223. SECTION THROUGH THE CLASSROOM AT ANY FLOOR.

Weight per Square Foot of Floor.

3-in. concrete slab	=	37 lb. per sq. ft.
2-in. wood floor on nailing strips	=	10
Suspended ceiling	=	10
Total	=	57 lb. per sq. ft.

Load per Lineal Foot of Joist.

Floor only — 57 × 2.33	=	133 lb. per ft.
Joist stem — 8 in. × 10 in.	=	82
Live load — 75 × 2.33	=	175
Total	=	390 lb. per ft.

End Shear for the 21-ft. Span.

Uniform load — 0.5(390 × 21)	=	4,100 lb.
Partition load at center of span * — 0.5(13.75 × 2.33 × 25)	=	400
Total	=	4,500 lb.

Bending Moment for the 21-ft. Span (simple beam).

Uniform load — 0.125 × 8200 × 21 × 12	=	258,000 in-lb.
Partition load — 0.25 × 800 × 21 × 12	=	51,000
Total	=	309,000 in-lb.

Working Stresses. (Slightly different values are given by 1940 Joint Committee.)

Medium carbon rods, 20,000 lb. per sq. in.
Ultimate concrete stress, $f_c' = 3000$ lb. per sq. in.
$f_c = 0.4f_c' = 1200$ lb. per sq. in.
$v = 0.02f_c' = 60$ lb. per sq. in. (ordinary anchorage).
$u = 0.05f_c' = 150$ lb. per sq. in. (deformed bars).

Steel Area (assume j to be 0.9).

$$A_s = \frac{M}{f_s jd} = \frac{309{,}000}{20{,}000 \times 0.9 \times 11} = 1.56 \text{ sq. in.}$$

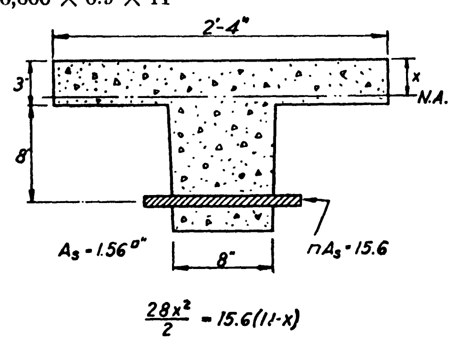


FIG. 224. TRANSFORMED SECTION.

Concrete Stress (see Fig. 224).

$$f_c = \frac{2M}{(kd)(jd)b} = \frac{2 \times 309{,}000}{3.0 \times 10.0 \times 28} = 735 \text{ lb. per sq. in.}$$

* Partitions may be only 12 to 13 ft. high; weight estimate for 13.75 ft. is on safe side.

Unit Shear.

$$v = \frac{V}{b(jd)} = \frac{4500}{8 \times 10.0} = 56 \text{ lb. per sq. in.}$$

Bond Stress.

$$u = \frac{V}{\Sigma O(jd)} \text{ or } \Sigma O = \frac{V}{ujd} = \frac{4500}{150 \times 10.0} = 3.0 \text{ in.}$$

(Any combination of bar sizes will provide adequate perimeter for bond.)

Remarks about Joists. The joist stem may appear somewhat larger than necessary since the compressive concrete stress is low. However, the width of 8 in. is no more than adequate to accommodate two ⅞-in. round bars and one ¾-in. round bar that provide the required area of steel. The depth to the steel could not be decreased, but the amount of cover below the steel might be reduced from 2 in. to 1½ in. under some building codes. The required steel area could be reduced by constructing the *joists as continuous beams.* A rough design shows that the steel might be reduced to four ⅝-in. round bars, two of which could be bent up and continued over the wind girders at the support. No reduction in size of stem would be possible, however, since an 8-in. width would still be needed to accommodate the four ⅝-in. bars and a depth of 11 in. to the steel would be required to hold the compressive concrete stress near the support down to 1200 lb. per sq. in. Therefore, in this particular instance, there is little to be gained by arranging the joists as continuous beams.

The joist as designed above will be found satisfactory for use in the corridor where the uniform live load is increased from 75 to 100 lb. per sq. ft. and the partition load does not occur.

Beam and Girder Selection

193. Spandrel Beams and Partition Beams. Steel beams will be used in the long direction of the building to support the exterior walls and the partition along the hallway. These beams also serve as struts between columns for erecting the steel frame. They might be designed as simple beams because ordinary clip-angle connections would provide adequate wind resistance in the long direction of the building. However, this presents an ideal situation for taking *advantage of moment reduction from continuity* since the loading is nearly all dead load, the spans are of one length, and the heavy columns will practically fix the ends of even the end spans. All spans therefore resist only the fixed-end moments of $\frac{1}{12}WL$.

Spandrel Beam. The spandrel beam carries the exterior wall, a 10-in. width of floor loading, and its own weight. See Fig. 222. The exterior wall consists of 13-in. brick masonry spandrels with plaster finish, weighing 150 lb. per sq. ft. of wall surface, with large glass-block and steel-sash

panels weighing 20 lb. per sq. ft. Since one half of the wall surface is to be glass, the average weight of the exterior wall will be about 85 lb. per sq. ft. The dimension 22 ft. will be reduced in weight calculations to 20.5 ft. to allow for the average width of column with fireproofing.

Wall weight on spandrel beam	$= 20.5 \times 14 \times 85$	=	24,400 lb.
Floor dead load	$= 20.5 \times 0.83 \times 57$	=	970
Floor live load	$= 20.5 \times 0.83 \times 100$	=	1,700
Weight of spandrel beam	$= 22 \times 30$	=	660
	Total load	=	27,730 lb.

Negative design moment ($\frac{1}{12}WL$) =
$\frac{1}{12} \times 27{,}730 \times 22 \times 12 = 610{,}000$ in-lb.

The required modulus of 30.5 is furnished by a $12WF28$ beam that also provides the flange thickness of over $\frac{3}{8}$ in. needed for structural welding. An end connection to develop the moment resistance of 610,000 in-lb. can be made by welding. It will consist of a *seat angle* welded to the lower flange and a *tie plate* to the upper flange. At 3750 lb. per lineal inch for a $\frac{3}{8}$-in. weld, the length of weld needed for each flange will be

610,000 ÷ (12 × 3750) = 14 lineal inches of weld. (See Fig. 225.)

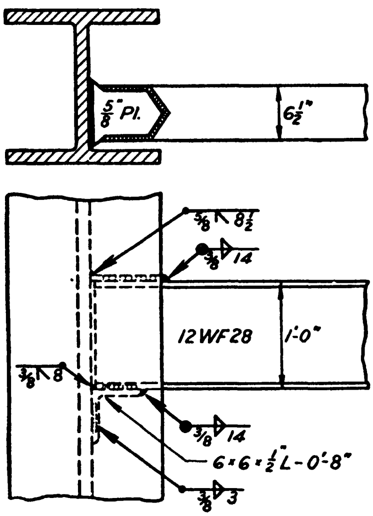


FIG. 225. SPANDREL BEAM CONNECTION.

Moment of Eccentricity. The spandrel beam as shown in Fig. 222 and Fig. 223 is framed into the column with an eccentricity of 6 in. The resulting applied moment at each floor is

$$M_e = 27{,}730 \times 6.0 = 166{,}000 \text{ in-lb.}$$

Partition Beam. The partition forming one side of the hallway is supported by steel beams that frame between pairs of interior columns. This beam carries the weight of an 8-in. gypsum tile partition at 30 lb. per sq. ft., the weight of a strip of floor and of suspended ceiling 2 ft.-5 in. wide (see Fig. 222), the live load over a strip of floor 1 ft.-9 in. wide remaining after we deduct the width of the partition, and also the weight of the steel beam and its fireproofing. In making the weight estimate, longitudinal dimensions will be reduced by 18 in. to allow roughly for the width of the column with fireproofing.

Partition load on beam	$= 12.75 \times 20.5 \times 30$	=	7,830 lb.
Floor dead load	$= 2.42 \times 20.5 \times 57$	=	2,830
Floor live load	$= 1.75 \times 20.5 \times 100$	=	3,590
Weight of beam	$= 22 \times 25$	=	550
Weight of fireproofing	$= 20.5 \times 125$	=	2,560
	Total load	=	17,360 lb.

$$\text{End design moment} = \tfrac{1}{12} \times 17{,}360 \times 22 \times 12 = 382{,}000 \text{ in-lb.}$$

The required modulus of 19.1 is furnished by a $10WF21$ section. The length of a ⅜-in. flange weld to develop the necessary end moment resistance is

$$382{,}000 \div (9.9 \times 3750) = 10.3 \text{ in.}$$

Since the spandrel beams and the partition beams have very small wind stresses, *a single weight of beam will serve at all floor levels.*

194. Main Girders in Upper Floors. These girders extend between the columns in the short direction of the building. They resist *heavy wind moments* and will therefore increase in size and weight from the upper floors downward. For the two upper floors, however, the wind moment will not increase the combined dead load and live load moment by as much as 33 per cent and, therefore, the sections may be chosen without consideration of the wind moments. Since each girder has more than 200 sq. ft. of floor area tributary to it, advantage will be taken of a common specification which permits the live load on such girders to be reduced 15 per cent.

15th Floor Girder across Corridor, AB. These girders will be rigidly connected to the columns, but they will not be entirely fixed thereby. Hence, we will make the preliminary design on the basis of a controlling moment of $\frac{1}{10}WL^2$ where L will represent the approximate clear span. With the column changing section frequently and the girder remaining of fixed size for several stories, we will choose 12 ft.-6 in. for the *maximum clear span,* which allows for a 15-in. depth for the interior column and a 14-in. depth for the wall column. The corridor live loading is 85 per cent of 100 or 85 lb. per sq. ft., while in the classroom this reduced live load is 64 lb. per sq. ft.

Joist reaction	$= (57 \times 2.33 + 82)21 + (85 \times 2.33 \times 22)$	=	8,870 lb.
Load from five joist reactions	$= 5 \times 8870$	=	44,400
Weight of steel beam	$= 12.5 \times 30$	=	370
Weight of fireproofing	$= 12.2 \times 175$	=	2,130
	Total	=	46,900 lb.

$$\text{Bending moment} = \tfrac{1}{10} \times 46{,}900 \times 12.5 \times 12 = 705{,}000 \text{ in-lb.}$$

The required section modulus of 35.2 is furnished by a $14WF30$ beam.

15th Floor Girder across Classroom, BC (clear span, 31 ft.-2 in.)

Joist reaction = $(57 \times 2.33 + 82)21 + (64 \times 2.33 \times 21)$	=	7,650
Load from thirteen joist reactions = 13×7650	=	99,600
Weight of two 6-in. partitions = $2(13.75 \times 31.2 \times 25)$	=	21,500
Weight of steel beam = 31.2×110	=	3,400
Weight of fireproofing = 30.9×400	=	12,400
Total	=	136,900 lb.

Bending moment = $\frac{1}{10} \times 136{,}900 \times 31.2 \times 12 = 5{,}130{,}000$ in-lb.

The required modulus of 257 is furnished by a 27*WF*106 beam

Column Selection

195. Column Load Increments per Story. For purposes of a weight estimate, the dead weight of the roof will be considered to be 67 per cent of the weight of a floor. The live load carried to a column will be reduced 15 per cent on the top floor, 20 per cent on the next lower floor, etc., until only 50 per cent of the live load is brought to the columns below the 9th floor. (See Fig. 221.)

Wall Column in Corridor, A.

D.L. reaction of spandrel beams	=	26,000 lb.
D.L. reaction of girder in corridor	=	12,600
Total	=	38,600 lb.

L.L. from floor without reduction = $6.7 \times 22.0 \times 100 = 14{,}700$ lb.

Interior Column, B.

D.L. reaction from partition beams	=	13,800 lb
D.L. reaction from girder in corridor	=	12,600
D.L. reaction from girder in classroom (considering one partition only)	=	42,600
Total	=	69,000 lb.

L.L. from floor without reduction = $14{,}700 + (16.0 \times 21.3 \times 75) = 40{,}200$ lb.

Wall Column in Classroom, C.

D.L. reaction from spandrel beams	=	26,000 lb.
D.L. reaction from girder in classroom	=	42,600
Total	=	68,600 lb.

L.L. from floor without reduction = $16.0 \times 21.3 \times 75 = 25{,}500$ lb.

By use of these standard increments of column loading and the percentage reductions mentioned above, we can estimate quite accurately the load to be carried by any column in any story. Due allowance, of course, must be made for the weight of the column section itself and for its fireproofing.

196. Column Sizes for D.L. and L.L. Columns for a building are designed from the top downward so that the dead weight above any story will be known when we start the design of the columns for that story.

In this preliminary design, we will choose the column sections at only two levels; first we will select the sections to be used between the 12th and 13th floors (this section being used clear to the roof), and, finally, we will determine the sections needed at the basement level. Dead weights of column and fireproofing above the lower level will be computed by averaging the known weight in the 12th story with the estimated weight in the basement since the column section increases gradually from the roof to the basement. The weight of fireproofing will be estimated on the basis that columns are boxed and poured solid, all parts of the section being covered to a depth of at least 2 in. with concrete. The outside faces of exterior columns are fireproofed with 4 in. of brickwork plus a varying thickness of concrete depending upon the *depth* of the steel section.

Column Moments from D.L. and L.L. It has frequently been the case that the influence of the vertical loading in producing column moments has been neglected. If we use rigid wind connections, we cannot justify the neglect of these moments. As soon as approximate stiffness factors are known, the column moments can be found by the process of moment distribution. Until sections are available on which to base a reasonable choice of stiffness factors, we must estimate such moments rather crudely. For an exterior column, the adjacent negative moment in the girder is somewhat less than $\frac{1}{12}WL$ which is resisted by column moments of about $\frac{1}{24}WL$. In this case the load W is the sum of the dead load and the live load acting on the girder. The live load for an interior column should be placed only on the longer adjacent girder span. Then the difference of the two girder moments may be divided equally between the columns above and below the joint. This procedure is conservative since the unloaded girder will actually resist a part of this moment. Thus, for the interior column we may write

$$M_{\text{col.}} = \tfrac{1}{24}W_1^{\text{DL+LL}}L_1 - \tfrac{1}{24}W_2^{\text{DL}}L_2.$$

By following this method and by using span lengths from center to center of columns, we obtain

Moment in the column A (D.L. + L.L.) $= \frac{1}{24} \times 46{,}900 \times 13.7 \times 12 = 322{,}000$ in-lb.

Moment in the column A (D.L. only) $= \frac{1}{24} \times 25{,}200 \times 13.7 \times 12 = 173{,}000$ in-lb.

Moment in the column C (D.L. + L.L.) $= \frac{1}{24} \times 136{,}900 \times 32.4 \times 12 = 2{,}220{,}000$ in-lb.

Moment in the column $B = 2{,}220{,}000 - 173{,}000 = 2{,}047{,}000$ in-lb.

These calculations may be checked from the data of § 194 where the girder loads were calculated.

Design Moments. From the moments computed above for the exterior columns, we may subtract the moment of eccentricity caused by the ec-

centrically connected spandrel beams. This moment was computed to be $27{,}700 \times 6.0 = 166{,}000$ in-lb., but it divides itself between the two columns above and below the joint. Hence, above the level where wind moments become serious, we should design the columns to resist the following bending moments.

Design moment for the column $A = 322{,}000 - 83{,}000 = 239{,}000$ in-lb.
Design moment for the column $B = 2{,}047{,}000$ in-lb.
Design moment for the column $C = 2{,}220{,}000 - 83{,}000 = 2{,}137{,}000$ in-lb.

197. Column Sections between the 12th and 13th Floors. We will take the weight of the roof and the roof load to be 67 per cent of the corresponding weight for a floor. The column will be considered to be of a constant section above the 12th floor. Moments will be introduced from dead load and live load, but the wind moments are assumed to be negligibly small at this level. Bending moments will be reduced to an equivalent central load from the relation $P = M \div (S/A)$. The factor S/A varies but slightly for the $14 \times 16WF$ sections, the range being from 5.4 to 5.6 only. Its variation is always small.

The column loads will be computed from the column load increments as determined in § 195, the column moments being those calculated in § 196.

Wall Column in Corridor, A.

D.L. for 3.67 floors $= 3.67 \times 38{,}600$	= 141,500 lb.
L.L. for 3.67 floors $= (0.67 + 0.85 + 0.80 + 0.75)14{,}700$	= 45,200
Column weight with fireproofing $= 4 \times 14(60 + 250)$	= 17,300
Dead and live load	= 204,000 lb.
Moment reduced to an equivalent central load $= 239{,}000 \div 5.1$	= 47,000
Design load	= 251,000 lb.

Try $14 \times 10WF61$; $S/A = 5.2$; $L/r = 168 \div 2.45 = 69$;
$P = 14{,}690 \times 17.94 = 263{,}000$ lb.

Interior Column, B.

D.L. for 3.67 floors $= 3.67 \times 69{,}000$	= 253,500 lb.
L.L. for 3.67 floors $= (0.67 + 0.85 + 0.80 + 0.75)40{,}200$	= 123,500
Column weight with fireproofing $= 4 \times 14(160 + 375)$	= 30,000
Dead and live load	= 407,000 lb.
Moment reduced to an equivalent central load $= 2{,}047{,}000 \div 5.5$	= 373,000
Design load	= 780,000 lb.

Try $14 \times 16WF158$; $S/A = 5.5$; $L/r = 168 \div 4.0 = 42$;
$P = 16{,}140 \times 46.47 = 750{,}000$ lb.

Use $14 \times 16WF167$; $S/A = 5.5$; $L/r = 168 \div 4.01 = 42$;
$P = 16{,}140 \times 49.09 = 792{,}000$ lb.

Wall Column in Classroom, C.

D.L. for 3.67 floors = 3.67 × 68,600	= 251,700 lb.
L.L. for 3.67 floors = (0.67 + 0.85 + 0.80 + 0.75)25,500	= 78,300
Column weight with fireproofing = 4 × 14(160 + 375)	= 30,000
Dead and live load	= 360,000 lb.
Moment reduced to an equivalent central load = 2,137,000 ÷ 5.5	= 388,000
Design load	= 748,000 lb.

Try $14 \times 16 WF158$; $S/A = 5.5$; $L/r = 168 \div 4.0 = 42$;
$P = 16{,}140 \times 46.47 = 750{,}000$ lb.

Wind Resistance by Statics

198. Wind-Stress Analysis by the Cantilever Method. An estimate of the wind moments to be resisted by the columns and girders at the level of the first floor is needed. Then we will redesign the girders so that their weights may be brought properly into the estimate of the column dead loads at the basement level. Actually, the cantilever method of analysis * depends upon relative column areas which cannot be known until the columns are selected. However, this method of analysis is only intended to be a crude approximation. Therefore, we can make a satisfactory estimate of relative column areas at the first floor level from our computations in the previous section. (Column weights found there were 61, 167, and 158 lb. respectively.) The smallest column (A) will be considered to have a unit area (1.0) and then it will be assumed that each of the columns B and C will have an area of 2.3. This allows for a relative increase in the column A due to wind. If this estimate is radically wrong, the wind moments can be recomputed after the column areas have been determined by calculation.

Center of Gravity of the Bent. The calculations are made for the dimensions given on Fig. 226.

$$e = \frac{2.3 \times 32.4 - 1.0 \times 13.7}{1.0 + 2.3 + 2.3} = 10.9 \text{ ft.}$$

$$I = 1.0 \times 24.6^2 + 2.3(10.9^2 + 21.5^2) = 1940.$$

Wind Shears and Overturning Moments. The wind pressure is 20 lb. per sq. ft. acting on the exposed width of 22 ft. per bent and over the height of 217 ft. above the ground surface.

Wind shear at the basement level	= 20 × 22 × 217 = 95,400 lb.
Wind shear above the first floor	= 20 × 22 × 203 = 89,200 lb.
Overturning moment in the basement story (mid-height)	= 95,400 × 108.5 = 10,350,000 ft-lb.
Overturning moment in the first story (mid-height)	= 89,200 × 101.5 = 9,040,000 ft-lb.

* *Theory of Modern Steel Structures*, Vol. 1, pp. 260–262

Column Direct Stresses and Girder Shears. Use is made of the beam-flexure formula (Mc/I) for the calculation of the column direct stresses. With the units used in calculating I, direct stress will be obtained in pounds per unit of area.

Column A, wall column in corridor; area = 1.0.

$$\text{In the basement, } P = \frac{1.0 \times 10{,}350{,}000 \times 24.6}{1940} = 131{,}000 \text{ lb.}$$

$$\text{In the first story, } P = \frac{1.0 \times 9{,}040{,}000 \times 24.6}{1940} = 115{,}000 \text{ lb.}$$

$$\text{Girder shear in } AB = 16{,}000 \text{ lb.}$$

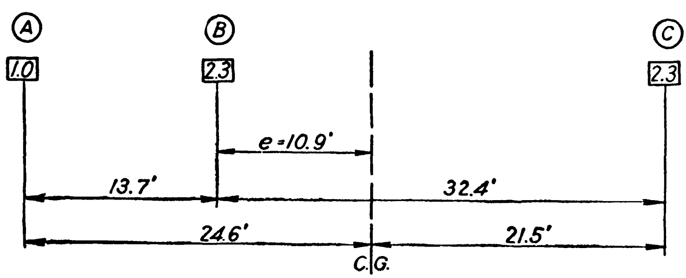


FIG. 226. CANTILEVER ANALYSIS.

Column B, interior column; area = 2.3.

$$\text{In the basement, } P = \frac{2.3 \times 10{,}350{,}000 \times 10.9}{1940} = 134{,}000 \text{ lb.}$$

$$\text{In the first story, } P = \frac{2.3 \times 9{,}040{,}000 \times 10.9}{1940} = 117{,}000 \text{ lb.}$$

Column C, wall column in classroom; area = 2.3.

$$\text{In the basement, } P = \frac{2.3 \times 10{,}350{,}000 \times 21.5}{1940} = 264{,}000 \text{ lb.}$$

$$\text{In the first story, } P = \frac{2.3 \times 9{,}040{,}000 \times 21.5}{1940} = 231{,}000 \text{ lb.}$$

$$\text{Girder shear in } BC = 33{,}000 \text{ lb.}$$

Girder and Column Moments. Girder moments are computed from the shears and lever arms shown in Fig. 227. The span length used in computing girder moments will be from center to center of columns. Thus, we will compensate in part for expected shift of the points of contraflexure away from the centers of the girders.

Moment for the girder AB = 16,000 × 6.8 × 12 = 1,300,000 in-lb.
Moment for the girder BC = 33,000 × 16.2 × 12 = 6,400,000 in-lb.

Girder moments will be divided between the columns at a joint in proportion to the story shears above and below. It is found that the division of moment to the columns at the first floor level should be 52 per cent and 48 per cent.

Wind moment for the column A = 0.52 × 1,300,000 = 680,000 in-lb.
Wind moment for the column B = 0.52(1,300,000 + 6,400,000) = 4,000,000 in-lb.
Wind moment for the column C = 0.52 × 6,400,000 = 3,300,000 in-lb.

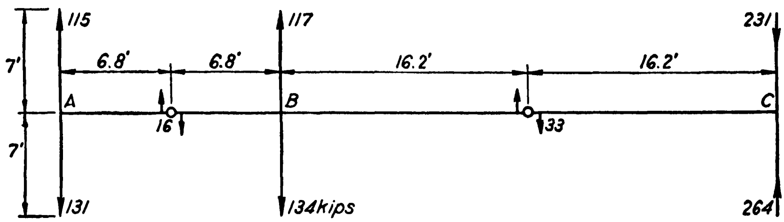


FIG. 227. WIND-STRESS ANALYSIS BY THE CANTILEVER METHOD.

199. 1st Floor Girders Designed for Wind Moment. The girders will be redesigned for moments caused by dead load, live load, and wind at the 1st floor level. Increase working stresses 33.3 per cent to 26,700 lb. per sq. in.

Girder across the Corridor, AB.

Moment computed for D.L. and L.L. (§ 194)	=	705,000 in-lb.
Moment caused by the estimated increase in D.L.	=	13,000
Moment due to wind (§ 198)	=	1,300,000
Design moment	=	2,018,000 in-lb.

The required section modulus of 76 will be furnished by an 18WF47 section. A 14WF30 section was used in the upper floors where wind was not considered. The increased weight of girder and fireproofing is about 70 lb. per ft.

Girder across the Classroom, BC.

Moment computed for D.L. and L.L. (§ 194)	=	5,130,000 in-lb.
Moment caused by the estimated increase in D.L.	=	210,000
Moment due to wind (§ 198)	=	6,400,000
Design moment	=	11,740,000 in-lb.

The required section modulus of 440 will be furnished by the 33WF141 section, but, in order to avoid excessive projection below the ceiling, we will use the 27WF163 section. A 27WF106 section was used in the upper floors where wind was not considered. The increased weight of girder and fireproofing is about 180 lb. per ft.

Revision of Average D.L. per Floor. The added weight of these girder sections and their fireproofing is found to increase the *average* floor dead load tributary to the column A by 300 lb., the total being 38,900 lb.; the average dead load tributary to the column B is increased by 1800 lb., the

total being 70,800 lb.; the average dead load tributary to the column C is increased by 1500 lb., the total being 70,100 lb. These corrections are really insignificant.

200. Column Sections at the Basement Level. It now becomes possible to select the maximum column sections for this building. The choice will be made for direct stress and moment caused by dead and live load with the use of *ordinary working stresses;* or, for direct stress and moment caused by dead load, live load, and wind with *working stresses increased 33⅓ per cent.* In other words, wind is to be considered only if it requires an increase of column section.

Wall Column in Corridor, A.

D.L. of 15.67 floors $= 15.67 \times 38{,}900$ $=$ 610,000 lb.
L.L. for 15.67 floors $= 9.57 \times 14{,}700$ $=$ 141,000
$(9.57 = 0.67 + 0.85 + 0.80 + 0.75 + 0.70 + 0.65 + 0.60 + 0.55 + 8 \times 0.50)$

Column with fireproofing $= 16 \times 14 \dfrac{(61 + 250) + (200 + 375)}{2} =$ 99,000

Dead and live load $=$ 850,000 lb.

D.L. and L.L. moment reduced to equivalent central load $= 239{,}000 \div 5.5$ (§ 196) $=$ 44,000

Design load $=$ 894,000 lb.

Wind direct stress $=$ 131,000 lb.
Wind moment reduced to an equivalent central load $= 680{,}000 \div 5.5$ (§ 198) $=$ 124,000

Total wind load $=$ 255,000 lb.

Wind being less than 33% of D.L. + L.L. is neglected.

Try $14 \times 16 WF 193$; $S/A = 5.5$; $L/r = 168 \div 4.05 = 41$;
$P = 16{,}190 \times 56.73 = 918{,}000$ lb.

Interior Column, B.

D.L. of 15.67 floors $= 15.67 \times 70{,}800$ $=$ 1,110,000 lb.
L.L. for 15.67 floors $= 9.57 \times 40{,}200$ $=$ 385,000

Column with fireproofing $= 16 \times 14 \dfrac{(167 + 375) + (425 + 450)}{2} =$ 158,000

Dead and live load $=$ 1,653,000 lb.

D.L. and L.L. moment reduced to an equivalent central load $= 2{,}047{,}000 \div 5.6$ (§ 196) $=$ 366,000

Equivalent dead and live load $=$ 2,019,000 lb.

Wind direct stress $=$ 134,000 lb.
Wind moment reduced to an equivalent central load $= 4{,}000{,}000 \div 5.6$ (§ 198) $=$ 715,000

Total wind load $=$ 849,000 lb.

Wind, being more than 33% of D.L. + L.L., must be included $=$ 849,000

Design load with increased working stress $=$ 2,868,000 lb.

Try $14 \times 16WF426$; $S/A = 5.6$; $L/r = 168 \div 4.34 = 38$;

$$P = 16{,}300 \times 125.25 \times 1.33 = 2{,}730{,}000 \text{ lb.}$$

Use $14 \times 16WF426 + 2$ web pls. $7 \times \frac{1}{2}$ in. welded;

$$P = 16{,}300 \times 132.25 \times 1.33 = 2{,}870{,}000 \text{ lb.}$$

Wall Column in Classroom, C.

D.L. of 15.67 floors $= 15.67 \times 70{,}100$ $= 1{,}100{,}000$ lb.

L.L. for 15.67 floors $= 9.57 \times 25{,}500$ $= 244{,}000$

Column with fireproofing $= 16 \times 14 \dfrac{(158 + 375) + (425 + 450)}{2} = 157{,}000$

Dead and live load $= 1{,}501{,}000$ lb.

D.L. and L.L. moment reduced to an equivalent central load $= 2{,}137{,}000 \div 5.6$ (§ 196) $= 380{,}000$

Equivalent dead and live load $= 1{,}881{,}000$ lb.

Wind direct stress $= 264{,}000$ lb.

Wind moment reduced to an equivalent central load $= 3{,}300{,}000 \div 5.6$ (§ 198) $= 589{,}000$

Total wind load $= 853{,}000$ lb.

Wind, being more than 33% of D.L. + L.L., must be included $= 853{,}000$

Design load with increased working stress $= 2{,}734{,}000$ lb.

Use $14 \times 16WF426$; $S/A = 5.6$; $L/r = 168 \div 4.34 = 38$;

$$P = 16{,}300 \times 125.25 \times 1.33 = 2{,}730{,}000 \text{ lb.}$$

NOTE: The ratio of the cross-sectional areas of each of the columns B and C to the area of the column A is close enough to the ratio of 2.3 used in the wind stress analysis so that a revision will not be necessary.

CHECKING THE PRELIMINARY DESIGN

201. Calculation of Moments. Unless the building is designed in detail from the top downwards, no better estimate of the column dead loads can be made than those already obtained. Also, it is impossible to compute correctly the column direct stresses caused by wind unless an analysis is made for the *entire structure.* However, we can determine rather simply the bending moments produced by dead load and live load at the first floor level by the process of *balancing moments;* we can also compute the wind moments by a similar process of *balancing K-values* discussed in Vol. II, *Theory of Modern Steel Structures,* page 168.

202. Maximum Moments from Dead and Live Load. There are several possible loadings that might be studied for the purpose of determining maximum moments: (1) dead loading; (2) live load on the long span, (*a*) on one floor only, (*b*) on all floors; (3) live load on the short span, (*a*) on one floor only, (*b*) on all floors. However, since we are not attempting to obtain an *exact* analysis, it should be satisfactory to determine approximate

maximum moments by a study of two conditions. As shown in Fig. 228, we will investigate the cases of live load over each span separately. The

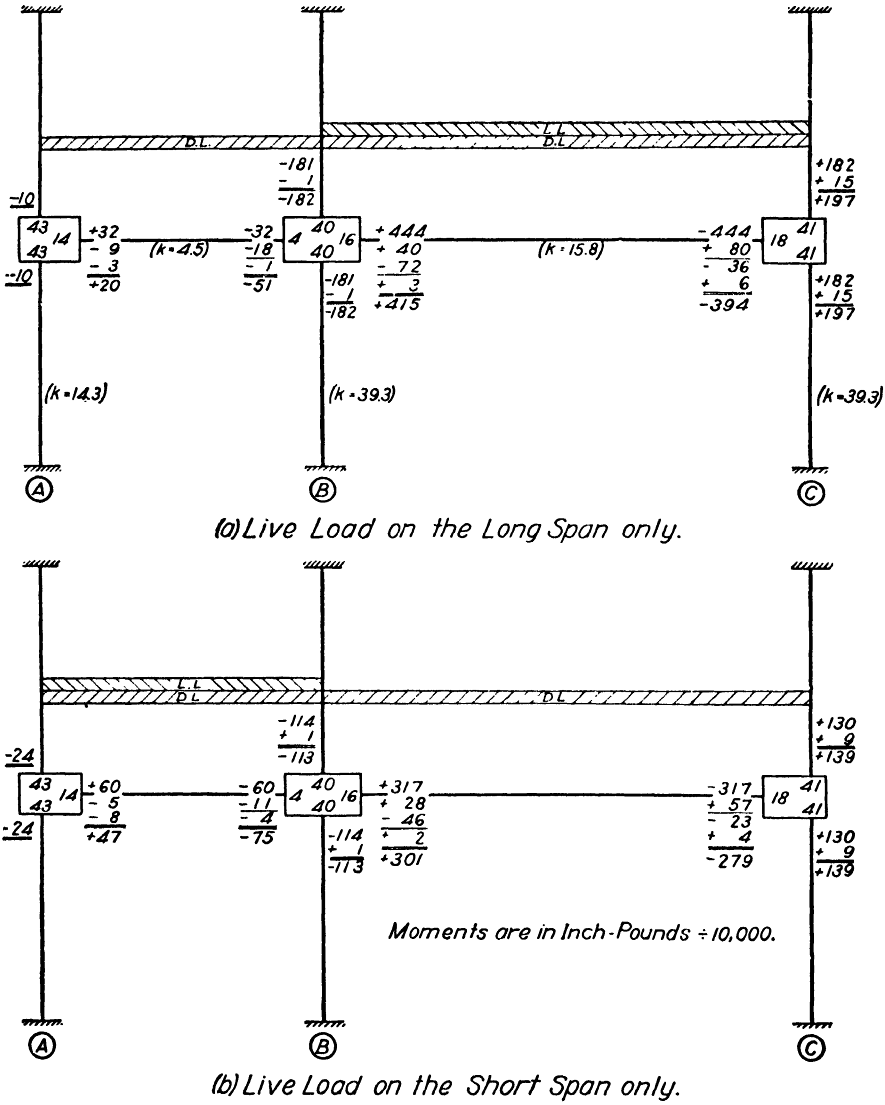


FIG. 228. BALANCING MOMENTS FOR THE DEAD AND LIVE LOADING.

columns are assumed to be fixed at the floor above and at the basement floor, and, of course, both spans are covered by dead load.

Fixed-End Moments. Span lengths again will be taken as 31.2 ft. and 12.5 ft. which are the maximum clear girder spans. Actually, the spans

are about 2 in. less at the first floor level. The fixed-end moments are needed for dead and live loadings. These loads are taken from the data of § 194 with an extra allowance of 33 per cent for the weights of the beams with fireproofing.

For the short span AB, we write:

$$M_{DL} = \tfrac{1}{12} \times 25{,}900 \times 12.5 \times 12 = 324{,}000 \text{ in-lb.}$$
$$M_{LL} = \tfrac{1}{12} \times 21{,}800 \times 12.5 \times 12 = \underline{272{,}000}$$
$$\text{Total for short span} = 596{,}000 \text{ in-lb.}$$

For the long span BC, we obtain:

$$M_{DL} = \tfrac{1}{12} \times 101{,}500 \times 31.2 \times 12 = 3{,}170{,}000 \text{ in-lb.}$$
$$M_{LL} = \tfrac{1}{12} \times 40{,}700 \times 31.2 \times 12 = \underline{1{,}270{,}000}$$
$$\text{Total for long span} = 4{,}440{,}000 \text{ in-lb.}$$

Girder Moments. The fixed-end moments are balanced in Fig. 228. The maximum girder moment is found to be 4,150,000 in-lb. for the long span and 750,000 in-lb. for the short span. The corresponding moments that were used in the revised preliminary design (§ 199) were 5,340,000 in-lb. and 718,000 in-lb.

Column Moments. The maximum column moments are 240,000 in-lb. for the column A, 1,820,000 in-lb. for the column B, and 1,970,000 in-lb. for the column C. In our preliminary analysis (§ 196) we had obtained 322,000 in-lb. for A, 2,047,000 in-lb. for B, and 2,220,000 in-lb. for C. Hence, we have clearly overdesigned the columns in this regard. Furthermore, we should *reduce* the moments in the wall columns A and C by the moment (83,000 in-lb.) introduced by the eccentrically connected spandrel beams.

203. Wind Moments. The method of analysis of wind moments will be the balancing of K-values. Since the basement columns are fixed at their bases by the dead load itself, a deflection estimate would show a much smaller lateral wind deflection in the basement story than in the first story. To attempt to account for this influence, we may introduce correction moments (negative K-values) into the basement story even before the balancing process is started. Previous experience has shown that negative corrections for basement stories of from 30 to 50 per cent of the K-values are usually effective in shortening the balancing process. We will therefore introduce negative K-value corrections equal to 40 per cent of the K-values of the columns. Actually, in Fig. 229 all K-values and corrections are multiplied by 3.0 in order to improve the accuracy of the analysis without using three digit numbers.

In order to make the calculations for two stories only, the columns of the second story are shown with pin ends at their midheights. This represents approximately their *true condition* and any small error involved will have a quite negligible influence upon the story below. Note that

stiffness (resistance to end rotation) is increased 50 per cent when the member is shortened 50 per cent and is pin connected at its far end.

Design Revisions

204. Final Revised Sections. The dead load and live load moments taken from Fig. 228 will be combined with the wind moments taken from Fig. 229 to revise the choice of girder and column sections at the first floor

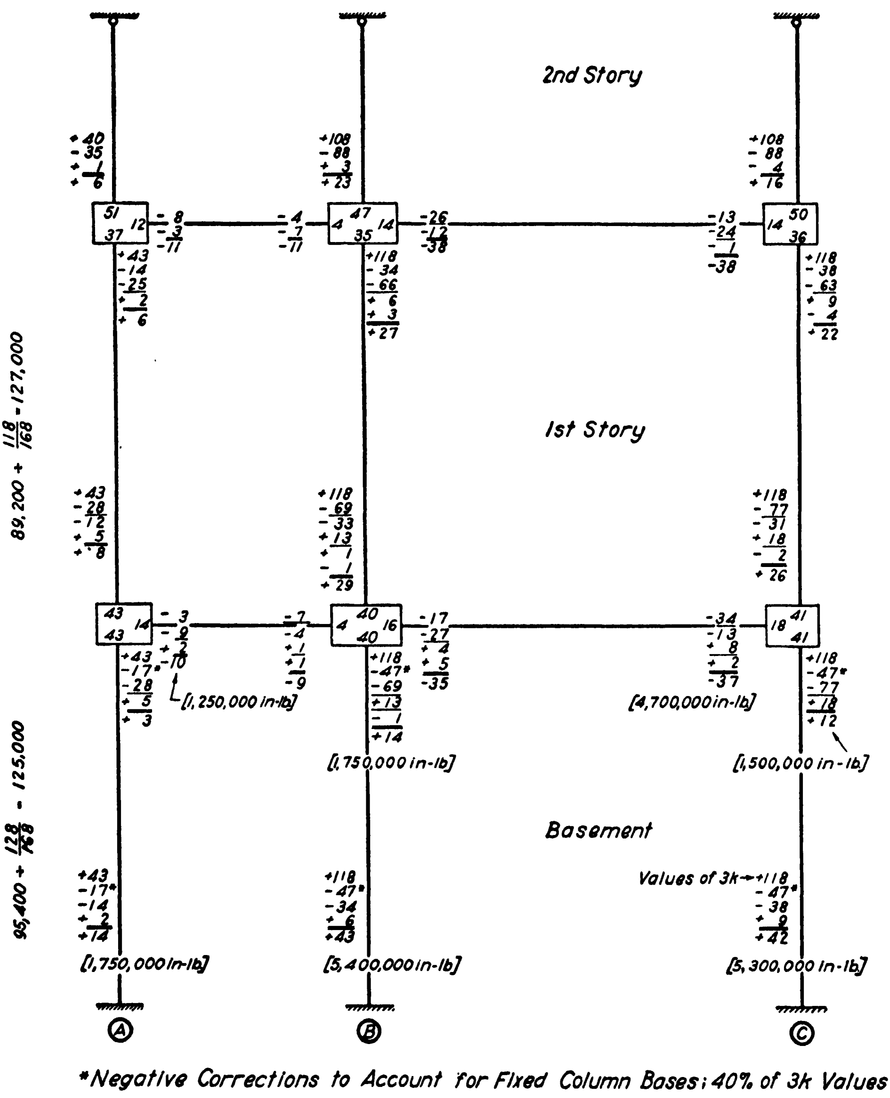


FIG. 229. WIND MOMENTS CALCULATED BY BALANCING K-VALUES.

of the building. The wind moments for the girders from Fig. 229 will be reduced for clear span since the girders are designed for the moments at the column faces, but *unreduced column moments* will be used because lateral buckling might be started by failure within the depth of the floor. For the same reason, the factor L/r has been computed for a length equal to the full story height. Obviously, this is a conservative procedure.

Girder Sections at the First Floor.

Maximum wind moment for the short girder, AB =

$$1{,}250{,}000 \times \frac{12.3}{13.7} = 1{,}120{,}000 \text{ in-lb.}$$

Maximum wind moment for the long girder, BC =

$$4{,}700{,}000 \times \frac{30.8}{32.4} = 4{,}470{,}000 \text{ in-lb.}$$

Wind moments are combined with the moments from dead load and live load in Fig. 230. From these moments we may compute the section moduli for the girders as follows.

Short span girder, AB. $S = 1{,}770{,}000 \div 26{,}700 = 67$.
The preliminary section (18*WF*47) may be reduced to a 16*WF*45 beam.

Long span girder, BC. $S = 8{,}410{,}000 \div 26{,}700 = 315$.
The preliminary section (27*WF*163) may be reduced to a 24*WF*130 beam.

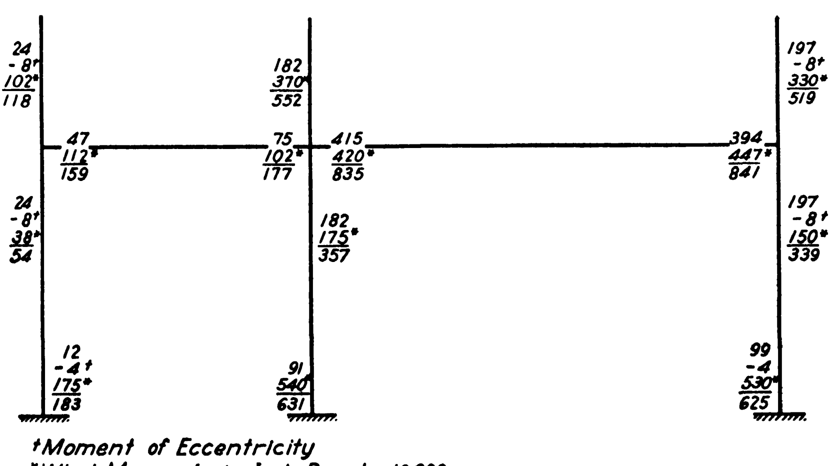


FIG. 230. FINAL MOMENTS FOR LOWER STORY.

Column Sections. A revision of the column sections will be made. We will use the direct stress for dead load, live load, and wind, as used previously in § 200. Moments have been summed in Fig. 230. One moment recorded there has not been given consideration. That is the moment

produced by the eccentric connections of the spandrel beams. One half of the value of this moment was used to reduce the dead load and live load moments for the preliminary design of the wall columns. However, before advantage is taken of this reduction of moment in the final design, it is desirable to analyze the action of such eccentric loads on a building column. As shown in Fig. 231, a moment of eccentricity Pe applied at *each story level* produces effective moments of $Pe/2$ in the columns at *each floor level* when the story heights and column sections are the same. However, the most serious stress condition for the basement columns will be found to occur at the fixed base where the moment of eccentricity is reduced to not more than $Pe/4$. This is but 42,000 in-lb. Hence, for the design of the columns, we should use the following moments from dead and live loading. Wind moments will be considered separately.

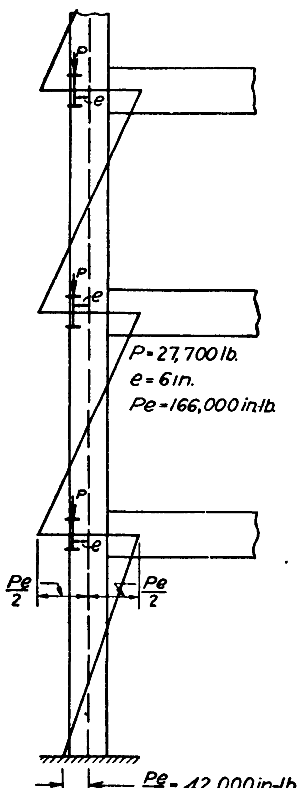


FIG. 231. MOMENTS FROM ECCENTRIC LOADING.

Column A. $M = 120{,}000 - 40{,}000 = 80{,}000$ in-lb.
Column B. $M = 910{,}000$ in-lb.
Column C. $M = 990{,}000 - 40{,}000 = 950{,}000$ in-lb.

Wall Column in Corridor, A.

D.L. and L.L. direct stress from § 200	=	850,000 lb.
D.L. and L.L. moment reduced to an equivalent central load = 80,000 ÷ 5.5	=	15,000
Total dead and live load	=	865,000 lb.
Wind direct stress from § 200	=	131,000 lb.
Wind moment reduced to an equivalent central load = 1,750,000 ÷ 5.5	=	319,000
Total wind load	=	450,000 lb.
Wind being over 33% of D.L. + L.L. must be included	=	450,000
Design load with increased working stress	=	1,315,000 lb.

Try 14×16WF211; $S/A = 5.5$; $L/r = 168 \div 4.07 = 41$;

$$P = 16{,}190 \times 1.33 \times 62.07 = 1{,}340{,}000 \text{ lb.}$$

This is an increase from the preliminary 14×16WF193 section.

Interior Column, B.

D.L. and L.L. direct stress from § 200	=	1,653,000 lb.
D.L. and L.L. moment reduced to an equivalent central load = 910,000 ÷ 5.6	=	163,000
Total dead and live load	=	1,816,000 lb

Wind direct stress from § 200	= 134,000 lb.		
Wind moment reduced to an equivalent central load = 5,400,000 ÷ 5.6	= 965,000		
Total wind load	= 1,099,000 lb.		
Wind being over 33% of D.L. + L.L. must be included			= 1,099,000
Design load with increased working stress			= 2,915,000 lb.

Use 14×16*WF*426 + 2 web pls. 9 × ½ in. welded;
$P = 16{,}300 \times 134.25 \times 1.33 = 2{,}920{,}000$ lb.

This is only a slight increase from the 14×16*WF*426 + 2 web pls. 7 × ½ in. used for the preliminary design.

Wall Column in Classroom, C.

D.L. and L.L. direct stress from § 200		= 1,501,000 lb.
D.L. and L.L. moment reduced to an equivalent central load = 950,000 ÷ 5.6		= 170,000
Total dead and live load		= 1,671,000 lb.
Wind direct stress from § 200	= 264,000 lb.	
Wind moment reduced to an equivalent central load = 5,300,000 ÷ 5.6	= 946,000	
Total wind load	= 1,210,000 lb.	
Wind being over 33% of D.L. + L.L. must be included		= 1,210,000
Design load with increased working stress		= 2,881,000 lb.

Use 14×16*WF*426 + 2 web pls. 8 × ½ in. welded;
$P = 16{,}300 \times 133.25 \times 1.33 = 2{,}890{,}000$ lb.

In the preliminary design this section was used without the web plates. For convenient fabrication the sections for the columns B and C could be made identical.

205. Comparisons of the Preliminary and Final Designs. It is significant that even the incomplete analysis made on Figs. 228 and 229 by *the methods of indeterminate structures pointed to a considerable revision of sections.* The short girder was lightened slightly but the long girder was reduced considerably in weight, in fact, by 20 per cent even though the depth was reduced from 27 to 24 in. All column sections had to be increased in weight at the basement level since the wind stress analysis showed that there would be heavy wind moments at the basement floor level accompanying an upward shift in the points of contraflexure of the basement columns. However, we observe that the wind moment and wind uplift are not sufficient to reverse from compression to tension the direct column stress caused by dead load. This means that the columns will remain fixed at the basement floor level. Hence, we can not feel safe in neglecting the heavy wind moments indicated by the analysis of Fig. 229 To make this point clear, it will be necessary to compute the fiber stresses

for the wall column in the classroom (column C) where the effect of wind is the most serious.

Dead load at basement level from § 200	=	1,257,000 lb.
Uplift from wind, § 198	=	264,000
Minimum net compressive load	=	993,000 lb.

Column area = 133.25; column modulus = 711.7

$$\text{Resultant fiber stress} = -\frac{993{,}000}{133.25} + \frac{5{,}300{,}000}{711.7}$$

$$= -7450 + 7450 = \text{zero.}$$

Hence, the column C remains fixed at the basement floor level even without anchor bolts.

Similar computations for the other columns will show that there is a resultant minimum compression of 4100 lb. per sq. in. for the column A and 800 lb. per sq. in. for the column B. Live load direct stress and moment are omitted from these calculations because the live load may or may not exist. Dead load column moment was not considered, although, for the column C analyzed above, it would have acted to reduce the wind moment accompanying wind uplift and thus tend to help fix the column base.

The Importance of Proper Analysis. The result of this study confirms the opinion, rapidly becoming universal, that rigid frame structures cannot be designed properly without an investigation by the methods of analysis of indeterminate structures. Certainly, this building frame is a simple one and yet our preliminary design (which is more thorough than many practical designers in past years had even considered necessary) was not an entirely satisfactory one. How much more important a proper analysis would be for an irregular building frame, the reader is left to judge. It should be made clear, however, that the computations of Fig. 228 and Fig. 229 are not as complete as such computations should be, when made in a practical design office. These analyses are themselves approximations in that they are limited to consideration of only two stories, and the member sizes above the second floor were not even available for the determination of stiffness factors. Therefore, columns had to be considered fixed at their upper ends, or some similar assumption had to be introduced into these studies. A complete design from the top of the building downward would make it possible for us to analyze properly for all moments and direct stresses. Whether the frame obtained here is the most economical one obtainable, raises a question that is too difficult to be answered. It seems reasonable to assume, however, that the most economical frame will be one in which each member is stressed to the maximum value permitted by the specifications.

PROBLEMS

223. Estimate the weight of structural steel for the building designed in this chapter. Assume that the weight of columns and girders changes in a straight line relationship from the basement to the roof. Finally, compute the weight of structural steel per cubic foot of volume and per square foot of floor space.

224. Redesign the columns at the basement level from § 204 using an allowable stress for combined direct compression and flexure to be determined from the relation $[(P/A) \div f_c + (M/S) \div f_b] < 1$ where f_c is the allowable stress from the column formula and f_b is the allowable stress in beam flexure. This is the formula given by the *AISC* specifications for checking a section that resists combined stress.

225. Redesign the building studied in this chapter according to the detailed specifications of a particular city building code. Increase the wind load by 30 per cent.

226. Extend the building designed in this chapter to twenty stories and make a complete redesign.

227. Study available building codes and equipment catalogues including Sweet's architectural and engineering catalogues. Design the vertical shafts (that enclose the elevators, ducts and stairways) for the building designed here. Meet the fire regulations of the Building Code of the National Board of Fire Underwriters. The problem is to make the building safe and convenient for the use of 1000 students and 200 employees.

228. Lay out and design a single college building to be placed on a lot of size 100 ft. × 250 ft. to accommodate an engineering student body of 750. Make your own estimate of space requirements and cover no more than one half of the lot area. Use a local building code or follow the specifications given in this book.

229. Design a building more than 15 stories high to serve a function set up by yourself. State all functional requirements before the design is started and then hold to these requirements *rigidly*. Choose any standard set of specifications, but make your building meet all of its requirements rigorously.

206. Observations Regarding Building Design. There are many matters of judgment that are left to the decision of the designer. In the example given in this chapter, most of these decisions were reached from the conservative point of view. It is probable that *a practical office designer would reduce the weight of the structural frame* of our building considerably. For example, consider the determination of working stress for column design. A considerable part of the actual stress was produced by moment, but we determined the working stress from the column formula. Many specifications permit an increase of working stress for combined direct stress and flexure. Again, the unsupported length of the column for lateral buckling was taken as the story height. This seemed desirable because the spandrel beam was attached only to one flange of the column. Probably most designers would be willing to reduce the unsupported length by the depth of the spandrel beam or perhaps by an even greater amount because of the support provided by the masonry part of the wall.

In particular, selection of the basement columns in the illustrative design was performed quite conservatively. With a working stress selected for failure by buckling, we chose sections at the basement floor level that

were stressed to a considerable extent by flexure. It seems evident that buckling could hardly start at a fixed base and, since the column above the base is more lightly stressed at every point, the working stress could certainly be increased at the base without danger. The following relation would have been used except that it seemed desirable to keep the illustrative example as simple as possible.

$$\frac{P/A}{f_{\text{column}}} + \frac{M/S}{f_{\text{beam}}} < 1. \quad \text{(Spec. 4).}$$

By taking advantage of all justifiable reductions of section, the careful designer will save a great deal of structural steel in a large building. The frequent repetition of sections makes a small saving per member important.

CHAPTER 16

DESIGN OF CONTINUOUS BEAMS*

207. Design Procedures. The preceding discussion has dealt with statically determinate structures. In such structures the stresses and bending moments are controlled by statics. For such systems we may calculate exactly the controlling moments or the axial stresses *before the sizes of the various members are known.* This *simple* process cannot be applied to indeterminate structures because the sizes of the members influence the stress distribution. For the continuous beam, the particular case to be studied here, this means that the moment diagram is not dependent solely upon the lengths of the spans, the amount and positions of the loads, and the manner of support, but also upon the relative stiffnesses of the several spans (controlled by the moments of inertia of the beams themselves). Evidently, then, the design of a continuous beam would normally be accomplished by a "guess and check" procedure. Relative moments of inertia could be assumed for the several spans and a moment diagram could be determined for this condition. Then, when beam sections had been selected to furnish the necessary section moduli, a new and improved set of stiffness factors for the several spans would be available. These moduli would be used as a basis for a second analysis giving rise to revised bending moments. By repeating this process several times, a continuous beam would finally be obtained that would provide relative stiffnesses consistent with those used in its analysis and design. Thus the process would converge to a final answer.

The only occasion where a process similar to the one discussed above has been regularly used for simple structures is in the design of column sections. In that instance, the working stress is always controlled by a formula which includes the radius of gyration of the section as one of its terms. Thus, it is necessary to start with an *estimate* of the radius of gyration (or of the working stress itself) and then to select a trial section to be revised once or twice until the radius of gyration of the section used checks the value introduced into the column formula for the determination of the allowable stress.

* If the author's book entitled *Automatic Design of Continuous Frames in Steel and Reinforced Concrete* is available, the reader will find the information of this chapter presented there in greater detail. Many other applications of this design procedure are also given in that volume. The chapter presented here will serve as an introduction.

208. Automatic Design. The author introduced in 1939 a new method of design for continuous beams and frames which has been termed automatic design in contrast to the "guess and check" process described above. This method, made dependent upon a procedure of balancing section moduli, is *a self contained design routine* that includes the necessary steps of analysis as an integrated part of the whole. By thorough organization it eliminates unnecessary calculations, avoids errors, and condenses the calculations so that they may usually be made on a single sheet. The title *Automatic Design* was suggested by the fact that the calculations are essentially routine and the process, when followed step by step, produces the most economical structure irrespective of the designer's experience or judgment. Nevertheless, the method permits the designer freedom to use his own experience to speed the series convergence.

209. Balancing Section Moduli. If the reader is familiar with the process of balancing moments described in Vol. 2 of the *Theory of Modern Steel Structures*, pp. 140–145, he will follow the method of balancing moduli readily. Otherwise, it will be necessary for him to study the conceptions involved in the balancing process and to practice its application to many simple cases before he attempts to use it as a design tool.

Definitions of Special Terms. A group of terms will be defined so that they may be used without confusion. These terms will be quite similar to those that have become commonplace among designers who use the process of balancing moments (or moment distribution) for the analysis of continuous structures. Beams are assumed to be of constant section from ènd to end of a single span, that is, the members are *prismatic.*

Fixed-End Modulus. This is the section modulus required to resist the moment at one end of a particular loaded span when end rotation is not permitted at either end of the span. In other words, the fixed-end modulus is the corresponding fixed-end moment divided by the allowable fiber stress.

Stiffness Factor. The stiffness or resistance to end rotation is the ratio of the moment of inertia to the length of the beam, that is, the I/L value. Relative values of I/L are as useful as absolute values. For rolled beams, for which the section modulus I/c is known, a relative value of I/L is conveniently obtained from the relation $2I/L = I/c \div L/d$, where L/d is known as the prismatic ratio.

Distribution Factors. When two beams of unlike sections or of unequal lengths are rigidly joined together at a support, they must rotate together. A moment applied to the structure at their connection will be distributed between the two beams in proportion to their individual resistances to end rotation or in proportion to their I/L values. Thus, the ratios of the individual I/L values to the sum of the I/L values at the joint become distribution factors for moment or *modulus.*

Unbalanced Modulus. If the fixed-end moments or the *fixed-end moduli* on the two sides of a support of a continuous beam total zero (with signs properly considered), the joint is in equilibrium and rotation will not occur. On the other hand, this sum may not be zero, and the resulting unbalanced modulus will then have to distribute itself between the connecting beams.

Balancing Modulus. The unbalanced modulus at a joint multiplied by the distribution factors at the joint gives rise to *balancing moduli* to be recorded at the joint. This is merely a process of expressing the result upon the structure of joint rotation.

Carry-over Factor. If one end of a prismatic member that is fixed at the far end is rotated by an applied moment, a moment of one half of this value appears at the far (fixed) end. The carry-over factor is therefore 0.5 for a prismatic member. This statement applies either to moment or *modulus distribution.*

Carry-over Modulus. The carry-over modulus to the fixed or far end of a member is the balancing modulus at the near end times the carry-over factor.

Correction Modulus. If it becomes necessary to revise a section and therefore *to change the stiffness* of a beam during the process of balancing section moduli, we will find that *the balancing moduli and the carry-over moduli at both ends of this span are increased or decreased in direct proportion to the change in stiffness. Correction* moduli are introduced to account for these changes. Evidently, *the fixed-end moduli are not dependent upon stiffness* (end rotation) and therefore are revised only when the loading is increased or decreased.

Sign Convention. Long experience with procedures of moment and modulus distribution has shown clearly that the most convenient sign convention is defined by the following statement: *a positive end moment (or modulus) is one that tends to rotate the adjacent joint clockwise.* By this sign convention, the total recorded moduli on the two sides of a balanced joint will add up (algebraically) to zero. Also, a carry-over modulus will be of the *same* sign as the balancing modulus which produces it. Fixed-end moduli for a horizontal span with downward acting loads will be positive at the left and negative at the right. These signs are illustrated by Fig. 232.

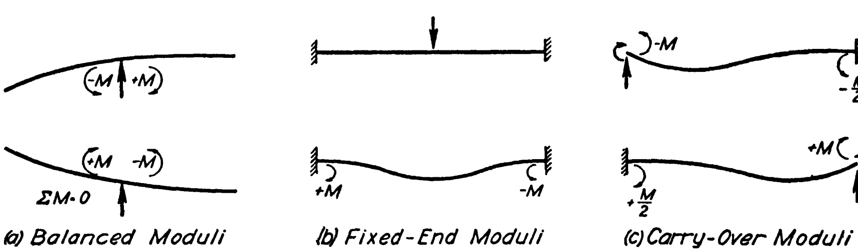


FIG. 232. SIGN CONVENTION FOR BALANCING MODULI.

210. Simple Design Problem Illustrating Balancing Procedure. The example (Fig. 233) will be used for describing the automatic procedure of design. This two-span beam has but one joint free to rotate. Therefore, it is an elementary design problem. As usual in design, the objective is to obtain the lightest weight of standard rolled beam sections that can be used in each span. Then, if the difficulty of joining spans of unequal depths is considered serious, we can revise the sections for a *constant depth* with full knowledge of the waste involved in added metal.

It is necessary first to compute the fixed-end moments as $\frac{1}{12}wL^2$ for the 40-ft. span and from the formula Pab^2/L^2 for the 30-ft. span. These are given at the top of Fig. 233. Then, these fixed-end moments are divided by the working stress of 20,000 lb. per sq. in. for flexure to obtain the fixed-end moduli. Since the maximum fixed-end moduli in these spans are 266 and 240, we conclude that 27*WF* beams are the most probable sections.* The first estimate of relative stiffness is therefore obtained as

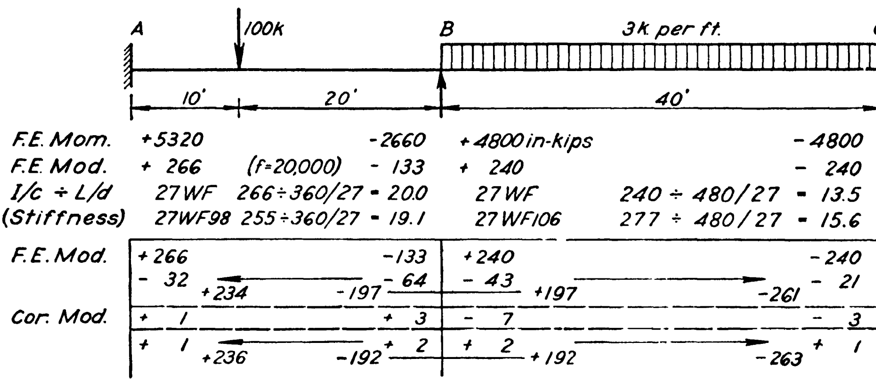


FIG. 233. SIMPLE DESIGN TO ILLUSTRATE PROCEDURE.

$I/c \div L/d$ for a value of d of 27 in. for each span. In these preliminary calculations of stiffness factors, the section modulus I/c is crudely taken as the maximum fixed-end modulus for each span.

Balancing Process. An inspection shows that the joint B is unbalanced since the fixed-end modulus to the left is -133 while the fixed-end modulus to the right is $+240$. For the sign convention used here, the moduli on the two sides of the joint must total *zero.* Hence, the unbalanced modulus of $(+240 - 133)$ or $+107$ must be balanced by a pair of balancing moduli whose sum will equal -107. These balancing moduli are -64 and -43 as recorded in Fig. 233. They bear the same ratio to each other as do the corresponding stiffness factors of 20.0 and 13.5. The procedure by which they were obtained is as follows:

$$\text{Bal. mod. at } B \text{ for } AB = -107\frac{20.0}{33.5} = -64;$$

$$\text{Bal. mod. at } B \text{ for } BC = -107\frac{13.5}{33.5} = -43.$$

As soon as the balancing moduli are recorded in Fig. 233, these moduli are carried over to the far (fixed) ends of the spans AB and BC. The

* Use should be made of the economy table of rolled beam sizes available in the *AISC* handbook. *Steel Construction.*

carry-over moduli are of the same signs as the corresponding balancing moduli but of one half their values. After a single balance of the joint that is free to rotate, totals are obtained for each column of moduli. The resulting totals are 234 at A, 197 at B, and 261 at C. The economic section to furnish the controlling modulus of 234 for the span AB is the $27WF98$ section. Similarly, the $27WF106$ section is chosen to furnish the controlling modulus of 261 for the span BC. The corresponding relative stiffness factors are found to be 19.1 and 15.6 respectively.

Correction Moduli. The span BC has been changed in stiffness from 13.5 to 15.6, an increase of 15.5 per cent. At each end of this span the balancing moduli and the carry-over moduli will be increased by the same percentage. The correction moduli for the span BC are therefore obtained as follows:

$$\text{Correction mod. at } C = 0.155 \times -21 = -3;$$

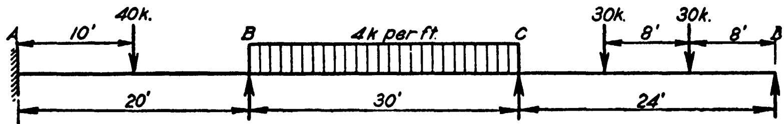
$$\text{Correction mod. at } B = 0.155 \times -43 = -7.$$

Corresponding correction moduli of $+1$ and $+3$ are obtained for the span AB, the signs being opposite to the signs of the balancing moduli and carry-over moduli because the stiffness of this span is *decreased* instead of increased.

Final Balance. After the addition of correction moduli, the unbalanced modulus at the joint B is again balanced — this time by use of the revised stiffness factors. Following this, the carry-over moduli are recorded at A and C. Columns are then summed again and total moduli requirements are compared with the moduli furnished to determine whether sections need be further revised. In Fig. 233 it is found that no additional revisions of sections are needed; hence, the $27WF98$ and $27WF106$ sections become final. For more complicated designs, where several joints must be permitted to rotate, we will find that stiffnesses will need to be revised about three times and correction moduli will have to be introduced two or three times to obtain a satisfactory convergence of sections. Naturally, the process can be speeded up by our ability to " guess sections closely " and it will be slowed down by attempts at guessing sections that prove to be unsuccessful, but the resulting *accuracy will not be influenced thereby.* The routine procedure is fast enough to discourage us from attempting to shorten it by outguessing the process of convergence except where the result becomes almost obvious.

211. Design of a Continuous Beam of Three Spans. The illustrative example Fig. 234 is a typical one where sections are revised several times with corresponding additions of correction moduli. The proper treatment of a simple end support where the modulus requirement is repeatedly reduced to zero by the balancing process is illustrated. In computing

stiffness factors as $I/c \div L/d$, the first values of section moduli I/c are the maximum fixed-end moduli in each span. The proper depths are chosen from a table of economic sections, and stiffnesses are then computed. After each joint except the fixed-end A has been balanced once, and after the carry-over moduli have been recorded, columns of moduli are totaled. Controlling moduli, selected for each span, are used to revise depths.



F.E. Mom. $\frac{PL}{8} = 1200$ in-kips $\frac{WL}{12} = 3600$ in-kips $\frac{PL}{9} = 1920$ in-kips

Stiffness:

Span AB	Span BC	Span CD
(16WF) 60÷240/16=4.0	(24WF) 180÷360/24=12.0	(18WF) 96÷288/18=6.0
(18WF) 93÷240/18=7.0	(24WF) 180÷360/24=12.0	(24WF) 180÷288/24=15.0
(21WF59) 119.3÷240/21=10.5	(24WF80) 185.8÷360/24=12.4	(24WF80) 185.8÷288/24=15.5
(21WF63) 128.0÷240/21=11.2	(ditto)	(ditto)

	A	B (AB)	B (BC)	C (BC)	C (CD)	D
F.E. Mod.	+60 (f=20,000)	-60	+180	-180	+96	-96
					+48	+96
			+12	+24	+12	+6
	-16	-33	-99	-49		
	+44	-93	+93	-205 (avg=180)	+156	+6
1st C.M.	-12	-25	0	0	+90	+153
					-79	-159
			+8	+17	+21	+10
	+3	+6	+11	+5		
	+35	-112	+112	-183 (avg=185)	+188	+10
2nd C.M.	-12	-26	-2	0	+3	+4
	+6	+12	+16	+8	-7	-14
			-2	-4	-5	-2
	+29	-126	+124	-179	+179	-2
3rd C.M.	-2	-4	0	0	0	0
	+1	+3	+3	+1	+1	+2
				-1	-1	
	+28	-127	+127	-179	+179	0

FIG. 234. AUTOMATIC DESIGN OF A CONTINUOUS BEAM.

Correction moduli are then introduced to account for changes of stiffness. When sections are again revised after each joint has been rebalanced and columns of moduli have been retotaled, we select sections not only as to depth but for *least weight* as well. Thus, the process gives rise to the correct economic design when the convergence is completed.

Economy and Variable Depths. The choice of beam sections of least weight may result (as in Fig. 234) in variations of depth from span to span. There is no objection to this variation if the beams frame into columns. On the other hand, connections may be difficult to arrange between spans of different depths for an ordinary continuous beam. Nevertheless, a knowledge of *the proper design for least weight* is very useful even if a revision for constant depth in all spans is considered necessary

as a practical expedient. The designer will then know just how much steel has been wasted, and his preliminary economic design will give him the information upon which to base his choice of the most economic single depth for all spans.

	47%	53%	44%	56%	
+60	-60	+180	-180	+96	-96
-28	-57	-63	-31	+48	+96
		+15	+30	+37	+18
-3	-7	-8	-4	-9	-18
		+3	+6	+7	+3
	-1	-2	-1	-1	-3
			+1	+1	
+29	-125	+125	-179	+179	0

FIG. 235. CHECKING THE DESIGN OF FIG. 234.

CHECK ANALYSIS. The check analysis of Fig. 235 shows that there have been no errors in the balancing process of Fig. 234. The check analysis is more conveniently performed with moduli than moments since the results are then in the same terms as the automatic design. The distribution factors are obtained, of course, from *final* values of stiffness.

SPECIAL DESIGN PROBLEMS

212. Automatic Design When Mid-Span Moduli Control Sections. Evidently, there is always the possibility that the modulus requirement

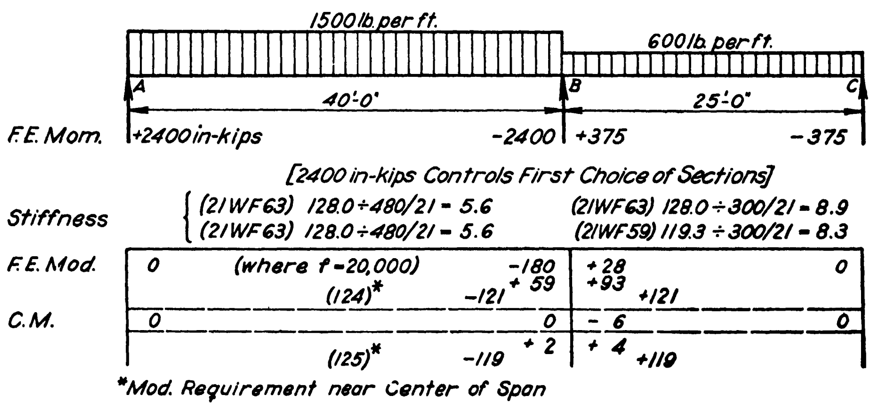


FIG. 236. DESIGN CONTROLLED BY A MID-SPAN MODULUS.

near the mid-span may be greater than the maximum modulus requirement at either support of the span under consideration.

This condition occurs in Fig. 236 where the final controlling modulus for the span AB is 125 near the mid-span rather than 119 at the support B. Either by mental arithmetic or by the use of plotted simple-span moment (or modulus) curves, we can readily determine the modulus requirement

near the mid-span as soon as the end moduli requirements have been expressed as percentages of the simple-span modulus requirement. This process is illustrated by Fig. 237 for the special case discussed.

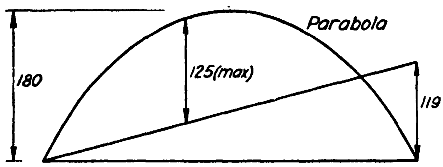


FIG. 237. MID-SPAN MODULUS FOR FIG. 236.

It is also of interest in Fig. 236 that there are no fixed-end moduli shown at A or C (simply supported ends). It will be observed that the fixed-end moduli at B are increased (50 per cent in this case) to account for the simple supports at A and C. Also, it would be proper *to reduce stiffness factors 25 per cent* to allow for the decrease in stiffness that accompanies a simply supported far end, but, since both spans have simply supported ends, this influence can be neglected without disturbing the balancing process.

213. Design Including Dead Weight. The design problem Fig. 238 is introduced to show how the influence of the dead weight of the steel beam and its concrete fireproofing or weatherproofing can be introduced into

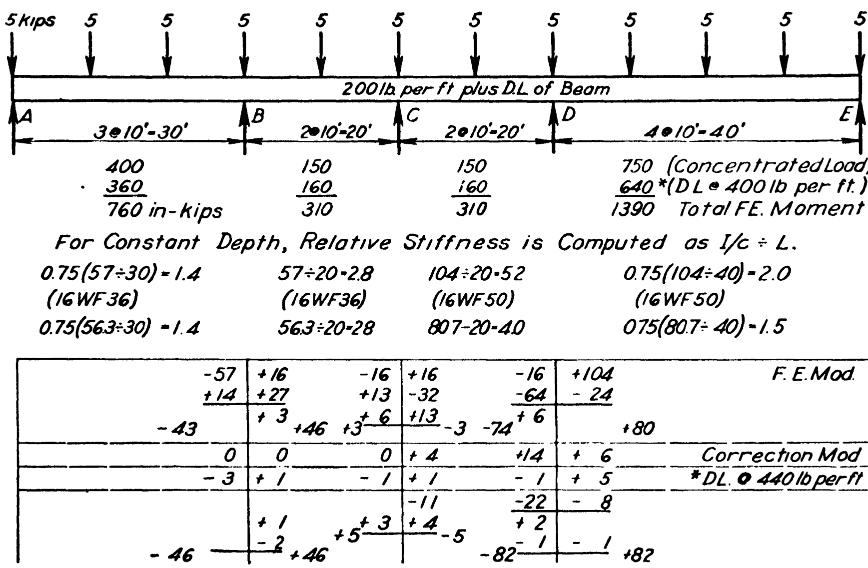


FIG. 238. DESIGN INCLUDING THE EFFECT OF DEAD WEIGHT.

the design. The dead weight of the beam was first estimated at 200 lb. per ft. This load was added to the fixed uniform load of 200 lb. per ft. to produce a total uniformly distributed loading of 400 lb. per ft. After the first balance, the dead weight of the *fireproofed beam* was calculated and found to be 240 lb. per ft. The excess load of 40 lb. per ft. was used

to calculate additional fixed-end moduli which were then introduced into the balancing process. This particular design problem converged so rapidly that only one set of correction moduli was needed. Again, as in Fig. 236, there are no moduli recorded at the simply supported ends of the beam. Note, however, that stiffnesses for these end spans are properly reduced 25 per cent by introduction of the factor 0.75 in the stiffness computations.

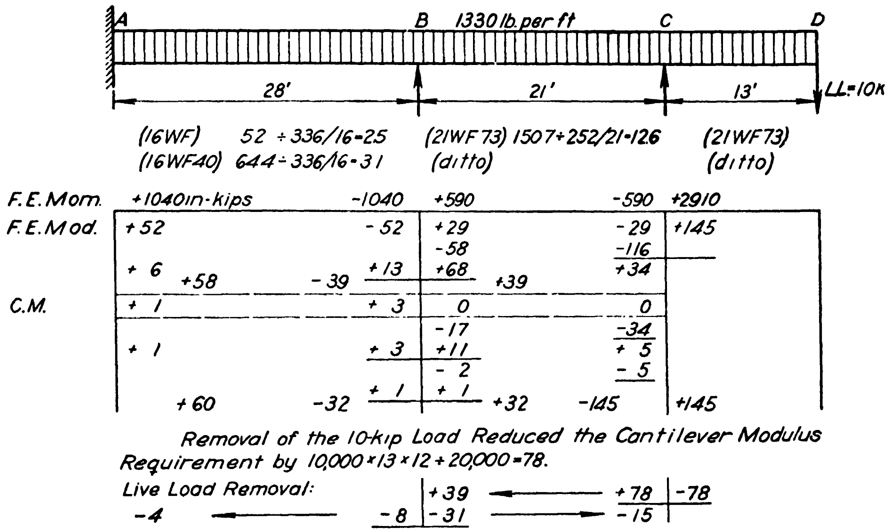


FIG. 239. DESIGN FOR VARIABLE DEPTH WITH A LIVE LOAD.

Since the controlling moduli for all spans are reduced by removal of the concentrated live load, the sections are correct. As an exercise this beam may be revised for a constant depth.

214. Design Including Live Loads. The influence of a removable concentrated load upon the design of a continuous beam is shown by Fig. 239. The design is first made for the load in place and with variable depths. Then, the concentrated live load is removed to determine its influence upon sections. Since all moduli are reduced thereby, the sections remain unchanged. As an exercise, the reader should revise this beam for a constant depth.

Uniform Live Load for Full Spans. The procedure of design, as illustrated by Fig. 240, is the use of simultaneous design sheets for each loaded span. Whenever a section is to be revised, the controlling modulus is determined by summing the corresponding moduli requirements from the several design sheets. For example, the final modulus at A (+90) is obtained as the sum of +75 and +15. The modulus +75 is required by the live load on the span AB while the modulus +15 is required for live load on the span CD. The influence of live load on the span BC would

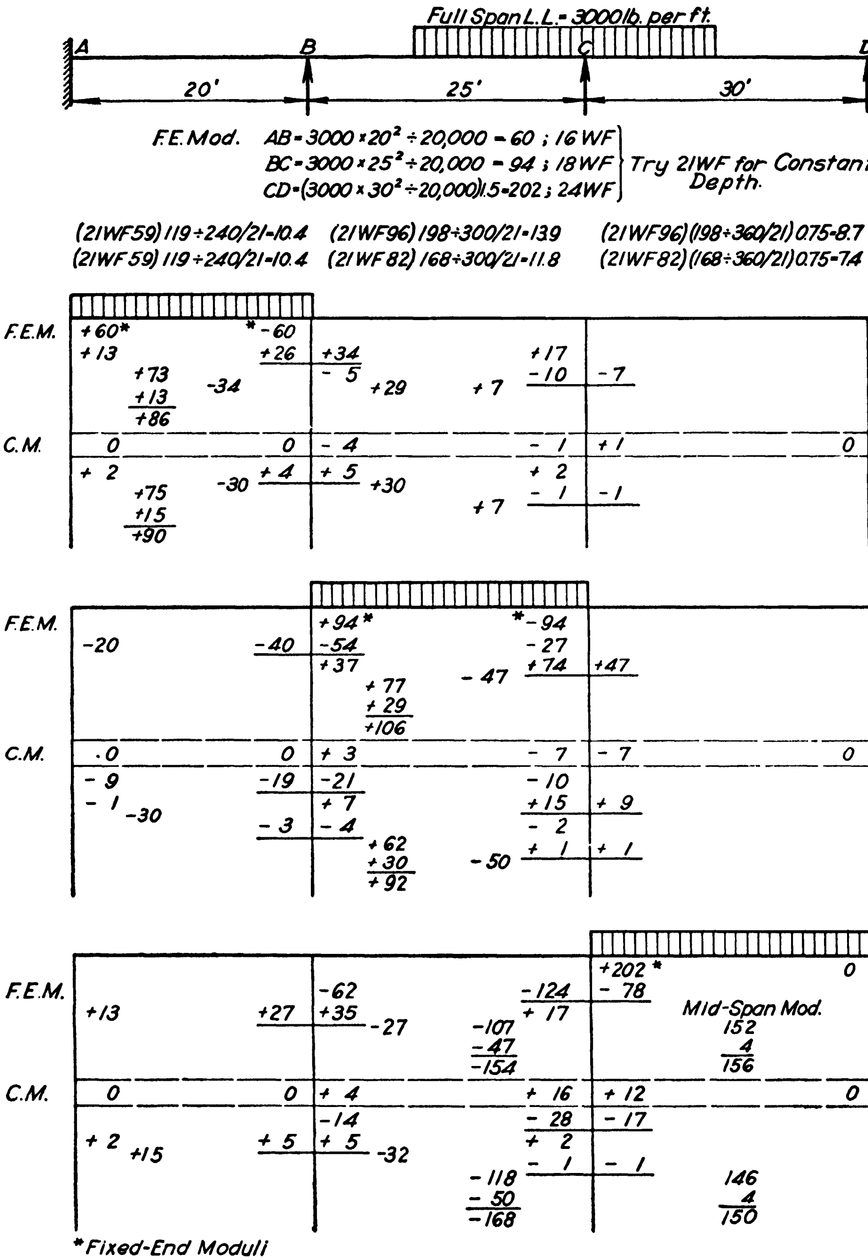


FIG. 240. DESIGN OF A CONTINUOUS BEAM FOR A UNIFORM LIVE LOAD.

be negative at A and it is therefore not considered. The mid-span modulus requirement was checked in the span CD. For a practical case, the dead load would also need to be considered. It could be handled on a fourth design sheet and it should be introduced into *all* controlling moduli irrespective of sign since the dead load always exists on all spans. The influence of a uniform dead load could also be found by summing the moduli requirements of an equal uniform live loading over each span.

Other Design Problems. The design process discussed here can be applied to reinforced concrete beams, to movable live load concentrations, and to continuous frames as well as continuous beams with haunches. These problems and others, such as simplified determinations of influence lines, have been discussed by the author in a book entitled *Automatic Design of Continuous Frames of Steel and Reinforced Concrete.* The process of automatic design can actually be applied advantageously to all structures composed of straight members (beams and columns). For example, its application to tall building design was worked out by W. M. Simpson as a thesis study under the writer's direction.*

* *Design of Tall Building Frames*, W. M. Simpson, Thesis for the degree of Doctor of Philosophy in Engineering, Illinois Institute of Technology, 1942.

CHAPTER 17

SPECIFICATIONS

AMERICAN INSTITUTE OF STEEL CONSTRUCTION

215. Abbreviated* *AISC* Specifications for Buildings

Loads and Stresses

1. Loads and Forces. Steel structures shall be designed to sustain the following loads and forces:

1. Dead Load.
2. Live Load.
3. Impact.
4. Wind and other Lateral Forces.
5. Erection Loads.
6. Other Forces.

2. Wind. Proper provision shall be made for stresses caused by wind both during erection and after completion of the building. The wind pressure is dependent upon the conditions of exposure and geographical location of the structure. The allowable stresses specified in Spec. 6 and Spec. 7 are based upon the steel frame being designed to carry a wind pressure of not less than 20 lb. per sq. ft. on the vertical projection of exposed surfaces during erection, and 15 lb. per sq. ft. on the vertical projection of the finished structure.

3. Reversal of Stress. Members subject to live loads producing alternating tensile and compressive stresses shall be proportioned as follows:

To the net total compressive and tensile stresses add 50 per cent of the smaller of the two and proportion the member to resist either of the increased stresses resulting therefrom.

Connections shall be proportioned to resist the larger of the two increased stresses.

4. Combined Stresses. Members subject to both axial and bending stresses shall be so proportioned that the quantity

$$\frac{f_a}{F_a} + \frac{f_b}{F_b} \text{ shall not exceed } \textit{unity}.$$

F_a = axial unit stress that would be permitted by this Specification if axial stress only existed.

F_b = bending unit stress that would be permitted by this Specification if bending stress only existed.

f_a = axial unit stress (actual) = axial stress divided by area of member.

f_b = bending unit stress (actual) = bending moment divided by section modulus of member.

5. Rivets. Rivets subject to shearing and tensile forces shall be so proportioned that the combined unit stress will not exceed the allowable unit stress for rivets in tension only.

* For complete specifications, consult the *AISC* Code.

6. Wind and Other Forces. Members subject to stresses produced by a combination of wind and other loads may be proportioned for unit stresses 33⅓ per cent greater than those specified in Spec. 10, provided the section thus required is not less than that required for the combination of dead load, live load, and impact (if any).

7. Wind Only. Members subject only to stresses produced by wind forces may be proportioned for unit stresses 33⅓ per cent greater than those specified in Spec. 10.

Effective Span Length

8. Simple Spans. Beams, girders, and trusses shall ordinarily be designed on the basis of simple spans whose effective length is equal to the distance between *centers of gravity* of the members to which they deliver their end reactions.

9. End Restraint. When designed on the assumption of end restraint, full or partial, based on continuous or cantilever action, beams, girders, and trusses, as well as the sections of the members to which they connect, shall be designed to carry the shears and moments so introduced, in addition to all other forces, without exceeding at any point the unit stresses prescribed in Spec. 10.

Allowable Unit Stresses

10. Structural and Rivet Steel. All parts of the structure shall be so proportioned that the unit stress in pounds per square inch shall not exceed the following:

Tension.

Structural steel, net section 20,000
Rivets, on area based on nominal diameter......................... 15,000

Compression.

Columns, gross section:
For axially loaded columns with values of L/r not greater than 120.. $17{,}000 - 0.485\dfrac{L^2}{r^2}$

For axially loaded columns with values of L/r greater than 120... $\dfrac{18{,}000}{1 + \dfrac{L^2}{18{,}000r^2}}$

in which L is the unbraced length of the column, and r is the corresponding radius of gyration of the section, both in inches.

Plate girder stiffeners, gross section 20,000
Webs of rolled sections at toe of fillet [Crimpling, see Spec. 46] 24,000

Flexure.

Tension on extreme fibers of rolled sections, plate girders, and built-up members.
[See Spec. 41] 20,000

Compression on extreme fibers of rolled sections, plate girders, and built-up members, for values of L/b not greater than 40.... $\dfrac{22{,}500}{1 + \dfrac{L^2}{1800b^2}}$

(with a maximum of 20,000) in which L is the laterally unsupported length of the member, and b is the width of the compression flange, both in inches.

Stress on extreme fibers of pins.................................. 30,000

Shear.

Rivets.. 15,000
Pins, and turned bolts in reamed or drilled holes.................. 15,000
Unfinished bolts.. 10,000
Webs of beams and plate girders, gross section..................... 13,000

Bearing.	Double Shear	Single Shear
Rivets	40,000	32,000
Turned bolts in reamed or drilled holes	40,000	32,000
Unfinished bolts	25,000	20,000

Pins	32,000
Contact area	
Milled stiffeners and other milled surfaces	30,000
Fitted stiffeners	27,000
Expansion rollers and rockers (pounds per linear inch)	$600d$

in which d is diameter of roller or rocker in inches.

11. Cast Steel. Compression and bearing same as for structural steel. Other unit stresses, 75 per cent of those for structural steel.

12. Masonry [Bearing]

Granite	800
Sandstone and limestone	400
Concrete, unless otherwise specified	600
Hard brick in cement mortar	250

Design

13. Slenderness Ratio. The ratio of unbraced length to least radius of gyration $\frac{L}{r}$ for compression members shall not exceed:

For main compression members	120
For bracing and other secondary members in compression	200

14. Unsupported Compression Flanges. The ratio of *unbraced length* to width of flange L/b for compression flanges of rolled sections, plate girders, and built-up members subject to bending, shall not exceed 40.

Minimum Thickness of Material

15. Main Material. The minimum thickness of steel except for linings, fillers, and the webs of rolled beams and channels, shall be: for *exterior* construction — $\frac{5}{16}$ in.; for *interior* construction — $\frac{1}{4}$ in. (These provisions do not apply to light structures such as skylights, marquees, fire-escapes, light one-story buildings, or light miscellaneous steelwork.)

16. Gusset Plates. Gusset plates for trusses with end reactions greater than 35,000 lb. shall be not less than $\frac{3}{8}$ in. thick.

17. Angles. The widths of the *outstanding legs* of angles in compression (except where reinforced by plates) shall not exceed 12 times the thickness for girder flanges and 16 times the thickness for other members.

18. Compression Members. In compression members consisting of segments connected by cover plates or lacing, or segments connected by webs, the thickness of the webs of the segments shall be not less than $\frac{1}{32}$ of the unsupported distance between the nearest rivet lines, or the roots of the flanges in case of rolled sections. The thickness of the cover plates or webs connecting the segments shall be not less than $\frac{1}{40}$ of the unsupported distance between the nearest lines of their connecting rivets, or the roots of their flanges in case of rolled sections.

Gross and Net Sections

19. Net Width. In the case of a chain of holes extending across a part in any *diagonal or zigzag line*, the net width of the part shall be obtained by deducting from the gross width the sum of the diameters of all the holes in the chain, and adding, for each gage space in the chain, the quantity $\frac{s^2}{4g}$.

s = longitudinal spacing (pitch) in inches of any two successive holes.

g = transverse spacing (gage) in inches of the same two holes.

The critical net section of the part is obtained from that chain which gives the least net width.

20. Angles. For angles, the gross width shall be the sum of the widths of the legs less the thickness. The gage for holes in opposite legs shall be the sum of the gages from back of angle less the thickness.

21. Size of Holes. In computing net area, the diameter of a rivet hole shall be taken as 1/8 in. greater than the nominal diameter of the rivet.

22. Pin Holes. In pin connected tension members, the net section across the pin hole, transverse to the axis of the member, shall be not less than 140 per cent, and the net section beyond the pin hole, parallel with the axis of the member, not less than 100 per cent of the net section of the body of the member.

In all pin connected riveted members, the net width across the pin hole, transverse to the axis of the member, shall preferably not exceed 12 times the thickness of the member at the pin.

Riveted Connections

23. Minimum Connections. Connections carrying calculated stresses, except for lacing, sag bars, and girts, shall have not fewer than *2 rivets*.

24. Eccentric Connections. Members meeting at a point shall have their gravity axes meet at a point if practicable; if not, provision shall be made for their eccentricity.

25. Rivets. The rivets at the ends of any member transmitting stresses into that member should preferably have their centers of gravity on the gravity axis of the member; otherwise, provision shall be made for the effect of the resulting eccentricity. Pins may be so placed as to counteract the effect of bending due to dead load.

26. Unrestrained Members. When beams, girders, or trusses are designed on the basis of simple spans in accordance with Spec. 8, their end connections may ordinarily be designed for the reaction shears only. If, however, the eccentricity of the connection is excessive, provision shall be made for the resulting moment.

27. Fillers. In truss construction when rivets carrying computed stress pass through fillers, the fillers shall be extended beyond the connected member and the extension secured by sufficient rivets to develop the stress in the filler.

28. Splices. Compression members when faced for bearing shall be spliced sufficiently to hold the connecting members accurately in place. Other joints in riveted work, whether in tension or compression, shall be spliced so as to transfer the stress to which the member is subject.

29. Diameter. In proportioning and spacing rivets, the nominal diameter of the undriven rivet shall be used.

30. Long Grips. Rivets carrying calculated stress, and whose grip exceeds five diameters, shall have their number increased 1 per cent for each additional 1/16 in. of the rivet grip.

31. Use of Unfinished Bolts. All field connections may be made with unfinished bolts, except as provided in Spec. 32.

32. Use of Rivets. Rivets shall be used for the following connections:

Column splices in all tier structures 200 ft. or more in height above curb.

Column splices in tier structures 100 to 200 ft. in height, if the least horizontal dimension is less than 40 per cent of the height.

Column splices in tier structures less than 100 ft. in height, if the least horizontal dimension is less than 25 per cent of the height.

Connections of all beams and girders to columns, and of any other beams and girders on which the bracing of columns is dependent, in structures over 125 ft. in height.

Roof-truss splices and connections of trusses to columns, column splices, column bracing, and crane supports, in all structures carrying cranes of over 5-ton capacity.

Connections for supports of running machinery, or of other live loads which produce impact or reversal.

Any other connections stipulated on the design plans.

33. Use of Turned Bolts. Turned bolts in reamed or drilled holes may be used in shop or field work where it is impossible to drive satisfactory rivets. The finished shank shall be long enough to provide full bearing, and washers shall be used under the nuts to give full grip when the nuts are turned tight.

34. Minimum Pitch. The preferable minimum distance between centers of rivet holes shall be not less than 4½ in. for 1¼-in. rivets, 4 in. for 1⅛-in. rivets, 3½ in. for 1-in. rivets, 3 in. for ⅞-in. rivets, 2½ in. for ¾-in. rivets, 2 in. for ⅝-in. rivets, and 1¾ in. for ½-in. rivets, but in no case shall it be less than 3 times the diameter of the rivet.

35. Maximum Pitch for Compression Members. The maximum pitch in the line of stress of compression members composed of plates and shapes shall not exceed 16 times the thickness of the thinnest outside plate or shape, nor 20 times the thinnest enclosed plate or shape with a maximum of 12 in., and at right angles to the direction of stress the distance between lines of rivets shall not exceed 30 times the thickness of the thinnest plate or shape. For angles in built sections with two gage lines, with rivets staggered, the maximum pitch in the line of stress in each gage line shall not exceed 24 times the thickness of the thinnest plate with a maximum of 18 in.

36. End Pitch for Compression Members. The pitch of rivets at the ends of built compression members shall not exceed 4 diameters of the rivets for a length equal to 1½ times the maximum width of the member.

37. Two-Angle Members. In tension members composed of *2 angles*, a pitch of 3 ft.-6 in. will be allowed, and in compression members, 2 ft.-0 in., but the ratio L/r for each angle between rivets shall be not more than ¾ of that for the whole member.

38. Minimum Edge Distance. The minimum distance from the center of any punched rivet hole to any edge shall be that given in the table.

Rivet Diameter, Inches	Minimum Edge Distance (Inches) for Punched Holes		
	In Sheared Edge	In Rolled Edge of Plates and Sections with Parallel Flanges	In Rolled Edge of Sections with Sloping Flanges
½	1	⅞	¾[a]
⅝	1⅛	1	⅞[a]
¾	1¼	1⅛	1[a]
⅞	1½	1¼	1⅛[a]
1	1¾	1½	1¼[a]
1⅛	2	1¾	1½[a]
1¼	2¼	2	1¾[a]

[a] May be decreased ⅛ in. when holes are near end of beam.

39. Minimum Edge Distance in Line of Stress. The distance from the center of any rivet under computed stress, and that end or other boundary of the connected member toward which the pressure of the rivet is directed, shall be not less than the shearing área of the rivet shank (single or double shear respectively) divided by the plate thickness.

This end distance may however be decreased in such proportion as the stress per rivet is less than that permitted under Spec. 10; and the requirement may be disregarded in case the rivet in question is one of three or more in a line parallel to the direction of stress.

40. Maximum Edge Distance. The maximum distance from the center of any rivet to the near edge shall be 12 times the thickness of the plate, but shall not exceed 6 in.

Plate Girders and Rolled Beams

41. Proportioning. Riveted plate girders, cover-plated beams, and rolled beams shall in general be proportioned by the moment of inertia of the gross section. No deduction shall be made for standard shop or field rivet holes in either flange; except that in special cases where the reduction of the area of either flange by such rivet holes, calculated in accordance with the provisions of Spec. 19, exceeds 15 per cent of the gross flange area, the excess shall be deducted. If such members contain other holes, as for bolts, pins, or countersunk rivets, the full deduction for such holes shall be made. The deductions thus applicable to either flange shall be made also for the opposite flange if the corresponding holes are there present.

42. Web. Plate-girder webs shall have a thickness of not less than $\frac{1}{170}$ of the unsupported distance between flanges.

43. Flanges. Cover plates, when required, shall be equal in thickness or shall diminish in thickness from the flange angles outward. No plate shall be thicker than the flange angles.

Unstiffened cover plates shall not extend more than 6 in. nor more than 12 times the thickness of the thinnest plate beyond the outer row of rivets connecting them to the angles.

The total cross-sectional area of cover plates shall not exceed 70 per cent of the total flange area.

44. Rivets. Rivets connecting the flanges to the web shall be proportioned to resist the horizontal shear due to bending as well as any loads applied directly to the flange.

45. Stiffeners. Stiffeners shall be placed on the webs of plate girders at the ends and at points of concentrated loads. Such stiffeners shall have a close bearing against the flanges, shall extend as closely as possible to the edge of the flange angles, and shall not be crimped. They shall be connected to the web by enough rivets to transmit the stress. Only that portion of the outstanding legs outside of the fillets of the flange angles shall be considered effective in bearing.

If h/t is equal to or greater than 70, intermediate stiffeners shall be required at all points where h/t exceeds $\dfrac{8000}{\sqrt{v}}$.

h = the clear depth between flanges, in inches.

t = the thickness of the web, in inches.

v = greatest unit shear in panel, in pounds per square inch under any condition of complete or partial loading.

The clear distance between intermediate stiffeners, when stiffeners are required by the foregoing, shall not exceed 84 in. or that given by the formula

$$d = \frac{270{,}000t}{v}\sqrt[3]{\frac{vt}{h}}.$$

d = the clear distance between stiffeners, in inches.

Intermediate stiffeners may be crimped over the flange angles.

Plate girder stiffeners shall be in pairs, one on each side of the web, and shall be connected to the web by rivets spaced not more than 8 times their nominal diameter.

46. Web Crimpling of Beams. Rolled beams shall be so proportioned that the compression stress at the web toe of the fillets, resulting from concentrated loads, shall not exceed the value of 24,000 lb. per sq. in. allowed in Spec. 10. Governing formulas shall be:

$$\text{For interior loads, } \frac{R}{t(N + 2k)} = \text{not over } 24{,}000,$$

$$\text{For end reactions, } \frac{R}{t(N + k)} = \text{not over } 24{,}000.$$

R = concentrated interior load or end reaction, in pounds.
t = thickness of web, in inches.
N = length of bearing, in inches.
k = distance from outer face of flange to web toe of fillet, in inches.

Tie Plates and Lacing

47. Compression Members. The open sides of compression members shall be provided with *lacing* having *tie plates at each end*, and at intermediate points if the lacing is interrupted. Tie plates shall be as near the ends as practicable. In main members carrying calculated stresses, the end tie plates shall have a length of not less than the distance between the lines of rivets connecting them to the segments of the member, and intermediate ones of not less than ½ of this distance. The thickness of tie plates shall be not less than 1/50 of the distance between the lines of rivets connecting them to the segments of the members, and the rivet pitch shall be not more than 4 diameters. Tie plates shall be connected to each segment by at least 3 rivets.

48. Tension Members. *Tie plates* shall be used to secure the parts of tension members composed of shapes. They shall have a length not less than ⅔ of the length specified for tie plates in compression members. The thickness shall be not less than 1/50 of the distance between the lines of rivets connecting them to the segments of the member and they shall be connected to each segment by at least 3 rivets.

49. Spacing. Lacing bars of compression members shall be so spaced that the ratio L/r of the flange included between their connections shall be not over ¾ of that of the member as a whole.

50. Proportioning. Lacing bars shall be proportioned to resist a shearing stress normal to the axis of the member equal to *2 per cent* of the total compressive stress in the member. In determining the section required, the compression formula shall be used, L being taken as the length of the bar between the outside rivets connecting it to the segment for single lacing and 70 per cent of that distance for double lacing. The ratio L/r shall not exceed 140 for single lacing nor 200 for double lacing.

51. Minimum Proportions. The *thickness of lacing bars* shall be not less than 1/40 for single lacing, and 1/60 for double lacing, of the distance between end rivets; their minimum *width* shall be 3 times the diameter of the rivets connecting them to the segments.

52. Inclination. The inclination of lacing bars to the axis of the members shall preferably be not less than 45° for double lacing and 60° for single lacing. When the distance between the rivet lines in the flanges is more than 15 in., the lacing shall be double and riveted at the intersection if bars are used, or else shall be made of angles.

53. Initial Stress. The total initial stress in any adjustable member shall be assumed as not less than 5000 lb

AMERICAN ASSOCIATION OF STATE HIGHWAY OFFICIALS

216. Abbreviated* *AASHO* Specifications for Highway Bridges

LOADS AND FORCES

54. Loads. Structures shall be proportioned for the following loads and forces:

(*a*) Dead load.
(*b*) Live load.
(*c*) Impact or dynamic effect of the live load.
(*d*) Lateral forces.
(*e*) Other forces, when they exist, as follows:

Longitudinal force, centrifugal force, and thermal forces.

55. Snow and Ice. The snow and ice load is considered to be offset by an accompanying decrease in live load and impact and shall not be included except under special conditions.

56. Highway Loadings. The highway loading shall be of three classes, namely, *H*-20, *H*-15, and *H*-10. Loadings *H*-15 and *H*-10 are 75 per cent and 50 per cent, respec-

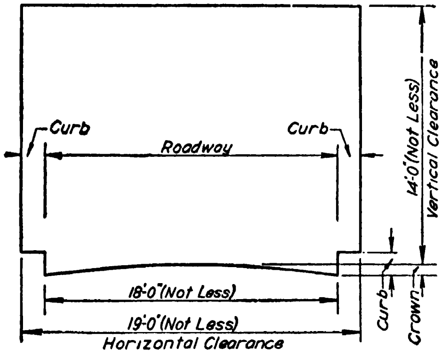


FIG. 241. CLEARANCE DIAGRAM.

tively, of loading *H*-20. These loadings shall consist of either truck trains or equivalent loadings as described below. (See Figs. 242–244.)

57. Truck Train Loadings. The truck train loadings shall be as shown in Fig. 243 and shall be used for loaded lengths of less than 60 ft., but may be used for greater loaded lengths. The loaded length for transverse members such as floor beams shall be considered as the combined lengths of the adjacent panels.

The trucks in adjacent lanes shall be considered as headed in the same direction.

* For complete specifications, the 1935 *AASHO* Code should be consulted. See p. 418 for 1941 specifications.

58. Equivalent Loadings. The equivalent loadings shall be as shown in Fig. 244, and shall be used only for loaded lengths of 60 ft. or greater. Each *lane loading* shall consist of a uniform load per linear foot of traffic lane combined with a single concentrated load so placed on the span as to produce maximum stress. The concentrated load shall be considered as uniformly distributed across the lane on a line normal to the center line of the lane. For the computation of moments and shears, different concentrated loads shall be used as indicated in Fig. 244. The *lighter* concentrated load shall be used in computing the stresses in members in which the greater part of the stress is produced by bending moments. The *heavier* concentrated load shall be used when the greater part of the stress in a member is produced by shearing forces.

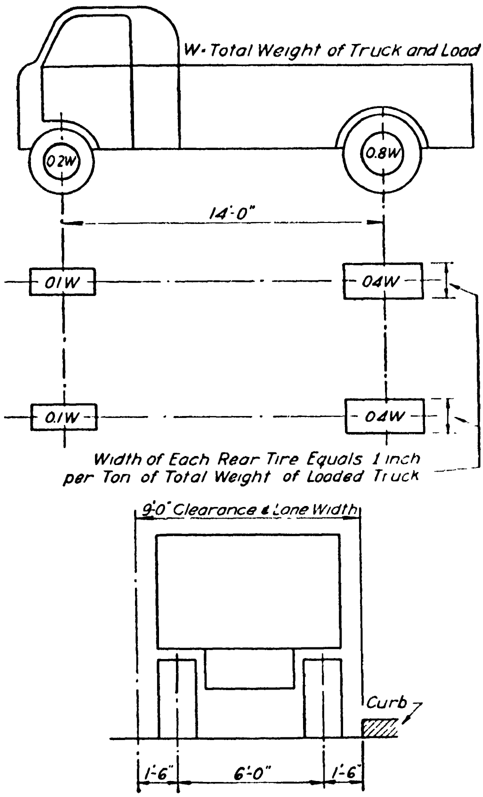


FIG. 242. STANDARD TRUCK LOADING.

59. Application of Loadings. The loadings shall be applied by that one of the following methods which produces the greater maximum stress in the member considered, due allowance being made for the reduced load intensities hereinafter specified for roadways having loaded widths in excess of 18 ft.

(1) Each traffic lane loading shall be considered as a unit, and the number and position of the loaded lanes shall be such as will produce maximum stress.

(2) The roadway shall be considered as loaded over its entire width with a load per foot of width equal to $\frac{1}{9}$ of the load of one traffic lane. This shall apply to both uniform and concentrated loads.

60. Reduction in Load Intensity. If the loaded width of the roadway exceeds the two lane width of 18 ft., the specified loads shall be reduced 1 per cent for each foot of loaded roadway width in excess of 18 ft. with a maximum reduction of 25 per cent, corresponding to a loaded roadway width of 43 ft. If the loads are lane loads, the loaded width of the roadway shall be the aggregate width of the lanes considered; if the loads are distributed over the entire width of the roadway, the loaded width of the roadway shall be the full width of roadway between curbs.

61. Sidewalk Loading. Sidewalk floors, stringers, and their immediate supports shall be designed for a live load of not less than 100 lb. per sq. ft. of sidewalk area.

62. Impact. Live load stresses, except those due to sidewalk loads and centrifugal,

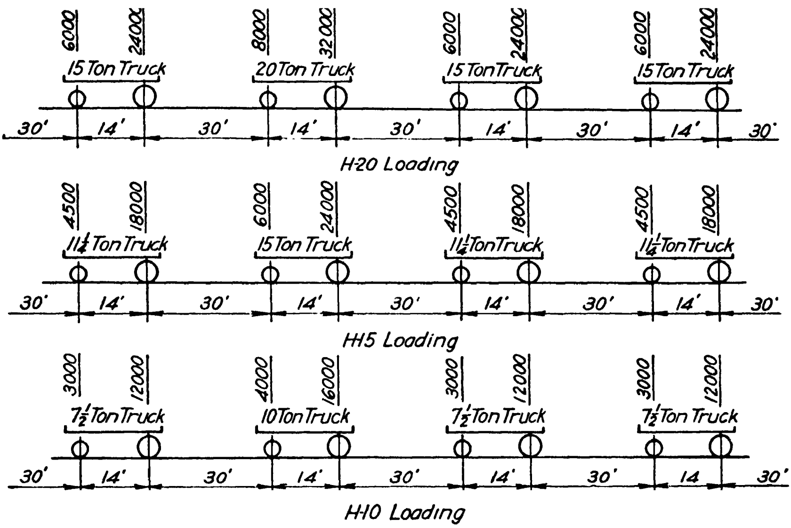


FIG. 243. HIGHWAY BRIDGE LOADINGS.

tractive, and wind forces, shall be increased by an allowance for *dynamic vibratory and impact effects*, provided, however, that this impact allowance shall not be applied to stresses in *timber* since the working stresses for timber given in these specifications are chosen sufficiently low to allow for impact effects.

The amount of this allowance or increment is expressed as a fraction of the live load stress, and for both electric railway and highway loadings shall be determined by the formula:

$$I = \frac{50}{L + 125}.$$

I = impact fraction.
L = the length in feet of the portion of the span which is loaded to produce the maximum stress in the member considered.

63. Longitudinal Force. Provision shall be made for the effect of a longitudinal force of 10 per cent of the live load on the structure, acting 4 ft. above the floor.

64. Lateral Forces. (*a*) The wind force on the structure shall be assumed as a moving horizontal load equal to 30 lb. per sq. ft. on $1\frac{1}{2}$ times the area of the structure as seen in elevation, including the floor system and railings and on $\frac{1}{2}$ the area of all trusses or girders in excess of two in the span.

(*b*) The lateral force due to the moving live load and the wind force against this load shall be considered as acting 6 ft. above the roadway and shall be as follows:

Highway bridges, 200 lb. per linear ft.

Highway bridges carrying electric railway traffic, 300 lb. per linear ft.

(*c*) The total assumed wind force shall be not less than 300 lb. per linear ft. in the plane of the loaded chord and 150 lb. per linear ft. in the plane of the unloaded chord on truss spans, and not less than 300 lb. per linear ft. on girder spans.

(*d*) In calculating the uplift, due to the foregoing lateral forces, in the posts and anchorages of viaduct towers, highway viaducts shall be considered as loaded on the leeward traffic lane with a uniform load of 400 lb. per linear ft. of lane, and viaducts carrying electric railway traffic in addition to highway traffic shall be considered as loaded on the leeward track with a uniform load of 800 lb. per linear ft. of track.

(*e*) A wind pressure of 50 lb. per sq. ft. on the unloaded structure, applied as specified above in paragraph (*a*), shall be used if it produces greater stress than the combined wind and lateral forces of paragraphs (*a*) and (*b*).

Distribution of Loads

65. Shear. In calculating end shears and end reactions in transverse floor beams and longitudinal beams and stringers, no lateral or longitudinal distribution of the wheel load shall be assumed.

66. Bending Moment in Stringers. In calculating bending moments in longitudinal beams or stringers, no longitudinal distribution of the wheel loads shall be assumed. The lateral distribution shall be determined as follows:

(*a*) *Interior Stringers.* Interior stringers shall be proportioned for loads determined in accordance with the following table, except that when the limiting stringer spacings are exceeded, the stringer loads shall be determined by the reactions of the truck wheels, assuming the flooring between stringers to act as a simple beam.

Kind of Floor	Floor Designed for One Traffic Lane		Floor Designed for Two or More Traffic Lanes	
	Fraction of a Wheel Load to Each Stringer	Limiting Stringer Spacing, in Feet	Fraction of a Wheel Load to Each Stringer	Limiting Stringer Spacing, in Feet
Plank	$\frac{S}{4.0}$	4.0	$\frac{S}{3.5}$	5.0
Strip 4 in. in thickness or wood block on 4-in. plank subfloor	$\frac{S}{4.5}$	4.5	$\frac{S}{3.75}$	5.5
Strip 6 in. or more in thickness	$\frac{S}{5.0}$	5.0	$\frac{S}{4.0}$	6.0
Concrete	$\frac{S}{6.0}$	6.0	$\frac{S}{4.5}$	10.0

S = spacing of stringers in feet

(*b*) *Outside Stringers.* The live load supported by outside stringers shall be the reaction of the truck wheels, assuming the flooring to act as a simple beam between stringers.

(*c*) *Total Capacity of Stringers.* The combined load capacity of the beams in a panel shall not be less than the total live and dead load in the panel.

67. Bending Moment in Floor Beams. In calculating bending moments in floor beams no transverse distribution of the wheel loads shall be assumed.

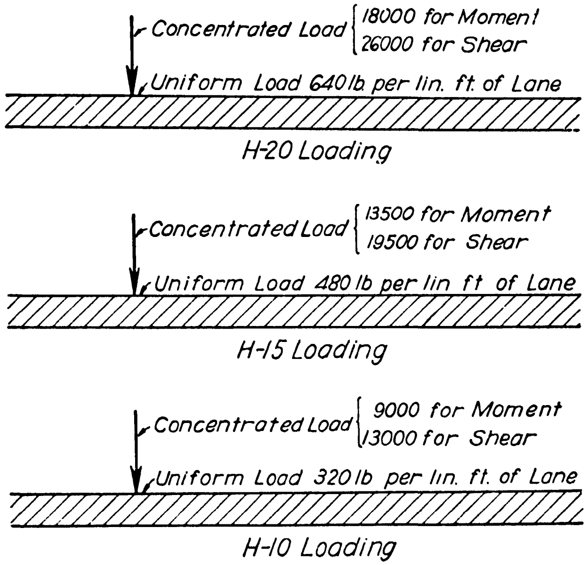


FIG. 244. SIMPLIFIED DESIGN LOADINGS.

If longitudinal stringers are omitted and the floor is supported directly on the floor beams, the latter shall be proportioned for a fraction of the wheel loads, as indicated in the following table, except that when the limiting floor beam spacing is exceeded the floor beam loads shall be determined by the reactions of the truck wheels, assuming the flooring between floor beams to act as a *simple beam*.

KIND OF FLOOR	FRACTION OF WHEEL LOADS TO EACH FLOOR BEAM	LIMITING FLOOR BEAM SPACING, IN FEET
Plank	$\frac{S}{4.0}$	4.0
Strip 4 in. in thickness or wood block on 4-in. plank subfloor	$\frac{S}{4.5}$	4.5
Strip 6 in. or more in thickness	$\frac{S}{5.0}$	5.0
Concrete	$\frac{S}{6.0}$	6.0

S = spacing of floor beams in feet

68. Distribution of Wheel Loads on Concrete Slabs.* *Bending Moment.* In calculating bending stresses due to wheel loads on floor slabs, no distribution in the direction of the span of the slab shall be assumed. In the direction perpendicular to the span of the slab, the wheel load shall be considered as distributed uniformly over a width of slab which is termed the "effective width." We define the following terms:

S = span of slab in feet.
W = width of tire with a maximum value of 1.25 ft.
E = effective width of slab in feet for one wheel load.

CASE I. *Single Load at Center of Span.*

$$E = 0.6S + 2W.$$

CASE II. *More Than One Load on the Same Element of Slab.*

In calculating the bending moment for more than one load on the same element of a slab, the loads shall be placed as in calculating the maximum moment for a simple beam. This process determines the number of loads that may occur on the element. The moment shall then be calculated for a single load at the center of the span as in Case I and increased for each additional load on the element as follows, where D = distance between the load nearest the center of the span and each additional load:

RATIO	INCREMENT
$\frac{D}{S} = 0$	100 per cent
$\frac{D}{S} = 0.1$	40 per cent
$\frac{D}{S} = 0.3$	15 per cent
$\frac{D}{S} = 0.6$	0 per cent

Increases for intermediate values of $\frac{D}{S}$ may be obtained by interpolation in the above table.

CASE III. *Loads on Parallel Elements of a Slab.*

The maximum bending moment shall be calculated as in Case I or Case II and increased by the following percentages where B = distance between the parallel loaded elements:

RATIO	INCREMENT
$\frac{B}{S} = 0$	100 per cent
$\frac{B}{S} = 0.1$	60 per cent
$\frac{B}{S} = 0.4$	30 per cent
$\frac{B}{S} = 1.0$	10 per cent
$\frac{B}{S} = 1.4$	0 per cent

* More exact methods of computing stresses in floor slabs due to concentrated loads may be found in "Public Roads" for March, 1930, in an article by H. M. Westergaard entitled *Computation of Stresses in Bridge Slabs Due to Wheel Loads.*

Increases for intermediate values of B/S may be obtained by interpolation in the above table.

The design assumptions of this article do not provide for the effect of loads near unsupported edges. Therefore, at the ends of the bridge and at intermediate points where the continuity of the slab is broken, the edges of the slab shall be supported by diaphragms or other suitable means.

Unit Stresses — Steel

69. General. Except as modified elsewhere in these specifications, the several parts of a steel or concrete structure shall be so proportioned that the unit stresses shall not exceed those given below.

Members subject to stresses produced by a *combination of dead load, live load and impact, and with either lateral or longitudinal forces*, or with bending due to lateral or longitudinal forces may be proportioned for unit stresses 25 per cent greater than those given below.

70. Structural Grade and Rivet Steel.

Tension.

Axial tension, structural members, net section 18,000
Bolts, area at root of thread 10,000

Axial Compression.

Axial compression, gross section, for values of L/r not greater than 140.

Riveted ends .... $15{,}000 - \frac{1}{4}\left(\frac{L}{r}\right)^2$

Pin ends .... $15{,}000 - \frac{1}{3}\left(\frac{L}{r}\right)^2$

L = length of members in inches
r = radius of gyration of member in inches
Compression splice material, gross section 18,000

Bending on Extreme Fiber.

Compression on flanges of beams and plate girders .... $18{,}000 - 5\left(\frac{L}{b}\right)^2$

L = length in inches of the unsupported flange between lateral connections or knee braces
b = flange width in inches
Tension in rolled shapes, built sections and girders, net section 18,000
Pins 27,000

Diagonal Tension.

In webs of girders and rolled beams, at sections where maximum shear and bending occur simultaneously 18,000

Shear.

Girder webs, gross section 11,000
Pins and shop driven rivets 13,500
Power driven field rivets and turned bolts 11,000
Hand driven rivets and unfinished bolts 9,000

Bearing.

Pins, steel parts in contact and shop driven rivets 27,000
Power driven rivets and turned bolts 22,500
Hand driven rivets and unfinished bolts 18,000
Expansion rollers, pounds per linear inch .... $600d$
d = diameter of roller in inches

In proportioning rivets the nominal diameter shall be used.

The effective bearing area of a pin, a bolt, or a rivet shall be its diameter multiplied by the thickness of the metal on which it bears.

In metal 3/8 in. thick and over, one half the depth of countersink shall be omitted in calculating bearing area. In metal less than 3/8 in. thick *countersunk rivets* shall not be assumed to carry stress.

71. Steel Castings. For steel castings, 3/4 of the unit stresses specified above for structural grade steel shall apply.

72. Cast Iron.

Bending on extreme fiber	3,000
Shear	3,000
Direct compression (short columns)	12,000

73. Bronze.

Bearing on bronze expansion plates	2,000

74. Bearing on Masonry.

Bearing on granite masonry	800
Bearing on sandstone and limestone masonry	400
Bearing on concrete	600

Concrete Structures

75. Concrete.

Direct Compression.	
Columns reinforced with longitudinal bars and separate lateral ties	$600 - 15\left(\frac{L}{D}\right)$
but not to exceed	450
Compressive stress due to bending	900
Tension	0
Shear (diagonal tension)	
Beams without shear reinforcement:	
Longitudinal bars not anchored	60
Longitudinal bars anchored	90
Beams with shear reinforcement and anchorage	160
Punching shear	160

76. Reinforcement.

Reinforcing Bars.	
Tension	16,000
Compression	10 times the compression in the surrounding concrete
Bond.	
Bars not anchored	80
Bars adequately anchored by hooks or otherwise	120

The above unit stresses are based upon the use of concrete having an ultimate compressive strength at 28 days of 3000 lb. per sq. in. For concrete having a smaller strength, the unit stresses shall be proportionately reduced.

For floor slabs, where it is particularly desired to lessen the dead weight of the floor, and for arch rings or ribs, the above unit stresses may be modified as follows:

For special mixes, designed for high strength, the compressive stress may be taken as 30 per cent of the ultimate strength of the concrete at 28 days for combined dead load, live load, temperature and shrinkage stresses. The extreme upper limit of strength which may be used in computing the working stress shall be 4500 lb. per sq. in.

Timber Structures

77. Structural Grades of Timber. The unit stresses for structural grades of timber set forth in § 75 of Chapter 5, or reduced where conditions of exposure in the structure require, shall be used with computed stresses for dead load, normal live load, and lateral forces. No allowance for impact shall be added to the normal live load stresses.

78. Formulas for the Computation of Stresses in Timber.

Axial Compression in Columns.

For $\frac{P}{A}$ greater than ⅔ S.

$$\frac{P}{A} = S\left[1 - \frac{1}{3}\left(\frac{L}{Kd}\right)^4\right].$$

P = total load in pounds.
A = area of cross section in square inches.
S = safe stress in compression parallel to grain for short columns.
L = unsupported length in inches.
d = least dimension in inches.
$K = \frac{\pi}{2}\sqrt{\frac{E}{6S}}$ for any species or grade.
E = modulus of elasticity.

For $\frac{P}{A}$ not greater than ⅔ S.

$$\frac{P}{A} = \frac{\pi^2}{36}\frac{E}{\left(\frac{L}{d}\right)^2}.$$

$\frac{L}{d}$ shall not exceed 50.

Bearing on Inclined Surfaces.

$$N = \frac{PQ}{P\sin^2\theta + Q\cos^2\theta}.$$

N = unit bearing on an inclined surface.
P = unit stress in compression parallel to the grain.
Q = unit stress in compression perpendicular to the grain.
θ = angle in degrees between the load and the direction of grain (Fig. 74).

79. Horizontal Shear in Beams. Horizontal shear in beams shall be computed from the maximum shear occurring at a distance from the support equal to *three times the depth* of the beam, or at the *quarter point* whichever would be the lesser distance from the support.

Foundations

80. Bearing Power of Soils. For the design of foundations, the following unit bearing values may be assumed in the absence of definite information as to the actual bearing power of the foundation in question. In this tabulation it is intended to cover only broad basic groups of materials and to specify for these a maximum range in bearing power. These groups may be further subdivided to provide for special conditions.

Material	Safe Bearing Power Tons per Square Foot Min.	Max.
Alluvial soils	½	1
Clays	1	4
Sand, confined	1	4
Gravel	2	4
Cemented sand and gravel	5	10
Rock	5	..

Structural Steel Design

81. Effective Span. For the calculation of stresses, span lengths shall be assumed as follows:

Beams and girders — distance between centers of bearings.
Trusses — distance between centers of end pins or of bearings.
Floor beams — distance between centers of trusses or girders.
Stringers — distance between centers of floor beams.

82. Alternative Stresses. Members subject to alternate stresses of tension and compression, due to the combination of dead, live, impact and centrifugal stresses, shall be proportioned for the kind of stress requiring the larger section.

If the alternate stresses occur in succession during one passage of the live load, each shall be increased by 50 per cent of the smaller. The connections of such members shall be proportioned for the sum of the net alternate stresses not so increased.

If the live load and dead load stresses are of opposite sign, only 70 per cent of the dead load stress shall be considered as effective in counteracting the live load stress.

83. Combined Stresses. Members subject to both *axial and bending stresses* shall be proportioned so that the combined fiber stresses will not exceed the specified axial unit stress. If members are continuous over panel points, ¾ of the bending stress, computed as for a simple beam, shall be added to the axial stress.

84. Limiting Lengths of Members. For compression members, the ratio of unsupported length to the least radius of gyration shall not exceed 120 for main and stiffening members nor 140 for laterals and sway bracing. In proportioning the top chords of half-through trusses, the unsupported length shall be assumed as the length between laterally supported panel points.

For main riveted tension members, the ratio of length to least radius of gyration shall not exceed 200.

85. Effective Area of Angles in Tension. The effective area of a *single angle tension member*, or of each angle of a double tension member in which the angles are connected back to back on the same side of a gusset plate, shall be assumed as the net area of the connected leg plus one half of the area of the unconnected leg.

If a *double angle tension member* is connected with the angles back to back on opposite sides of a gusset plate, the full net area of the angles shall be considered as effective. If the angles connect to separate gusset plates, as in the case of a double-webbed truss, and the angles are connected by stay plates located as near the gussets as practicable, or by other effective means, the full net area of the angles shall be considered as effective. If the angles are not so connected, only 80 per cent of the net area shall be considered as effective.

Lug angles shall not be considered as effective in transmitting stress.

86. Minimum Thickness of Metal. Gusset plates shall be not less than ⅜ in. in thickness. Other structural steel, except for fillers and in railings, shall be not less than 5/16 in. in thickness.

Metal subjected to marked corrosive influence shall be increased in thickness or specially protected against *corrosion.*

87. Plates in Compression. The thickness of *web plates* of compression members shall be not less than $\frac{1}{30}$ of the transverse distance between the lines of rivets connecting them to the flanges. The thickness of *cover plates* of compression members and cover plates on the compression flanges of plate girders, preferably, shall be not less than $\frac{1}{40}$ of the transverse distance between the lines of rivets connecting them to the flanges, but the minimum may be $\frac{1}{50}$ of this distance, provided that the width of the plate between the connecting lines of rivets in excess of 40 times the thickness shall not be considered as effective in resisting stress.

88. Outstanding Legs of Angles. The widths of the outstanding legs of angles in compression (except where reinforced by plates) shall not exceed the following:

In girder flanges, 12 times the thickness.
In main members carrying axial stress, 12 times the thickness.
In bracing and other secondary members, 16 times the thickness.

89. Size of Pins. Pins shall be proportioned for the maximum shears and bending moments produced by the stresses in the members connected. If there are eyebars among the parts connected, the diameter of the pin shall be not less than $\frac{3}{4}$ of the width of the widest bar.

Riveted Connections

90. Size of Rivets. Rivets shall be of the size shown on the drawings but generally shall be $\frac{3}{4}$ in. or $\frac{7}{8}$ in. in diameter. Rivets $\frac{5}{8}$ in. in diameter shall not be used in members carrying calculated stress except in $2\frac{1}{2}$-in. legs of angles and in flanges of 6-in. and 7-in. beams and channels.

The diameter of rivets in angles carrying calculated stress shall not exceed $\frac{1}{4}$ of the width of the leg in which they are driven.

In angles whose size is not determined by calculated stress, $\frac{5}{8}$-in. rivets may be used in 2-in. legs, $\frac{3}{4}$-in. rivets in $2\frac{1}{2}$-in. legs, $\frac{7}{8}$-in. rivets in 3-in. legs, and 1-in. rivets in $3\frac{1}{2}$-in. legs.

Structural shapes which do not admit the use of $\frac{5}{8}$-in. diameter rivets shall not be used except in hand rails.

91. Pitch of Rivets. The minimum distance between centers of rivets shall be three times the diameter of the rivet but, preferably, shall be not less than the following:

For 1-in. rivets, $3\frac{1}{2}$ in.
For $\frac{7}{8}$-in. rivets, 3 in.
For $\frac{3}{4}$-in. rivets, $2\frac{1}{2}$ in.
For $\frac{5}{8}$-in. rivets, $2\frac{1}{4}$ in.

92. Pitch in Ends of Compression Members. In the ends of compression members the pitch of rivets connecting the component parts of the member shall not exceed 4 times the diameter of the rivet for a length equal to $1\frac{1}{2}$ times the maximum width of the member. Beyond this point the pitch shall be increased gradually for a length equal to $1\frac{1}{2}$ times the maximum width of the member until the maximum pitch is reached.

93. Maximum Pitch. The maximum pitch *in the line of stress* shall not exceed 6 in. or 16 times the thickness of the thinnest outside plate or angle connected, except that in angles having two gage lines with the rivets staggered, the pitch in each line may be twice that given by these rules, with a maximum of 10 in.

94. Stitch Rivets. If two or more web plates are in contact, they shall be held together by stitch rivets. In compression members, the stitch rivets shall be spaced

in the direction perpendicular to the line of stress not more than 24 times the thickness of the thinnest plate, and, in the line of stress, not more than 12 times the thickness of the thinnest plate. In tension members and girders, the stitch rivets shall be spaced, in either direction, not more than 24 times the thickness of the thinnest outer plate. In tension members composed of two angles in contact, the angles shall be held together by stitch rivets having a maximum pitch of 12 in.

95. Edge Distance of Rivets. The minimum distance from the center of any rivet to a *sheared edge* shall be:

For 1-in. rivets, 1¾ in.
For ⅞-in. rivets, 1½ in.
For ¾-in. rivets, 1¼ in.
For ⅝-in. rivets, 1⅛ in.

The minimum distance from a *rolled or planed edge*, except in flanges of beams and channels, shall be:

For 1-in. rivets, 1½ in.
For ⅞-in. rivets, 1¼ in.
For ¾-in. rivets, 1⅛ in.
For ⅝-in. rivets, 1 in.

The maximum distance from any edge shall be 8 times the thickness of the thinnest outside plate, but shall not exceed 5 in.

96. Long Rivets. Rivets subjected to calculated stress and having a grip in excess of 4½ diameters shall be increased in number at least 1 per cent for each additional 1/16 in. of grip. If the grip exceeds 6 times the diameter of the rivet, specially designed rivets shall be used.

97. Rivets in Tension. Rivets in direct tension shall, in general, not be used, but if so used their value shall be one half that permitted for rivets in shear. *Countersunk* rivets shall not be used in tension.

98. Strength of Connections. Unless otherwise provided, connections shall be proportioned to develop the *full strengths* of the members connected.

Connections shall be made symmetrical about the axes of the members insofar as practicable. Connections, except for lacing bars and handrails, shall contain not less than 3 rivets.

99. Splices. Compression members, such as chords and trestle columns, in riveted structures shall have *milled ends* and full contact bearing at the splices.

Splices, whether in tension or compression, shall be proportioned to develop the full strength of the members spliced and no allowance shall be made for the *bearing of milled ends* of compression members.

Splices in riveted columns and chord members shall be located as close to panel points as possible and, usually, shall be on that side of the panel point where the smaller stress occurs.

100. Indirect Splices. If splice plates are not in direct contact with the parts which they connect, the number of rivets on each side of the joint shall be in excess of the number required for a direct contact splice to the extent of 2 extra transverse lines of rivets for each intervening plate.

101. Fillers. If rivets carrying stress pass through fillers, the fillers shall be extended beyond the connected member and the extension secured by enough additional rivets to carry the stress passing through the fillers. If the filler is less than ¼-in. thick it shall not be extended beyond the splicing material.

102. Gusset Plates. The gusset plates shall be of ample thickness to resist shear, direct stress, and flexure, acting on the weakest or critical section of maximum stress.

Re-entrant cuts shall be avoided as far as practicable.

Stay Plates and Lacing

103. Stay Plates. The open sides of *compression members* shall be provided with lacing bars and shall have stay plates as near each end as practicable. Stay plates shall be provided at intermediate points where the lacing is interrupted. In main members, the length of the end stay plates between end rivets shall be not less than 1¼ times the distance between the inner lines of rivets connecting them to the flanges; the length of intermediate stay plates between end rivets shall be not less than ¾ of that distance. In *lateral struts and other secondary members*, the over-all length of end and intermediate stay plates shall be not less than ¾ of the distance between the inner lines of rivets connecting them to the flanges.

The separate segments of *tension members* composed of shapes may be connected by stay plates or end stay plates and lacing. End stay plates shall have the same minimum length as specified for end stay plates on main compression members and intermediate stay plates shall have a minimum length of ¾ of that specified for intermediate stay plates on main compression members. The clear distance between stay plates on tension members shall not exceed 3 ft.

The *thickness* of stay plates shall be not less than 1/50 of the distance between the inner rivet lines connecting them to the flanges. Stay plates shall be connected by not less than 3 rivets on each side and in members having lacing bars, the last rivet in the stay plate, preferably, shall also pass through the end of the adjacent bar.

104. Lacing Bars. The lacing of compression members shall be proportioned to resist shearing stresses normal to the member not less than those calculated by the formula:

$$V = \frac{P}{100}\left[\frac{100}{\frac{L}{r} + 10} + \frac{\frac{L}{r}}{100}\right]$$

V = normal shearing stress in pounds.
P = allowable compressive axial load on member, in pounds.
L = length of member, in inches.
r = radius of gyration of section about the axis perpendicular to the plane of the lacing, in inches.

If the lacing of a horizontal or inclined compression member is in a vertical plane, the shear in the lacing caused by the weight of the member shall be added to the shear calculated by the formula above.

The shear shall be considered as divided equally among all shear resisting elements in parallel planes, whether made up of continuous plates or of lacing. The size of the bar shall be determined by the formula for axial compression in which " L " shall be taken as the distance between the connections to the main sections.

The minimum *width of lacing bars* shall be:

For 1-in. rivets, 2¾ in.
For ⅞-in. rivets, 2½ in.
For ¾-in. rivets, 2¼ in.
For ⅝-in. rivets, 2 in.

The minimum *thickness of bars* shall be 1/40 of the distance between connections for single lacing, and 1/60 for double lacing, but not less than 5/16 in.

Lacing bars of compression members shall be so spaced that the L/r of the portion of the flanges included between lacing-bar connections will be not greater than 40, and not greater than ⅔ of the L/r of the member.

The angle between the lacing bars and the axis of the member shall be approximately 45° for double lacing and 60° for single lacing. If the distance between rivet lines in the flanges is more than 15 in., and a bar with a single rivet in the connection is used, the lacing shall be double and riveted at the intersections. Lacing bars having at least *2 rivets* in each end shall be used on flanges 5 in. or more in width.

Shapes of equal strength may be used instead of flats.

Net Section and Expansion

105. Net Section at Pin Holes. In pin connected riveted tension members, the net section across the pin hole shall be not less than 140 per cent and the net section back of the pin hole not less than 100 per cent of the net section of the body of the member.

106. Net Section of Riveted Tension Members. In calculating the required section of riveted tension members, net sections shall be used in all cases, and, in deducting rivet holes, the holes shall be taken as $\frac{1}{8}$ in. larger than the nominal diameter of the rivet.

The net section shall be the least area which can be obtained by deducting from the gross sectional area, the area of holes cut by any *straight or zigzag section* across the member, counting the full area of the first hole and a fractional part of each succeeding hole the fractional part being determined by the formula:

$$X = 1 - \frac{S^2}{4gh}.$$

X = fraction of rivet hole to be deducted.
S = stagger or longitudinal spacing of rivet with respect to rivet on last gage line.
g = distance between gage lines, or transverse spacing.
h = diameter of rivet holes, or nominal diameter of rivet plus $\frac{1}{8}$ in.

107. Expansion. Provision shall be made for expansion and contraction at the rate of $1\frac{1}{4}$ in. for every 100 ft. The expansion ends shall be secured against lateral movement.

108. Expansion Bearings. Spans of less than 70 ft. may be arranged to slide upon metal plates with smooth surfaces. Spans of 70 ft. and greater shall be provided with rollers or rockers, or else with bronze sliding expansion bearings.

Floor System

109. Stiffness of Floor Members. Floor members shall be designed with special reference to stiffness by making them as deep as economy or the limiting under clearances will permit.

110. Stringers. Stringers, preferably, shall be riveted between the floor beams.

111. End Connection of Floor Beams and Stringers. The end connection angles of floor beams and stringers shall be not less than $\frac{3}{8}$ in. in finished thickness. Except in cases of special end floor-beam details, each end connection for floor beams and stringers shall be made with *2 angles*. The length of these angles shall be as great as the flanges will permit. Bracket or shelf angles which may be used to furnish support during erection shall not be considered in determining the number of rivets required to transmit end shear.

Bracing

112. General. Bracing shall be composed of angles or other shapes and the connections shall be riveted.

If a double system of bracing is used, both systems may be considered effective

simultaneously if the members meet the requirements both as tension and compression members. The members shall be riveted at their intersections.

113. Minimum Size of Angles. The smallest angle used in bracing shall be 3 by 2½ in. There shall be not less than *3 rivets* in each end connection of the angles.

114. Lateral Bracing. Bottom lateral bracing shall be provided in all spans except I-beam spans and deck plate girder spans of 50 ft. or less. Bottom laterals shall be supported at their intersections by rigid *hangers*, if necessary, to prevent excessive deflection.

Top lateral bracing shall be provided in deck spans, and in through spans having sufficient headroom.

The lateral bracing of compression chords, preferably, shall be as deep as the chords and effectively connected to both flanges.

115. Portal and Sway Bracing. Through truss spans shall have portal bracing, preferably, of the 2-plane or box type, rigidly connected to the end post and the top chord flanges, and as deep as the clearance will allow.

Through truss spans shall have sway bracing at each intermediate panel point if the height of the trusses is such as to permit a depth of 5 ft. or more for the bracing. When the height of the trusses will not permit of such depth, the top lateral struts shall be provided with *knee braces.* Top lateral struts shall be at least as deep as the top chord.

116. Half-Through Truss Spans. The vertical truss members and the floor-beam connections of half-through truss spans shall be proportioned to resist a *lateral force,* applied at the top chord panel points of the truss, determined by the following equation:

$$R = 150\ (A + P).$$

R = lateral force in pounds.
A = area of cross-section of the chord in square inches.
P = panel length in feet.

This rigidity may be secured in part by extending one or both of the floor-beam connection angles upward along the inside of the post and by providing a solid web in the post. If outrigger brackets are used, they shall be effectively connected to the floor beam.

Trusses

117. Chords and End Posts. Top chords and end posts usually shall be made of two side segments with one cover plate, and with stay plates and lacing on the open side. In chords of light section, stay plates and lacing may be used in place of the cover plate.

If the shape of the truss permits, compression chords shall be continuous. Top and bottom chord splices shall be as near the panel points as practicable and, preferably, on the side of the panel point where the smaller stress occurs.

The top chord sections of half-through truss spans shall be so proportioned that the radius of gyration about the vertical axis of the member will be at least 1½ times that about the horizontal axis.

118. Working Lines and Gravity Axes. In compression members of unsymmetrical section, such as chord sections formed of side segments and a cover plate, the gravity axis of the section shall coincide as nearly as practicable with the working line, except that eccentricity may be introduced to counteract dead load bending. In 2-angle bottom chord or diagonal members, the working line may be taken as the *gage line nearest the back of the angle.*

119. Camber. The length of the truss members shall be such that the camber will be equal to or greater than the deflection produced by the dead load plus the full live load

without impact. Ordinarily this will be accomplished by increasing the length of the top chord approximately 3/16 in. for each 10 ft. of its horizontal projection.

120. Diaphragms. There shall be diaphragms in the trusses at the end connections of floor beams.

The gusset plates engaging the pedestal pin at the end of the truss shall be connected by a diaphragm. Similarly, the webs of the pedestal shall, if practicable, be connected by a diaphragm.

There shall be a diaphragm between gusset plates engaging main members if the end tie plate is 4 ft. or more from the point of intersection of the members.

AASHO Specifications for 1941

Revised Specifications. The specifications given in § 216 are the 1935 Standard Specifications for Highway Bridges of the American Association of State Highway Officials. These specifications were revised in 1941. Since the 1935 specifications are quite simple, they are retained for use with illustrative design problems in the text. The 1941 specifications will be presented here simply by pointing out the major changes from those already given.

Clearances. The two-lane roadway has been increased in width to not less than 22 ft. and not less than 4 ft. more than the approach pavement.

***H-S* Loadings.** The standard *H*-loadings are retained, but a truck and trailer loading (*H-S* series) is considered to be optional for superhighways. This loading consists of three axles — two on the truck and one on the trailer. It provides another rear axle load 14 ft. behind the rear axle of the standard *H*-loading. The axle load of the trailer is identical with the rear axle load of the standard truck. For example, the *H*-20 truck has a front axle load of 8000 lb. and a rear axle load of 32,000 lb. The corresponding *H*-20;*S*-16 loading adds another 32,000 lb. load 14 ft. behind the rear axle of the *H*-20 truck.

Equivalent Loading. The uniform load with a single floating concentration (selected for shear or moment) is used either for the *H*-loading or for the *H-S* loading. For the *H*-20 loading, the equivalent uniform load is 640 lb. per ft. of lane, the floating concentration being 18,000 lb. for moment or 26,000 lb. for shear. For the *H*-20;*S*-16 loading, the uniform load remains 640 lb. per ft. of lane but the concentration is increased to 32,000 lb. for moment or 40,000 lb. for shear. The *H*-15;*S*-12 loading is 75 per cent as great.

Application of Loadings. The lane width is set at 10 ft. and fractional lane loadings are no longer to be considered. The number and position of the loaded lanes are to be chosen to produce a maximum stress. This specification is much improved over Spec. 59 and Spec. 60.

Load Distribution to Beams. The load distributed to an interior stringer (Spec. 66) is reduced somewhat. If S is the stringer spacing in feet, the part of one front and one rear wheel (*H*-loading) considered to act as loads on one stringer are as follows: $S/3.75$ for plank floors; $S/4.0$ for wood-strip floor 4 in. thick; $S/5.0$ for concrete floor; $S/4.0$ for steel-grid floor less than 4 in. thick; $S/5.0$ for steel-grid floor more than 4 in. thick These ratios apply to bridges of two or more traffic lanes.

Allowable Stresses. There are no basic changes in allowable stresses except that the distinction between shop and field rivets is dropped. All power driven rivets are given the old shop rivet values of 13,500 lb. per sq. in. for shear and 27,000 lb. per sq. in. for bearing. High strength rivets (*ASTM-A*195 rivet steel) are permitted at unit stresses of 20,000 lb. per sq. in. for shear and 40,000 lb. per sq. in. for bearing.

Secondary Tension Members. The slenderness ratio or L/r value of secondary tension members (not eye bars or rods) is limited to 240.

Lug Angles. Lug angles may now be considered effective in transmitting stress if they are connected by each leg with at least $\frac{1}{3}$ more rivets than are required by the stress to be carried through the lug angle.

Thickness of Metal. The web thickness of rolled shapes is limited to a minimum value of 0.23 in.

Plates in Compression. Webs must not be less than $\frac{1}{32}$ of the unsupported distance between the nearest rivet lines or the roots of rolled flanges. This ratio was $\frac{1}{30}$ in Spec. 87.

Strength of Connections. Instead of a requirement that the connections shall develop the full strength of the member (Spec. 98), the new requirement is that connections shall be designed for the average of the calculated stress and the strength of the member. However, they shall not be designed for less than 75 per cent of the strength of the member. The same change applies to splices.

Gusset Plates. If the length of the unsupported edge of a gusset plate exceeds 60 times its thickness, it shall be stiffened.

Camber. The lengths of truss members shall be such that the camber will be equal to or greater than the deflection produced by dead load. This differs from Spec. 119 which required additional camber for live loading.

Concrete and Timber Parts. The revised code reflects changes in specifications for these materials.

AMERICAN WELDING SOCIETY

217. Abbreviated* *AWS* Code for Fusion Welding of Buildings and Bridges

Building Construction

121. General. Fusion welding may be substituted for or used in combination with riveting, bolting or other connecting means specified in the Building Code,† for connecting to one another or assembling the component parts of steel beams, girders, lintels, trusses, columns and other structural steel members of buildings, or for connecting steel to wrought-iron members of existing buildings, provided that such work be designed and executed in accordance with this Code.

122. Definitions. (*a*) *Fusion Welding.* The process of joining metal parts in the molten (or molten and vapor) state, without application of mechanical pressure or blows. Under this code, fusion welding is restricted to the electric-arc and gas-welding processes.

(*b*) *Root.* The zone at the bottom (or base) of the cross-sectional space provided to contain a fusion weld.

(*c*) *Throat.* The minimum thickness of a weld along a straight line passing through the root.

(*d*) *Throat Dimension.* The thickness of throat assumed for purposes of design. Under this code the throat dimension of a *fillet weld* is the altitude from the root to the opposite side of the largest isosceles triangle which can be constructed within the cross-section of the weld, the equal legs lying in the fused faces. In the case of a *butt weld,* the throat dimension is the thickness of the thinner of the parts joined.

(*e*) *Fillet Weld.* A weld of approximately triangular cross section, whose throat lies in a plane disposed (inclined) approximately 45° with respect to the surfaces of the parts joined. The size of a fillet weld shall be expressed as the dimensions of the equal legs of the isosceles triangle described in paragraph (*d*).

(*f*) *Butt Weld.* A weld whose throat lies in a plane disposed approximately 90° with respect to the surfaces of at least one of the parts joined. The size of a butt weld shall be expressed as the throat dimension.

(*g*) *Weld Length.* The unbroken over-all length of the full cross section of the weld exclusive of the length of any craters. Under this Code the length of the full cross section is termed the *effective length,* and shall be used in all specifications, calculations and drawings. In determining the effective length of a fillet weld, ¼ in. shall be deducted from the over-all length of the weld as an allowance for the rounded ends and the crater.

Materials

123. Base Metal. Structural steel to be welded under this code shall conform to Serial Designation *A*-9 (Steel for Buildings) of the current Standard Specifications of the American Society for Testing Materials.

* For complete specifications, consult the 1938 *AWS* Code.

† *Building Code,* wherever the expression occurs in this Code, refers to building law or specifications or other construction regulations in conjunction with which this Code is applied.

Tensile Requirements for Deposited Metal

Grade	Base Metal	Treatment of Welded Specimen[c]	All-Weld Tension Test: Tensile strength, min., lb. per sq. in.	All-Weld Tension Test: Elongation in 2 in., min., per cent
No. 2	*ASTM A*-70 firebox quality steel plate,[a] tensile strength 55,000 lb. per sq. in., min.; or *ASTM A*-7 steel for bridges,[b] tensile strength 60,000 lb. per sq. in., min., with carbon not over 0.25 per cent and manganese not over 1.00 per cent; or equivalent steels.	stress-relieved non-stress-relieved	80,000 80,000	20[d] 15[d]
No. 4	same as for Grade No. 2	stress-relieved non-stress-relieved	75,000 75,000	20[d] 15[d]
No. 10	same as for Grade No. 2	stress-relieved non-stress-relieved	60,000 60,000	25 20
No. 15	same as for Grade No. 2	stress-relieved non-stress-relieved	60,000 60,000	20 17
No. 20	same as for Grade No. 2	stress-relieved non-stress-relieved	52,000 55,000	10 7
No. 30	same as for Grade No. 2	stress-relieved non-stress-relieved	45,000 47,000	7 5
No. 40	same as for Grade No. 2	stress-relieved non-stress-relieved	45,000 47,000	5 3

[a] See *Standard Specifications for Carbon-Steel Plates for Stationary Boilers and Other Pressure Vessels* (*ASTM* Designation: A-70) of the American Society for Testing Materials, 1936 Book of *ASTM* Standards, Part I, p. 56.

[b] See *Standard Specifications for Steel for Bridges* (*ASTM* Designation: *A*-7) of the American Society for Testing Materials, 1936 Book of *ASTM* Standards, Part I, p. 1.

[c] Stress-relieving where called for in these specifications is for the purpose of developing the fundamental properties of the weld metal unaltered by locked-up stress. Values obtained from stress-relieved welded specimens are about 5 per cent lower in tensile strength and 10 to 20 per cent higher in ductility than those of non-stress-relieved specimens. The fact that a filler metal test requires stress-relief signifies only that it must develop the strength required regardless of stress-relief, and not that stress-relief must always be used in actual work. Stress-relieving shall be within the range of 1100 to 1200° F. for 1 hr. per 1 in. of thickness. Gas welded specimens may be heat-treated at higher temperatures than the temperature specified above for stress-relief.

[d] It is immaterial how these high physical properties are obtained, that is, whether by carbon-steel filler metal with manganese and silicon contents or by the use of some other alloying elements, but the total amount of such other elements shall be less than 4 per cent.

124. Weld Metal. Filler metal (arc-welding electrodes and gas-welding rods) shall conform to all general requirements, and to all special requirements for at least one of the grades of filler metal, provided by Serial Designation *A*-205–37T (Iron and Steel Filler Metal), as amended to date, issued jointly by the American Society for Testing Materials and the American Welding Society. The table on page 421 gives strength and elongation requirements.

Permissible Unit Stresses

125. Allowable Unit Stresses. Welded joints shall be proportioned so that the stresses caused therein by loads specified in the Building Code shall not exceed the following values, expressed in kips (thousands of pounds) per square inch:

Allowable Stresses in Welds

Kind of Stress	For Welds Made with Filler Metal of	
	Grade 2, 4, 10 or 15	Grade 20, 30 or 40
Shear on section through weld throat	13.6	11.3
Tension on section through weld throat	15.6	13.0
Compression (crushing) on section through throat of butt weld	18.0	18.0

Fiber stresses due to bending shall not exceed the values prescribed above for tension and compression, respectively. Stress in a fillet weld shall be considered as *shear*, for any direction of the applied stress.

In designing welded joints, adequate provision shall be made for bending stresses due to eccentricity, if any, in the disposition or sections of base-metal parts.

Building Design

126. Plate Girders. Girders shall be proportioned either by their moments of inertia or by the flange-area method; in the latter method, when applied to a welded girder having no holes in the web, 1/6 of the web area may be considered a part of the area of *each* flange. Stiffeners may be either angles or flat plates, welded to the web and flanges by intermittent or continuous fillet welds designed to transmit the stresses. Connection of component parts of flanges to each other and of flanges to web shall be by means of intermittent or continuous fillet welds designed to transmit the stresses.

127. Beams. The use of continuous beams and girders, designed in accordance with accepted engineering principles, shall be permitted provided that their welded connections be designed to transmit the stresses involved in continuous beam construction. At the ends of non-continuous beams, the connections shall be designed to avoid excessive secondary stresses due to bending.

128. Columns. Adjacent component parts of a built-up column shall be joined by 2 or more lines of continuous or intermittent welding in the direction of stress, such lines to be not further apart than 30 times the thickness of metal in the thinner part. In any line of intermittent welding, the clear distance between welds shall nowhere be more than 12 in., or more than 16 times the thickness of metal in the thinner part, or more than 1 in. for each kip of designed strength in either adjoining weld. Fillet welding within a distance from either end of the column equal to the least width of column shall be

continuous. Sufficient weld strength shall be provided to transmit the shearing stresses between joined parts caused by flexure due to long-column action, applied bending moments, and any beam reactions or other loads tending to compress the parts unequally.

129. Butt Joints. One or both edges of base-metal parts to be joined by a butt weld shall be *beveled* if the throat dimension exceeds ¼ in., except that beveling may be omitted if the weld is to transmit only compressive stress and if the space between the parts be made wide enough to permit sound welding and if the opening be backed up by a base-metal part or by sheet metal on the side further from the welding operator. For single and double-V joints, the angle of bevel of each part shall be not less than 30°, and for single and double-bevel joints not less than 45 degrees. The *clearance* between parts *at the root* of a beveled joint shall be from $\frac{1}{16}$ to $\frac{3}{16}$ in., except that when welding joints with heavily coated electrode, the clearance at the root may be made equal to the diameter of the electrode. Butt welds required to be beveled shall also be reinforced by making the thickness greater than the throat dimension defined previously; the exposed face of a single-V or single-bevel weld shall be reinforced at least 20 per cent of the throat dimension and each exposed face of a double-V or double-bevel weld shall be reinforced at least 12½ per cent of the throat dimension. A butt weld intended to transmit tensile stress shall be made only when one of the parts to be joined is free or is flexible enough to permit contraction of the weld metal.

130. Fillet Welds. The *length* of any fillet weld shall be made not less than 4 times the weld size or else the size of the weld shall be considered not to exceed ¼ of the length for purposes of calculating strength under this Code.

131. Welds in Slots or Holes. When welding inside a slot or hole in a plate or other part, in order to join same to an underlying part, fillet welding may be used along the wall or walls of the slot or hole, but the latter shall not be filled with weld metal or partially filled in such manner as to form a direct weld-metal connection between opposite walls, except that fillet welds along opposite walls may overlap each other for a distance of ¼ of their size. No slot or hole shall be less in width or diameter than 1½ times its depth.

Workmanship

132. Mill Scale. Surfaces to be welded shall be free from loose mill scale, rust, paint or other foreign matter, except that a thin coat of linseed oil, if present, need not be removed. This clause applies not only to new structures but also to cases where new steel is to be welded to members of existing structures.

133. Clamps. Component parts of built-up members shall be firmly secured together, by adequate clamps or other means, in preparation for assembly welding.

134. Separation. Where parts to be joined by a fillet weld are separated more than $\frac{1}{16}$ in., the excess above $\frac{1}{16}$ in. in amount of separation shall be added to the weld size required by the design.

135. Painting. Structural steel shall not be painted on any areas where shop or field welding is later to be performed, except that a coat of linseed oil without pigment may be used for temporary protection. However, this clause shall not prohibit welding of steel which has been painted, provided that the paint be first completely removed from the areas to be welded.

Erection

136. Bolting. In erecting a welded structure, adequate means shall be employed for temporarily fastening the members together and bracing the framework until the

joints are welded; such means shall consist of erection bolts or other positive devices imparting sufficient strength and stiffness to resist all temporary weights and lateral forces, including wind. Owing to the small number of bolts ordinarily employed for joints which are to be welded, the temporary support of heavy girders carrying columns should receive special attention.

137. Light Construction. In *tier-building erection*, members shall not be erected more than 4 tiers or more than 2 column lengths above any column connections yet unwelded. Light structures under 30 ft. high may be erected without the use of temporary joint fastenings, provided that the members be welded together sufficiently for temporary security at the time they are erected.

Design of New Bridges

138. General. Full and complete information regarding location, type, size and extent of all welds in accordance with the specifications, shall be clearly shown on the plans. The plans shall clearly distinguish between shop and field welds. The specifications given here are intended to provide against *fatigue failure.*

139. Maximum and Minimum Stresses. Maximum and minimum stresses (axial stress, bending moment, shear, etc., respectively) shall be computed in accordance with the requirements of the applicable general specifications. These will hereinafter be referred to as " Max." and " Min.," respectively. *Max.* refers to the numerically greater stress, of whichever sign, and is to be used in the design formulas of this Section with a plus sign. *Min.* refers to the numerically smaller stress; if it be of the same sign as *Max.*, it shall be used with a plus sign, and if it be of opposite sign to *Max.*, it shall be used with a minus sign, in the design formulas.

140. Required Base Material. Case 1. For beams, girders and axially stressed members not spliced or end connected by fillet welding, if *Max.* and *Min.* have the same sign (no reversal), the required base material shall be calculated by the unit stresses prescribed in the applicable general specifications.

Case 2. For beams and girders subject to reversal, for axially stressed members subject to reversal, and for axially stressed members spliced or end connected by fillet welding regardless of reversal, the required base material shall be calculated in accordance with the formulas of the first table in which —

Max. and *Min.* have the respective values and signs stated in Spec. 139.

Mom. = calculated external bending moment.

I/c = required section modulus for beams and girders (gross areas in the absence of rivet or bolt holes).

A = required cross-sectional area (gross section for tension members in the absence of rivet or bolt holes).

141. Required Weld Areas. The required weld areas shall be calculated in accordance with Formulas 7–9.

For butt welds, A = minimum cross-sectional area of weld transverse to the line of action of the stress.

For fillet welds, A = length of fillet times throat dimension.

Base Material

Formula No.	Type of Member	Type of Stress	Required Area[a]
1	Beams and girders	Bending (with reversal)	$\frac{I}{c} = \frac{\text{Max. (Mom.)} - \frac{1}{2}\text{Min. (Mom.)}}{18{,}000}$ Footnote[b]
2	Beams and girders	Shear (with reversal)	$A = \frac{\text{Max.} - \frac{1}{2}\text{Min.}}{13{,}500}$
3	Axially stressed, not connected by fillet welding	Tension (Max.) with reversal to compression (Min.)	$A = \frac{\text{Max.} - \frac{1}{2}\text{Min.}}{18{,}000}$ but not less than $\frac{\text{Min.} + \frac{1}{2}\text{Min.}}{p}$ Footnote[c]
4	Axially stressed, not connected by fillet welding	Compression (Max.) with reversal to tension (Min.)	$A = \frac{\text{Max.} - \frac{1}{2}\text{Min.}}{p}$ Footnote[c]
5	Axially stressed, connected by fillet welding	Tension (Max.). Either sign for Min.	$A = \frac{\text{Max.} - \frac{1}{2}\text{Min.}}{12{,}000}$ but not less than $\frac{\text{Max.}}{15{,}000}$ and not less than $\frac{\text{Min.} + \frac{1}{2}\text{Min.}}{p}$
6	Axially stressed, connected by fillet welding	Compression (Max.). Either sign for Min.	$A = \frac{\text{Max.} - \frac{1}{2}\text{Min.}}{12{,}000}$ but not less than $\frac{\text{Max.} - \frac{1}{2}\text{Min.}}{p}$ and not less than $\frac{\text{Max.}}{p}$

[a] The values herein stated assume 33,000 lb. per sq. in. to be the *minimum allowable yield point of base material*, and assume 18,000 lb. per. sq in. to be the basic allowable tension under the applicable general specifications. Otherwise all denominators to be modified proportionately, except that those of Formulas 5 and 6 are not to be increased.

[b] The denominator 18,000 is to be reduced, for the compression flange, as directed in the applicable general specifications. Thus for *AASHO* Hwy. Br. Spec. 1935, and for *AREA* Rwy. Br. Spec. 1935, use $18{,}000 - 5(L/b)^2$.

[c] p = allowable compressive unit stress from the applicable general specifications. Thus for *AASHO* Hwy. Br. Spec. 1935, and for *AREA* Rwy. Br. Spec. 1935, $p = 15{,}000 - \frac{1}{4}(L/r)^2$.

WELD AREAS

FORMULA No.	TYPE OF WELD	TYPE OF STRESS	REQUIRED WELD AREA
7[a]	Butt	Tension or Compression	$A = \dfrac{\text{Max.} - \frac{1}{2}\text{ Min.}}{13{,}500}$ but not less than $\dfrac{\text{Max.}}{16{,}000}$
8[a]	Butt	Shear	$A = \dfrac{\text{Max.} - \frac{1}{2}\text{ Min.}}{9000}$ but not less than $\dfrac{\text{Max.}}{12{,}000}$
9	Fillet	Tension, Compression or Shear	$A = \dfrac{\text{Max.} - \frac{1}{2}\text{ Min.}}{7200}$ but not less than $\dfrac{\text{Max.}}{9600}$

[a] Single-vee butt welds, except when under compression only, shall have the allowable stress reduced 15 per cent.

142. Design of Butt Welds. Formulas 7 and 8 for butt welds assume that there are no abrupt changes of stress distribution on the opposite sides of the weld, that the reinforcement is very moderate and merges smoothly into the base metal, and that the parts are so arranged and held at the time of welding that the weld metal may contract with practically entire freedom.

143. Design of Fillet Welds. Fillet welds placed transversely to the direction of stress shall be calculated as carrying shear.

By special design of fillet welded splices or end connections, proved by tests to alleviate or remove the susceptibility to *fatigue failure* of the base material adjacent to the ends of the fillets, the area of base material required by Formulas 5 and 6 may be reduced, the lower limit being the area required by Spec. 140, CASE 1, or by Formulas 3 and 4, whichever may apply.

AMERICAN RAILWAY ENGINEERING ASSOCIATION

218. Abbreviated* *AREA* Specifications for Steel Railway Bridges

General Features of Design

144. Materials. Structures shall be made wholly of structural steel except where otherwise specified. Rivet steel shall be used for rivets only. Forged steel shall be used for large pins, large expansion rollers, and other parts if specified by the Engineer. Preferably, cast steel shall be used for shoes, rockers, and bearings. Cast iron may be used only where specifically authorized by the Engineer. The accompanying table lists standard physical properties.

145. Spacing of Trusses, Girders, and Stringers. The distance between centers of trusses or girders shall be sufficient to prevent overturning by the specified lateral forces. In no case shall it be less than $\frac{1}{20}$ of the span for through spans, nor $\frac{1}{15}$ of the span for deck spans.

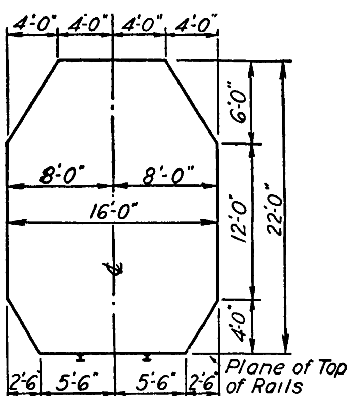


Fig. 245. Single Track — Clearance Diagram.

146. Depth Ratios. The depth of *trusses* preferably shall be not less than $\frac{1}{10}$ of the span. The depth of *plate girders* preferably shall be not less than $\frac{1}{12}$ of the span. The depth of *rolled beams* used as girders and the depth of *solid floors* preferably shall be not less than $\frac{1}{15}$ of the span.

147. Clearances. The clearances on straight track shall not be less than those shown in Fig. 245. On curved track the clearance shall be increased to allow for the overhanging and the tilting of a car 85 ft. long, 60 ft. between centers of trucks, and 14 ft. high.

* For complete design specifications consult the 1936 *AREA* Code.

Materials — Standard Physical Properties[a]

Designation of Material	Tensile Strength (Lb. per Sq. In.)	Yield Point		Min. Elongation in 8 in. (Per Cent)	Min. Elongation in 2 in. (Per Cent)	Min. Reduction of Area. (Per Cent)
		Relative	Min.			
Structural carbon steel	60,000–72,000	½ tens. str.	33,000	1,500,000 ÷ tens. str.	22%	
Rivet steel	52,000–62,000	½ tens. str.	28,000	1,500,000 ÷ tens. str.		
Carbon steel eye bars	60,000		33,000	full size test	12% in 18 ft.	
Structural silicon steel	80,000–95,000		45,000	1,500,000 ÷ tens. str.	1,600,000 ÷ tens. str.	30
Structural nickel steel	90,000–115,000	½ tens. str.	55,000	1,600,000 ÷ tens. str.	1,700,000 ÷ tens. str.	30
Nickel steel eye bars	85,000–100,000		48,000	full size test	10% in 18 ft.	30
Steel forgings 12 in. max.	60,000	½ tens. str.	33,000		1,700,000 ÷ tens. str. or 25%	2,700,000 ÷ tens. str. or 38%
Steel forgings 12 in. min.	"	"	"		1,600,000 ÷ tens. str. or 24%	2,520,000 ÷ tens. str. or 36%
Steel castings	66,000		33,000		22%	33%

[a] For limitations on thicknesses and testing speeds, see *AREA* and *ASTM* specifications.

Loads and Stresses

148. Loads and Forces. Bridges shall be proportioned for the following loads and forces.

(*a*) Dead Load.
(*b*) Live Load.
(*c*) Impact.
(*d*) Centrifugal force.
(*e*) Other lateral forces.
(*f*) Longitudinal force.

Stresses from each of these loads and forces shall be shown separately on the stress sheet.

149. Dead Load. In estimating the weight for the purpose of computing dead load stresses, the following unit weights shall be used:

	Pounds per Cubic Foot
Steel	490
Concrete	150
Sand, gravel, and ballast	120
Asphalt-mastic and bituminous macadam	150
Granite	170
Paving bricks	150
Timber	60

The track rails, inside guard rails, and fastenings shall be assumed to weigh 200 lb. per lineal ft. for each track.

150. Live Load. The recommended live load for each track is the Cooper's *E*-72 load shown in Fig. 246.

The Engineer shall specify the live load to be used, such load to be proportional to the recommended load, with the same axle spacing.

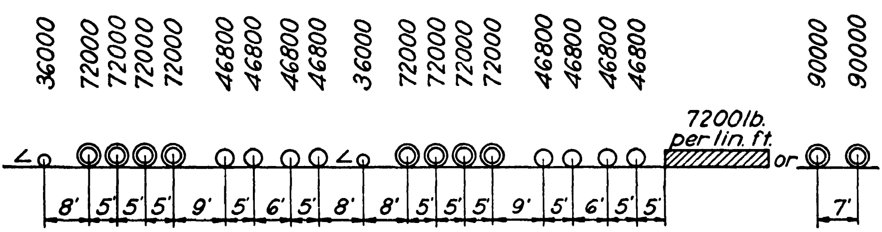


Fig. 246. Cooper's *E*-72 Loading.

151. Distribution of Live Load. (*a*) Floor beams that are spaced close enough to carry the track loads without stringers shall be designed for a proportion of the axle load equal to the ratio of the floor-beam spacing to the axle spacing. The floor beams shall be connected by solid-web diaphragms at intervals not exceeding 12 times the flange width, with at least one diaphragm for each track.

(*b*) For ballasted floor bridges, the lateral distribution of the live load on each track shall be taken as uniform over a width of 10 ft.

152. Impact. To the maximum computed static live load stresses, there shall be added the impact, consisting of:

(*a*) *The Lurching Effect.*

The lurching effect is due to the rolling of the live load from side to side. It shall be taken as increasing the static live load on one rail by 20 per cent, with an equal decrease on the other rail. These loads shall be distributed to the supporting members.

(*b*) *The Direct Vertical Effect.*

With steam locomotives (hammer blow, track irregularities, and car impact) a percentage of the static live load stress equal to:

For L less than 100 ft. .... $100 - 0.60L$

For L 100 ft. or more .... $\dfrac{1800}{L - 40} + 10$

With electric locomotives (track irregularities and car impact),

a percentage of the static live load stress equal to .... $\dfrac{360}{L} + 12.5$

L = length, in feet, center to center of supports for stringers, longitudinal girders, and trusses (chords and main members);

or, L = length of floor beams or transverse girders, in feet, for floor beams, floor-beam hangers, subdiagonals of trusses, transverse girders, and supports for transverse girders.

153. Multiple Tracks. For members receiving load from more than one track, the impact percentage shall be applied to the static live load on the number of tracks shown below.

(*a*) *Load from Two Tracks.*

For L less than 175 ft., full impact on two tracks.

For L from 175 ft. to 225 ft., full impact on one track and a percentage of full impact on the other as given by the formula, $450 - 2L$.

For L greater than 225 ft., full impact on one track and none on the other.

(*b*) *Load from More Than Two Tracks.*

For all values of L, full impact on any two tracks.

154. Centrifugal Force. On curves, the centrifugal force in percentage of the live load is $0.00117S^2D$.

S = speed in miles per hour.

D = degree of curve.

It shall be assumed to act 6 ft. above the rail and shall be taken without impact.

155. Wind on Loaded Bridge. The wind force shall be considered as a *moving load* acting in any horizontal direction. On the train it shall be taken at 300 lb. per linear ft. on one track, applied 8 ft. *above the top of rail.* On the bridge it shall be taken at 30 lb. per sq. ft. of the following surfaces:

(*a*) For girder spans, 1½ times the vertical projection of the span.

(*b*) For truss spans, the vertical projection of the span plus any portion of the leeward trusses not shielded by the floor system.

(*c*) For viaduct towers and bents, the vertical projections of the columns and tower bracing.

The wind force on girder spans and truss spans, however, shall not be taken at less than 200 lb. per linear ft. for the loaded chord or flange, and 150 lb. per linear ft. for the unloaded chord or flange.

156. Wind on Unloaded Bridge. If a wind force on the unloaded bridge of 50 lb. per sq. ft. of the surface defined in Spec. 155 combined with the dead load produces greater stresses than those produced by the wind forces specified in Spec. 155 combined with the stresses from dead load, live load, impact, and centrifugal force, the members should be designed for the greater stresses.

157. Stability of Spans and Towers. In calculating the stability of spans and towers, the live load on one track shall be 1200 lb. per lineal ft., taken without impact. On multiple track bridges, this live load shall be on the *leeward track.*

The lateral forces shall be those specified in Spec. 154 to Spec. 156.

158. Sway of Locomotives. The lateral force to provide for the effect of the sway of locomotives (in addition to the other lateral forces specified) shall be a moving concentrated load of 20,000 lb. applied at the *top of rail,* in either horizontal direction, at any point of the span. The resulting vertical forces shall be disregarded.

159. Bracing between Compression Members. The lateral bracing of the compression chords or flanges of trusses and deck girders and between the posts of viaduct towers shall be proportioned for a transverse shear in any panel equal to 2½ per cent of the total axial stress in both members in that panel, in addition to the shear from the specified lateral forces.

160. Longitudinal Force. The longitudinal force resulting from the starting and stopping of trains shall be the larger of:

(*a*) *Force Due to Braking.*

15 per cent of the live load without impact.

(*b*) *Force Due to Traction.*

25 per cent of the weight on the driving wheels, without impact.

The longitudinal force shall be taken on one track only and shall be assumed to act *6 ft. above the top of the rail.*

For bridges where, by reason of continuity of members or frictional resistance, much of the longitudinal force will be carried directly to the abutments (such as ballasted deck bridges of only 3 or 4 spans), only ½ of the longitudinal force shall be considered effective.

161. Reversal of Stress. Members subject to reversal of stress (whether axial, bending, or shearing) during the passage of the live load shall be proportioned as follows:

Determine the maximum stress of one sign and the maximum stress of the opposite sign and increase each by 50 per cent of the smaller. Proportion the member so that it will be capable of resisting either stress so increased. The connections shall be proportioned for the sum of the maximum stresses.

162. Combined Stresses. (*a*) Members subject to both axial and bending stresses shall be so proportioned that the combined fiber stresses will not exceed the allowed axial stress. In members continuous over panel points, only ¾ of the bending stress computed as for simple beams shall be added to the axial stress.

(*b*) Members subject to stresses produced by a combination of dead load, live load, impact, and centrifugal force, with other lateral forces and with longitudinal force, or with bending due to such forces, may be proportioned for unit stresses 25 per cent greater than those specified in Spec. 164; but the section of the member shall not be less than that required for the combination of dead load, live load, impact, and centrifugal force.

163. Secondary Stresses. The design and details shall be such that secondary stresses will be as small as practicable. Secondary stresses due to truss distortion or floorbeam deflection usually need not be considered in any member the width of which, measured parallel to the plane of distortion, is less than ⅒ of its length. If the secondary stress exceeds 4000 lb. per sq. in. for tension members and 3000 lb. per sq. in. for compression members, the excess shall be treated as a primary stress.

Allowable Unit Stresses

164. Unit Stresses. The allowable unit stresses to be used in proportioning the parts of a bridge shall be as follows:

	Pounds per Square Inch
(a) *Structural and Rivet Steel.*	
Axial tension, structural steel, net section	18,000
Tension in extreme fibers of rolled shapes, girders, and built sections, subject to bending	18,000
Axial compression, gross section:	
For stiffeners of plate girders	18,000
For compression members centrally loaded and with values of L/r not greater than 140:	
Riveted ends	$15{,}000 - \frac{1}{4}\frac{L^2}{r^2}$
Pin ends	$15{,}000 - \frac{1}{3}\frac{L^2}{r^2}$
L = length of member, in inches. r = least radius of gyration of member, in inches.	
For compression members with values of L/r greater than 140 and for compression members of known eccentricity, see Spec. 166.	
Compression in extreme fibers of rolled shapes, girders, and built sections, subject to bending (for values of L/b not greater than 40)	$18{,}000 - 5\frac{L^2}{b^2}$
L = length, in inches, of unsupported flange between lateral connections or knee braces. b = flange width, in inches.	
Diagonal tension in webs of girders and rolled beams at sections where maximum shear and bending occur simultaneously	18,000
Stress in extreme fibers of pins	27,000
Shear in plate girder webs, gross section	11,000
Shear in power driven rivets and pins	13,500
Shear in turned bolts and hand driven rivets	11,000
Bearing on pins	24,000
Bearing on power driven rivets, milled stiffeners, and other steel parts in contact	27,000
Rivets driven by pneumatically or electrically operated hammers are considered power driven.	
Bearing between rockers and rocker pins	12,000
Bearing on turned bolts and hand driven rivets	20,000
Bearing on expansion rollers and rockers, pounds per lineal inch:	
For diameters up to 25 in.	$\frac{y - 13{,}000}{20{,}000} 600d$
For diameters from 25 in. to 125 in.	$\frac{y - 13{,}000}{20{,}000} 3000\sqrt{d}$
d = diameter of roller or rocker, in inches. y = yield point in tension of the steel in the roller or the base, whichever is the lesser.	

(b) *Cast Steel.*

For cast steel shoes and pedestals, the allowable unit stresses in compression and bearing shall be the same as those for structural steel. Other allowable unit stresses shall be ¾ of those for structural steel.

(*c*) *Masonry.*

	Pounds per Square Inch
Bearing pressure:	
Granite	800
Concrete	600
Sandstone and limestone	400

(*d*) *Timber Cross Ties.*

Extreme fiber stress in bending:	
Yellow pine, dense structural grade	1,500
Douglas fir, close grain structural grade	1,400
White oak	1,200
White pine, Norway pine, and spruce	800

In computing the stresses in timber cross ties, the wheel load shall be considered as distributed over 3 ties and as applied without impact.

165. Unit Stresses for Alloy Steels. The allowable unit stresses to be used in proportioning the parts of a bridge shall be as follows:

	Pounds per Square Inch Silicon Steel	Nickel Steel
Structural Steel.		
Axial tension, structural steel, net section	24,000	30,000
Axial tension, eye bars		27,000
Tension in extreme fibers of rolled shapes, girders, and built sections, subject to bending	24,000	30,000
Axial compression, gross section:		
For stiffeners of plate girders	24,000	30,000

For compression members centrally loaded and with values of L/r not greater than 130 for silicon steel or 120 for nickel steel:

Riveted ends	$20{,}000 - 0.46\dfrac{L^2}{r^2}$	$24{,}000 - 0.66\dfrac{L^2}{r^2}$
Pin ends	$20{,}000 - 0.61\dfrac{L^2}{r^2}$	$24{,}000 - 0.90\dfrac{L^2}{r^2}$

L = length of member, in inches.

r = least radius of gyration of member, in inches.

For compression members of known eccentricity, see Spec. 166.

Compression in extreme fibers of rolled shapes, girders, and built sections, subject to bending (for values of L/b not greater than 40)

$$24{,}000 - 6.67\frac{L^2}{b^2} \qquad 30{,}000 - 8.33\frac{L^2}{b^2}$$

L = length, in inches, of unsupported flange between lateral connections or knee braces.

b = flange width, in inches.

	Silicon Steel	Nickel Steel
Diagonal tension in webs of girders and rolled beams at sections where maximum shear and bending occur simultaneously	24,000	27,000
Stress in extreme fibers of pins	36,000	44,000
Shear in plate girder webs, gross section	14,000	17,500
Shear in pins	18,000	22,000
Bearing on pins	32,000	40,000
Bearing on milled stiffeners and other steel parts in contact	36,000	44,000
Bearing between rockers and rocker pins	16,000	18,000

POUNDS PER LINEAR INCH

Bearing on expansion rollers and rockers:

For diameters up to 25 in. . . . . $\frac{y - 13{,}000}{20{,}000} 600d$

For diameters from 25 in. to 125 in. . . . . $\frac{y - 13{,}000}{20{,}000} 3000 \sqrt{d}$

d = diameter of roller or rocker, in inches.
y = yield point in tension of the steel in the roller or the base, whichever is least.

166. Formulas for Slender Compression Members. The permissible unit stresses in axial compression for centrally loaded members given by the parabolic formulas in Spec. 164 and Spec. 165 agree so closely with those obtained by the secant formula that they may be used without substantial error for slenderness ratios within the limits shown.

The corresponding secant formulas, which should be used when the slenderness ratio exceeds those limits, are:

(1) For riveted ends,
$$p = \frac{\frac{y}{f}}{1 + 0.25 \sec. \frac{0.75L}{2r}\sqrt{\frac{fp}{E}}}.$$

(2) For pin ends,
$$p = \frac{\frac{y}{f}}{1 + 0.25 \sec. \frac{0.875L}{2r}\sqrt{\frac{fp}{E}}}.$$

The formulas for compression members with *known eccentricity of loading*, for all slenderness ratios, are:

(3) For riveted ends,
$$p = \frac{\frac{y}{f}}{1 + \left(\frac{ec}{r^2} + 0.25\right) \sec. \frac{0.75L}{2r}\sqrt{\frac{fp}{E}}}.$$

(4) For pin ends,
$$p = \frac{\frac{y}{f}}{1 + \left(\frac{ec}{r^2} + 0.25\right) \sec. \frac{0.875L}{2r}\sqrt{\frac{fp}{E}}}.$$

p = allowable average compressive unit stress.
e = known eccentricity of applied load, in inches.
c = distance, in inches, from neutral axis to extreme fiber in the direction of the known eccentricity.
L = length of member, in inches.
r = least radius of gyration of member, in inches, for Formulas (1) and (2) and radius of gyration in the direction of the known eccentricity for Formulas (3) and (4).
E = modulus of elasticity = 29,400,000.
y = yield point in tension:

33,000 for structural steel.
45,000 for silicon steel.
55,000 for nickel steel.

f = factor of safety based on yield point:

1.76 for structural steel.
1.80 for silicon steel.
1.83 for nickel steel.

The coefficient 0.25 in the denominators of the formulas provides for inherent crookedness and unknown eccentricity.

When the eccentricity of the applied load is small and the radius of gyration normal to the direction of eccentricity is much less than that in the direction of eccentricity, Formulas (1) and (2), assuming the member as centrally loaded, may give smaller permissible unit stresses than Formulas (3) and (4).

167. Slenderness Ratio. The slenderness ratio (ratio of length to least radius of gyration) shall not exceed:

100 for main *compression members.*
120 for wind and *sway bracing* in compression.
140 for *single lacing.*
200 for *double lacing.*
200 for *tension members* other than eyebars.

Details of Design

168. Compression Members. Compression members shall be so designed that the main elements of the section will be connected directly to the gusset plates, pins, or other members.

Built-up sections shall be so arranged that the center of gravity will coincide as nearly as practicable with the center line of the section. Preferably the segments shall be connected by solid webs.

In members consisting of segments connected by cover plates or lacing, or segments connected by webs, which receive their full allowable unit stress, the thickness of the webs of the segments shall be not less than $\frac{1}{32}$ of the unsupported distance between the nearest lines of their connecting rivets or the roots of their rolled flanges. The thickness of the cover plates or of the webs connecting the segments shall be not less than $\frac{1}{40}$ of the unsupported distance between the nearest lines of their connecting rivets or the roots of their rolled flanges. For such members in which the stress is less than that allowable, the denominators 32 and 40 may be multiplied by the factor $\sqrt{p/f}$.

p = the allowable unit stress.
f = the unit stress in the member.

169. Outstanding Legs of Angles. The width of the outstanding legs of angles in compression, except those reinforced by plates, shall not exceed the following:

(*a*) For stringers and girders, where the ties rest on the flange, 10 times the thickness.
(*b*) For main members carrying axial stress, and for stringers and girders not included in (*a*), 12 times the thickness.
(*c*) For bracing and other secondary members, 14 times the thickness.

170. Strength of Connections. Connections shall have a strength not less than that of the member connected, based on the allowable unit stress in the member. Connections shall be made as nearly symmetrical about the axes of the members as practicable.

Effective Section

171. Net Section. The net section of a riveted tension member is the sum of the net sections of its component parts. The net section of a part is the product of the thickness of the part multiplied by its least net width.

The *net width* for any chain of holes extending progressively across the part shall be obtained by deducting from the gross width the sum of the diameters of all the holes in the chain and adding, for each gage space in the chain, the quantity,

$$\frac{s^2}{4g}.$$

s = pitch of any two successive holes in the chain.
g = gage of the same holes.

The net section of the part is obtained from that chain which gives the least net width.

For angles, the gross width shall be the sum of the widths of the legs less the thickness. The gage for holes in opposite legs shall be the sum of the gages from back of angle less the thickness.

For splice members, the thickness shall be only that part of the thickness of the member which has been developed by rivets beyond the section considered.

The diameter of the hole shall be taken as ⅛ in. greater than the nominal diameter of the rivet.

172. Effective Sections of Angles. If angles in tension are so connected that bending cannot occur in any direction, the effective section shall be the net section of the angle. If connected on one side of a gusset plate, the effective section shall be the net section of the connected leg plus ½ the section of the unconnected leg.

173. Section at Pin Holes. In pin connected riveted tension members the net section beyond the pin hole, parallel with the axis of the member, shall be not less than the net section of the member. The net section through the pin hole, transverse to the axis of the member, shall be at least 40 per cent greater than the net section of the member. The ratio of the net width (through the pin hole transverse to the axis of the member) to the thickness of the segment preferably shall not be more than 12.

Rivets

174. Grip of Rivets. If the grip of rivets carrying calculated stress exceeds 4½ times the diameter, the number of rivets shall be increased at least 1 per cent for each additional 1/16 in. of grip. If the grip exceeds 6 times the diameter, the shanks shall be specially designed to fill the holes completely when driven.

175. Pitch of Rivets. The pitch in the direction of stress for members composed of plates and shapes shall not exceed 7 times the diameter of the rivets except for web stitch rivets.

At the ends of built compression members, the pitch in the direction of stress shall not exceed 4 times the diameter of the rivets for a distance 1½ times the width of the member.

176. Stitch Rivets. Where two or more web plates are in contact, there shall be stitch rivets to make them act in unison. In compression members, the pitch of such rivets in the direction of stress shall not exceed 12 times the thickness of the thinnest outside plate connected, and the gage 24 times that thickness. In tension members, the maximum pitch or gage of such rivets shall be 24 times that thickness. In tension members composed of *2 angles* in contact, the pitch of the stitch rivets shall not exceed 12 in.

177. Minimum Spacing of Rivets. The distance between centers of rivets shall be not less than 3 times the diameter of the rivets.

178. Edge Distance of Rivets. The distance from the center of a rivet to a *sheared edge* shall not be less than 1¾ times the diameter, nor to a *rolled or planed edge* less than

1½ times the diameter, except in flanges of beams and channels, where the minimum distance may be 1¼ times the diameter.

The distance from the center of a rivet to the edge of a plate shall not exceed 8 times the thickness of the plate.

179. Sizes of Rivets in Angles. The diameter of the rivets in angles whose size is determined by calculated stress shall not exceed ¼ of the width of the leg in which they are driven. In angles whose size is not so determined, 1-in. rivets may be used in 3½-in. legs, ⅞-in. rivets in 3-in. legs, and ¾-in. rivets in 2½-in. legs.

180. Compression Splices. Members subject to compression only, if faced for bearing, shall be spliced on 4 sides sufficiently to hold the abutting parts true to place. The splice shall be as near a panel point as practicable and shall be designed to transmit at least ½ of the stress through the splice material. Members not faced for bearing shall be fully spliced.

181. Extra Rivets. If splice plates are *not in direct contact* with the parts which they connect, there shall be rivets on each side of the joint in excess of the number required in the case of direct contact, to the extent of 2 extra lines for each intervening plate.

If rivets carrying stress pass through *fillers*, the fillers shall be extended beyond the connected member and the extension secured by enough additional rivets to develop the value of the filler.

Riveted Connections

182. Stay Plates. On the open sides of *compression members*, the segments shall be connected by lacing bars and there shall be stay plates as near each end as practicable. There shall be stay plates at intermediate points where the lacing is interrupted. In main members the length of the *end stay plates* shall be not less than 1¼ times the distance between the lines of rivets connecting them to the outer flanges. The length of *intermediate stay plates* shall be not less than ¾ of that distance.

The segments of *tension members* composed of shapes shall be stayed together. The length of the stay plates shall be not less than ⅔ of the lengths specified for stay plates on compression members. They shall be connected to each segment by at least 3 rivets.

The *thickness of stay plates* shall be not less than $\frac{1}{50}$ of the distance between the lines of rivets connecting them to the outer flanges for main members, or $\frac{1}{60}$ of that distance for bracing members.

183. Lacing. Lacing bars of compression members shall be so spaced that the slenderness ratio of the portion of the flange included between the lacing bar connections will be not more than 40 nor more than ⅔ of the slenderness ratio of the member.

In compression members, the *shearing stress* normal to the member in the plane of the lacing shall be that obtained by the following formula:

$$V = \frac{P}{100}\left[\frac{100}{\frac{L}{r} + 10} + \frac{\frac{L}{r}}{100}\right]$$

V = normal shearing stress.
P = allowable compressive axial load on member.
L = length of member, in inches.
r = radius of gyration of section about the axis perpendicular to plane of lacing, in inches.

To the shear so determined shall be added any shear due to the weight of the member or to other forces, and the lacing shall be proportioned for the combined shear.

The shear shall be considered as divided equally among all parallel planes in which there are shear resisting elements, whether continuous plates or lacing. The section of the lacing bars shall be determined by the formula for axial compression in which L is taken as the distance along the bar between its connections to the main segments for single lacing, and as 70 per cent of that distance for double lacing.

If the distance across the member between rivet lines in the flanges is more than 15 in. and a bar with a single rivet in the connection is used, the lacing shall be double and riveted at the intersections.

The angle between the lacing bars and the axis of the member shall be approximately 45° for double lacing and 60° for single lacing.

Lacing bars may be shapes or flat bars. For main members the *minimum thickness* of flat bars shall be $1/40$ of the distance along the bar between its connections for single lacing, and $1/60$ for double lacing. For bracing members, the limits shall be $1/50$ for single lacing and $1/75$ for double lacing.

The diameter of the rivets in lacing bars shall not exceed $1/3$ of the width of the bar. There shall be at least 2 rivets in each end of lacing bars riveted to flanges more than 5 in. in width.

Pin Connections

184. Reinforcing Plates at Pin Holes. Where necessary for the required section or bearing area, the section at pin holes shall be increased on each segment by plates so arranged as to reduce the eccentricity of the segment to a minimum. One plate on each side shall be as wide as the outstanding flanges will allow. At least one full width plate on each segment shall extend to the far edge of the stay plate and the others not less than 6 in. beyond the near edge. These plates shall be connected by enough rivets to transmit the bearing pressure and so arranged as to distribute it uniformly over the full section.

185. Eye Bars. The *thickness of eye bars* shall be not less than 1 in. nor more than 2 in. The section of the head through the center of the pin hole shall exceed that of the body of the bar by at least 35 per cent. The form of the head shall be submitted to the Engineer for approval before the bars are made. The *diameter of the pin* shall be not less than $8/10$ of the width of the widest bar attached.

186. Eye-Bar Packing. The eye bars of a set shall be symmetrical about the central plane of the truss and as nearly parallel as practicable. The *inclination* of any bar to the plane of the truss shall not exceed $1/16$ in. to the foot. The bars shall be packed close, held against lateral movement, and so arranged that those in the same panel will not be in contact.

187. Pins. In pins more than 9 in. in diameter, there shall be a *hole* not less than 2 in. in diameter bored longitudinally on the center line.

The turned bodies of pins shall be long enough to extend at the ends $1/4$ in. beyond the outside faces of the parts connected. The pins shall be secured by chambered nuts or by solid nuts and washers. If the pins are bored, through rods with cap washers may be used. The screw ends shall be long enough to allow burring the threads.

Pin connected members shall be secured in such a way as to limit lateral movement on the pin. *Filler rings* shall be made of metal not less than $1/2$ in. thick.

Plate Girders

188. Proportioning Plate Girders. Plate girders, I-beams, and other members subject to bending that produces tension on one face, shall be proportioned by the moment-of-inertia method. The neutral axis shall be taken along the center of gravity of the gross section. The tensile stress shall be computed from the moment of inertia of the

entire *net section* and the compressive stress from the moment of inertia of the entire *gross section.*

189. Flange Section. In order to offset the effects of corrosion and the possible crookedness of the compression flange of a plate girder or a rolled beam, the gross section of the compression flange shall not be less than the gross section of the tension flange.

Flanges of plate girders preferably shall be made without cover plates or side plates unless angles of greater section than 6 × 6 × ⅞ in. would otherwise be required.

Cover plates shall be equal in thickness, or shall diminish in thickness from the flange angles outward. No plate shall be thicker than the flange angles. When cover plates are used, at least one plate on each flange shall extend the full length of the girder. Other flange plates shall extend far enough to allow 2 rows of rivets at each end of the plate, beyond the theoretical end, and there shall be enough rivets to develop the plate between its end and the theoretical end of the next plate outside.

In through bridges, there shall be end and corner cover plates.

190. Flange Rivets. The flanges of plate girders shall be connected to the web with enough rivets to transmit to the flange section the horizontal shear at any point together with any load that is applied directly on the flange. Where the ties rest on the flange, one wheel load shall be assumed to be distributed over 3 ft.

191. Flange Splices. Flange members that are spliced shall be covered by extra material equal in section to the member spliced. There shall be enough rivets on each side of the splice to transmit to the splice material the stress value of the part cut.

Flange angles shall be spliced with angles. No two members shall be spliced at the same cross-section.

192. Web Splices. Splices in the webs of plate girders shall be designed for the *full strength* of the web in both shear and bending.

193. Thickness of Web Plates. The thickness of the webs of plate girders shall be not less than $\frac{1}{170}$ of the *clear distance* between the flanges, except that if the extreme fiber stress in the compression flange is less than that allowable, the denominator 170 may be multiplied by the factor $\sqrt{p/f}$.

p = the allowable extreme fiber stress.
f = the extreme fiber stress in the compression flange.

194. Stiffeners at Points of Bearing. Stiffeners shall be placed at end bearings of plate girders and at points of bearing of concentrated loads. They shall extend as nearly as practicable to the edges of the flange angles and shall be connected to the web by enough rivets to transmit the stress. Such stiffeners shall not be crimped. Only that part of the stiffener cross-section which lies without the fillet of the flange angle shall be considered effective in bearing.

195. Intermediate Stiffeners. If the depth of the web between the flanges or side plates of a plate girder exceeds 60 times its thickness, it shall be stiffened by pairs of angles riveted to the web. The clear distance between stiffeners shall not exceed 72 in. nor that given by the formula:

$$d = \frac{255{,}000t}{S}\sqrt[3]{\frac{St}{a}}.$$

d = clear distance between stiffeners, in inches.
t = thickness of web, in inches.
a = clear depth of web between flanges or side plates, in inches.
S = unit shearing stress, gross section, in web at point considered.

The width of the outstanding leg of each angle shall be not more than 16 times its thickness and not less than 2 in. plus $\frac{1}{30}$ of the depth of the girder.

ENGINEERING DRAWINGS AND REPORTS

219. Instructions for Student Draftsmen. The instructions given below have been used in classes and drafting rooms. To be of value they must be definite. However, changes may be necessary to make them fit local conditions.

REPORTS

(a) All design calculations shall be neatly written on paper of good quality, size $8\frac{1}{2} \times 11$ in. The use of paper with faintly ruled lines, background cross-hatching, is recommended.

(b) Ruled margins shall be placed at least at the top and left hand side of the sheet and all calculations shall be kept inside of these margins.

(c) Each part of the report shall have a title such as *Dead Load Stresses* which shall be lettered and underlined to make it stand out. The use of lettered subheads is also desirable.

(d) Free hand sketches shall not be used for illustrative figures in an engineering report. See to it that vertical lines used in figures are reasonably parallel to the sides of the paper, etc.

(e) Whenever the report involves the calculation of stresses, always place the final stresses on a picture of the structure. Show all of your calculations; do not summarize.

(f) Bind your design sheets into a standard report cover using at least two staples. Do not put the staples through the top cover. Do not tie the sheets into the cover with string.

(g) Poor form, lack of neatness, or summarized calculations are sufficient reasons for rejection of a report. In revising a report always rewrite those sheets that have correction marks.

(h) Poor English, inconsistent use of punctuation, abbreviations, or symbols, and careless expression are as obvious to your superior and as much criticized as inaccurate calculations.

DRAWINGS

(a) *Kinds of Drafting Work.* Two types of drawings are in common use. In some cases you will work directly on vellum paper. Blue prints can be made from a pencil drawing on vellum provided that an F or H pencil is used. Some draftsmen prefer still softer pencils. It is necessary for you to use great care in protecting the vellum from dirt which ruins the print. Triangles and scales should be washed frequently. The other type of drawing is made on yellow detail paper and is traced. A somewhat harder pencil, 2H, may be used and less care need be taken with lettering. Notes are frequently filled in directly on the tracing after being written in long hand on the pencil drawing. However, unless you are a very good draftsman, these practices are not advisable.

(b) *Equipment.* The usual equipment required is as follows:

1 T square (about 36 in. long).
2 Triangles (45° and 60°, medium size).
2 Scales (Architects' and Engineers').
1 Ruling pen (must be able to make fine lines).
1 Ink and pencil compass.
1 Small ink compass.

(c) *Sheet Sizes.* All major drawings shall be made on 24-in. by 36-in. sheets. Two margins are used. The first shall be a ½-in. margin all around and the second shall be a ½-in. margin at top, bottom, and right hand side and a 1-in. margin at the left. The inside part of the sheet then becomes 22 in. by 33½ in. Where possible, it is desirable to keep all parts of the drawing at least 1 in. away from the margins. Details of structural members and connections may be made on a half-size sheet, 18 in. by 24 in.

(d) *Lettering.* Lettering shall be performed carefully with the use of faint guide lines. Preferably, main titles and sub-titles shall be vertical letters and the remainder inclined. Height of letters shall be as follows: *Main titles* 0.3 in. for main capitals and 0.20 in. for small capitals; *subtitles* 0.25 in. for capitals and numerals and 0.15 in. for lower case letters; *elsewhere* 0.15 in. for capitals and numerals and 0.10 in. for lower case letters. Use round letters of full size as simply made as possible.

(e) *Titles.* The title shall preferably be placed in the lower right hand corner of the sheet and shall consist of three parts: First, a main title, that is, DETAIL DRAWING — THROUGH PRATT HIGHWAY BRIDGE, of largest sized capital letters. Second, a reference to the place where the work is being done, that is, STATE BRIDGE DEPARTMENT, SANTA FE DISTRICT, in the next smaller size of capital letters. Third, a set of statements made in capitals and lower case letters giving the drawing number, the date, the name of the designer, and leaving a blank space for the approval or rejection mark of the checker.

(f) *Lines.* Only two weights of lines need be used on a drawing, a medium or heavy line for the outline of members, plates, etc., and a light line for dimensions. The dimension line shall be unbroken and the dimension in numerals shall be placed above the line. Dimension lines always occur in pairs, that is, a partial set of dimensions tied together by an overall dimension. The overall dimension shall never be omitted, and each partial dimension should be given. Both partial and overall dimensions should be complete from one main intersection point to the succeeding intersection point, as, for example, from one joint to the next joint of a truss. Succeeding dimension lines must not be offset one from the other. For instance, all partial dimension lines for the panels of the bottom chord of a bridge truss should form one straight line. Also, it is desirable to keep all partial dimension lines for the entire truss as nearly as possible at a fixed distance from the nearest edges of the members. The distance between partial and overall dimension lines shall be as nearly constant as is feasible.

Broken lines shall be reserved for indicating members that are hidden behind other members, for down-turned angle legs, and for beam and channel webs as on plan views of floor systems.

Center lines and gage lines are indicated by a light dot-and-dash line.

(g) *Rivets and Welds.* Use care in making rivet heads and open holes. For pencil drawings, obtain correct dimensions from a handbook and make rivet heads and open holes to scale. Rivets and open holes when made in ink look better if they are shown to about three fourths of the scale of the drawing. Different types of rivets and welds are represented by conventional signs which are reproduced elsewhere.

(h) *Laying Out Truss Members.* In laying out truss members, arrange to have the assumed center lines meet at a point. To take the place of the true center line of a symmetrical section, the gage line of an angle is used, and, for an unsymmetrical built-up section, the center of gravity is placed along the center line wherever possible. If an angle has two gage lines, the one nearer the back of the angle is placed on the center line.

(i) *Marking Dimensions and Member Sizes.* All dimensions shall be given in feet and inches to the nearest $\frac{1}{16}$ in., that is, 6′–8$\frac{7}{16}$″ or 1′–3$\frac{1}{2}$″. Dimensions shall be placed above the dimension line.

Mark all member sizes and lengths above the member wherever possible For angles, state *first* the angle leg that appears on the drawing.

Examples	1 angle	3 × 3 × $\frac{3}{8}$″ × 0′–7$\frac{1}{2}$″
	2 angles	2$\frac{1}{2}$ × 2 × $\frac{1}{4}$″ × 12′–3″
	1 channel	8″ at 16$\frac{1}{4}$# × 12′–11$\frac{1}{4}$″
	1 plate	18$\frac{1}{2}$″ × $\frac{3}{8}$″ × 3′–3$\frac{1}{2}$″
	1 filler	3″ × $\frac{3}{8}$″ × 0′–9″

The common signs are used to represent angles and channels, and the word "plate" is abbreviated to Pl. Note the sequence in which leg size, thickness, and length of an angle are given. Also, note that the width of a plate is first given in inches, then the thickness is given in inches, and, finally, the length is always to be given in feet and inches. Such details as these must be observed carefully.

(j) *Detailing Plates.* The plate size listed for irregularly shaped plates is that of the smallest rectangle from which the plate can be cut. It is only necessary to give such plate sizes to half inches. The size given should overrun, not underrun, since it may be used in making weight estimates and for ordering material.

In laying out plates, use as few sides as possible. It is customary to have the sides of the gusset plates meet on the center lines or on the gage lines of the members. For large members, there is a saving in cutting the plate perpendicularly across the width of the member. Re-entrant cuts in plates must be avoided because they can only be flame-cut economically. Plates are ordinarily cut in the shears.

Sufficient dimensions must be given for each plate detail so that it can be laid out and the rivet holes marked. When making a detail drawing, we should give the major plate dimensions and the distance out along the gage line from the joint intersection to each rivet hole. The shop detailer will take care of edge distances, but the designer must have given these requirements proper consideration because the plates will be ordered from the mill to fit the sizes that he specifies. Shop details for plates include sufficient dimensions to locate every cut and every rivet hole.

(k) *Detailing Members and Joints.* Care shall be taken by the designer and the detailer to be sure that all rivets shown on the drawing can be driven and

that all members can be erected readily. The clearances required are given in all steel handbooks. Where flattened heads or countersunk rivets are required, they shall be marked properly. Field rivets and field welds are to be reduced to a minimum. Overhead welds are to be avoided wherever possible.

Built-up members shall be detailed as fully on the drawing as space will allow. If the joints are shown to a larger scale than the dimensions of the truss (joints are frequently detailed to a ¾-in. scale while the truss may be shown to a ⅜-in. scale), it is necessary to break each member. Sufficient information must nevertheless be given so that the shop detailer can complete the details of the members. It is necessary to give the length, size, and thickness of each angle, the length, size, and weight of each channel and rolled beam, and the width, thickness, and length of each plate, including tie plates, batten plates, cover plates, and connection plates. The number and spacing of each type of plate must be shown. At least one plate of each type must be shown on each member so that the number, size, and spacing of rivets may be given. Where lacing bars are used, the size of the bar, the rivet spacing, and the starting point must be shown as well as the number of bars required for each member. End plates are used at the ends of all laced members and these plates should be detailed completely. By adjusting the lengths of end plates, the lacing may be made exactly 60° or 45°, if this seems desirable, or rivets may be spaced to even fractions of an inch by the same device.

Where exact bearing of the ends of compression members is desired, as at the hip joint of a truss, the members shall be marked 'MILL TO BEAR' or 'FINISH.' The exact angle of bevel in degrees and minutes shall be given.

Detailing of members and joints is not an absolutely fixed process. Considerable variation in ideas of detailing exists.

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